Example problems for homework assignment 1

1. (Chapter 2, Exercise 2, Parts a) and d))
   
a) If \( m < n \) then \( n > m \)
   
d) If John gave some money to \( x \) and Mary gave some money to \( y \) (with \( x \neq y \)) then \( x \) may not have wanted \( y \) to know that Andrea gave \( x \) additional money.

2. (Chapter 2, Exercise 3, Parts a), b), and h)) Describe each of the following sets, using the notation that explicitly lists out the set’s members.
   
   (a) \( \{1, 2, 3\} \cup \{3, 4, 5\} \)
   
   (b) \( \{1, 2, 3\} \cap \{3, 4, 5\} \)
   
   (h) \( (\{1, 2, 3\} \cup \{4\}) - \{3\} \)

   (a) \( \{1, 2, 3, 4, 5\} \)
   
   (b) \( \{3\} \)
   
   (h) \( \{1, 2, 4\} \)
3. Chapter 2 exercise 5 d)  
Prove that $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

This is a set identity, and as so often it is easiest to “prove both directions”.

First, we prove $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$

Suppose $W \in X \cap (Y \cup Z)$. Then $W \in X$ and $W \in (Y \cup Z)$

Since $W \in (Y \cup Z)$ we have that $W \in Y$ or $W \in Z$

a) Say $W \in Y$ Then $W \in X$ and $W \in Y$ so $W \in (X \cap Y)$. Weakening, we have that $W \in (X \cap Y)$ or $W \in (X \cap Z)$. Hence, $W \in (X \cap Y) \cup (X \cap Z)$

b) Say $W \in Z$ Then $W \in X$ and $W \in Z$ so $W \in (X \cap Z)$. Weakening, we have that $W \in (X \cap Y)$ or $W \in (X \cap Z)$. Hence, $W \in (X \cap Y) \cup (X \cap Z)$

One of a) or b) must be true, and on either one, $W \in (X \cap Y) \cup (X \cap Z)$, we conclude $W \in (X \cap Y) \cup (X \cap Z)$. This is the first direction.

Second, we prove $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$

Suppose $W \in (X \cap Y) \cup (X \cap Z)$. Then $W \in (X \cap Y)$ or $W \in (X \cap Z)$

a) Say $W \in (X \cap Y)$. Then $W \in X$ and $W \in Y$. Weakening, we have $W \in Y$ or $W \in Z$  
Hence $W \in (Y \cap Z)$. Combining $W \in X$ and $W \in (Y \cap Z)$ we have $W \in X \cap (Y \cup Z)$.

b) Say $W \in (X \cap Z)$. Then $W \in X$ and $W \in Z$. Weakening, we have $W \in Y$ or $W \in Z$  
Hence $W \in (Y \cap Z)$. Combining $W \in X$ and $W \in (Y \cap Z)$ we have $W \in X \cap (Y \cup Z)$.

One of a) or b) must be true, and on either one, $W \in X \cap (Y \cup Z)$ so we conclude $W \in X \cap (Y \cup Z)$. This proves the second direction.

Combining both directions, we can conclude $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$. 
4. (Chapter 2, Exercise 7 a) 
Show that for all sets $X$ and $Y$, $X \subseteq Y$ if and only if $X \cap Y = X$.

We want to show that for all sets $X$ and $Y$, $X \subseteq Y$ if and only if $X \cap Y = X$.

First, suppose that $X \subseteq Y$. Let $Z \in X$. Then $Z \in Y$, since $X \subseteq Y$, so $Z \in X \cap Y$ by the Intersection Principle. On the other hand, if $Z \in X \cap Y$ then $Z \in X$, by the Intersection Principle. So $X \cap Y = X$.

Second, suppose that $X \cap Y = X$. Let $Z \in X$. Then (since $X = X \cap Y$), $Z \in X \cap Y$. By the Intersection Principle, $Z \in Y$. Therefore, $X \subseteq Y$.

5. (Chapter 2, Exercise 26, Parts b), c), e), g), and i).) Working with strings over the alphabet \{a, b\}, write down five instances of each of the following patterns.

(b) $axbya$
(c) $axaxa$
(e) $axbybybzzza$
(g) $uax$
(h) $uu'xuu'$

(b) $aba\ aaba\ abaa$
   $ababa\ abaaa$
(c) $ababa\ aaa\ aaaaa$
   $aabaaba\ ababba$
(e) $abba\ abbaabaaa\ aabaabaaa$
   $abbbabaaa\ abbbbbbbbbba$
(g) $aa\ ba\ aaa$
   $baa\ bab$
(i) $aaaa\ abab$
   $baba\ abaab\ bbaaabb$

6. (Chapter 2, Exercise 27, Parts (a), (b), and (d).)

(a) $baaba\ x := a, u := a$
   $baaabb\ x := aa, u := b$
   $bbaba\ x := b, u := a$
   $baabb\ x := a, u := b$
   $baba\ x := \epsilon, u := a$
(b) $aa\ x := \epsilon, u := a$
   $bb\ x := \epsilon, u := b$
   $bbaa\ x := a, u := b$
   $bbbb\ x := b, u := b$
   $bbaaaa\ x := aa, u := a$
(d) $aaa \quad x := \epsilon, y := \epsilon, z := \epsilon$
    $aaaa \quad x := a, y := \epsilon, z := \epsilon$
    $aabba \quad x := \epsilon, y := b, z := \epsilon$
    $aaaabb \quad x := a, y := \epsilon, z := b$
    $aaaabbbbaababab \quad x := aa, y := bb, z := ab$

7. (Chapter 2, Exercise 28, Parts (a), (c), and (e).)

Note: In general, there are many correct answers for each of the questions.

(a) $xx$
(b) $axaxax$
(c) $xbybx$