Philosophy 303
Symbolic Logic
Winter, 2009

# Example problems for homework assignment 1

1. (Chapter 2, Exercise 2, Parts a) and d))

a) If $m < n$ then $n > m$

d) If John gave some money to $x$ and Mary gave some money to $y$ (with $x \neq y$) then $x$ may not have wanted $y$ to know that Andrea gave $x$ additional money.

2. (Chapter 2, Exercise 3, Parts a), b), and h)) Describe each of the following sets, using the notation that explicitly lists out the set's members.
(a) $\{1, 2, 3\} \cup \{3, 4, 5\}$
(b) $\{1, 2, 3\} \cap \{3, 4, 5\}$
(h) $(\{1, 2, 3\} \cup \{4\}) - \{3\}$

(a) $\{1, 2, 3, 4, 5\}$

(b) $\{3\}$

(h) $\{1, 2, 4\}$

3. Chapter 2 exercise 5 d)

   Prove that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

   This is a set identity, and as so often it is easiest to "prove both directions".

   First, we prove $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$

   Suppose $W \in X \cap (Y \cup Z)$. Then $W \in X$ *and* $W \in (Y \cup Z)$

   Since $W \in (Y \cup Z)$ we have that $W \in Y$ or $W \in Z$

   a) Say $W \in Y$ Then $W \in X$ and $W \in Y$ so $W \in (X \cap Y)$. Weakening, we have that $W \in (X \cap Y)$ *or* $W \in (X \cap Z)$. Hence, $W \in (X \cap Y) \cup (X \cap Z)$

   b) Say $W \in Z$ Then $W \in X$ and $W \in Z$ so $W \in (X \cap Z)$. Weakening, we have that $W \in (X \cap Y)$ *or* $W \in (X \cap Z)$. Hence, $W \in (X \cap Y) \cup (X \cap Z)$

   One of a) or b) must be true, and on either one, $W \in (X \cap Y) \cup (X \cap Z)$, we conclude $W \in (X \cap Y) \cup (X \cap Z)$. This is the first direction.

   Second, we prove $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$

   Suppose $W \in (X \cap Y) \cup (X \cap Z)$. Then $W \in (X \cap Y)$ *or* $W \in (X \cap Z)$

   a) Say $W \in (X \cap Y)$. Then $W \in X$ and $W \in Y$. Weakening, we have $W \in Y$ or $W \in Z$
   Hence $W \in (Y \cap Z)$. Combining $W \in X$ and $W \in (Y \cap Z)$ we have $W \in X \cap (Y \cup Z)$.

   b) Say $W \in (X \cap Z)$. Then $W \in X$ and $W \in Z$. Weakening, we have $W \in Y$ or $W \in Z$
   Hence $W \in (Y \cap Z)$. Combining $W \in X$ and $W \in (Y \cap Z)$ we have $W \in X \cap (Y \cup Z)$.

   One of a) or b) must be true, and on either one, $W \in X \cap (Y \cup Z)$ so we conclude $W \in X \cap (Y \cup Z)$. This proves the second direction.

   Combining both directions, we can conclude $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.

4. (Chapter 2, Exercise 7 a)

   Show that for all sets $X$ and $Y$, $X \subseteq Y$ if and only if $X \cap Y = X$.

   We want to show that for all sets $X$ and $Y$, $X \subseteq Y$ if and only if $X \cap Y = X$.

   First, suppose that $X \subseteq Y$. Let $Z \in X$. Then $Z \in Y$, since $X \subseteq Y$, so $Z \in X \cap Y$ by the Intersection Principle. On the other hand, if $Z \in X \cap Y$ then $Z \in X$, by the Intersection Principle. So $X \cap Y = X$.

   Second, suppose that $X \cap Y = X$. Let $Z \in X$. Then (since $X = X \cap Y$), $Z \in X \cap Y$. By the Intersection Principle, $Z \in Y$. Therefore, $X \subseteq Y$.

5. (Chapter 2, Exercise 26, Parts b), c), e), g), and i).) Working with strings over the alphabet $\{a, b\}$, write down five instances of each of the following patterns.

   (b) $axbya$
   (c) $axaxa$
   (e) $axbyybzzza$
   (g) $uax$
   (h) $uu'xuu'$

   (b) $aba$ $aaba$ $abaa$
   $ababa$ $abaaa$

   (c) $ababa$ $aaa$ $aaaaa$
   $aabaaba$ $abbabba$

   (e) $abba$ $abbaabaaaa$ $aabaabaaaa$
   $abbbaabaaaa$ $abbbbbbbba$

   (g) $aa$ $ba$ $aaa$
   $baa$ $bab$

   (i) $aaaa$ $abab$
   $baba$ $abaab$ $bbaaabb$

6. (Chapter 2, Exercise 27, Parts (a), (b), and (d).)

   (a) $baaba$ $x := a, u := a$
   $baaabb$ $x := aa, u := b$
   $bbaba$ $x := b, u := a$
   $baabb$ $x := a, u := b$
   $baba$ $x := \epsilon, u := a$

   (b) $aa$ $x := \epsilon, u := a$
   $bb$ $x := \epsilon, u := b$
   $bbaa$ $x := a, u := b$
   $bbbb$ $x := b, u := b$
   $bbaaaa$ $x := aa, u := a$

(d)

| | |
|---|---|
| $aaa$ | $x := \epsilon, y := \epsilon, z := \epsilon$ |
| $aaaa$ | $x := a, y := \epsilon, z := \epsilon$ |
| $aabba$ | $x := \epsilon, y := b, z := \epsilon$ |
| $aaaabbb$ | $x := a, y := \epsilon, z := b$ |
| $aaaabbbbaababab$ | $x := aa, y := bb, z := ab$ |

7. (Chapter 2, Exercise 28, Parts (a), (c), and (e).)

   *Note:* In general, there are many correct answers for each of the questions.

   (a) $xx$

   (c) $axaxax$

   (e) $xbybx$