Non-Potential Electric Field Model of Magnetosphere-Ionosphere Coupling

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Non-Potential Electric Field Model of Magnetosphere-Ionosphere Coupling

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A new model is proposed to describe the electrodynamic coupling between the magnetosphere and ionosphere. In contrast with existing models, the ionospheric electric field is not assumed to be a potential field. The equation coupling the electric currents flowing into the ionosphere to the ionospheric electric fields is integrated analytically. This approach results in a simple, local and physically reasonable boundary condition, coupling the local tangential plasma velocity values in the magnetosphere to the tangential magnetic field through a properly integrated ionospheric conductivity. For simplified test cases the simulation results are in good agreement with those obtained with a traditional ionospheric electric potential model. The proposed approach improves computational efficiency and also allows prediction of the electromotive forces acting on closed electric current loops at the surface of the Earth. Such electric current loops can be induced by non-potential electric fields, generated by rapid changes in the ionosphere and magnetosphere.

INTRODUCTION

To a large extent the state of the magnetosphere is controlled by conditions in the solar wind and in the ionosphere. In a first approximation the distant solar wind is unaffected by the presence of the magnetosphere: therefore a “one-way” coupling adequately describes the interaction with the magnetosphere-ionosphere system. The magnetosphere-ionosphere (M-I) coupling, on the other hand, is a highly non-linear two-way interaction, which strongly affects the large-scale behavior of both domains.

Self-consistent global magnetosphere models include some kind of dynamic ionosphere model which interacts with the magnetosphere and provides ionospheric boundary conditions that actively respond to changing magnetospheric conditions [Ogino and Walker, 1984; Lyon et al., 1986; Tanaka, 1995; Raeder et al., 1995; Janhunen, 1996; White et al., 1998; Powell et al., 1999].

While mass exchange between the ionosphere and the magnetosphere is undoubtedly of major importance, the dominant component of M-I coupling is a system of field-aligned currents (Birkeland currents) connecting the magnetosphere and the high-latitude ionosphere. These Birkeland currents carry momentum and energy along stretched magnetic field lines connecting the ionosphere and the magnetosphere. Self-consistent global magnetosphere models need to describe the generation and closure of these Birkeland currents through appropriate boundary conditions and embedded non-MHD models.

The most important current systems coupling the ionosphere and the magnetosphere are the so called Region 1 and Region 2 Birkeland currents. Region 1 currents, flowing near the open-closed magnetic field boundary, connect the magnetopause current to the ionosphere where they are closed through ionospheric Pedersen currents. Region 2 currents flow along closed magnetic field lines and connect to the ionosphere at lower magnetic latitudes than the Region 1 current. Region 2 currents are generated in the inner part of the plasma sheet and in the ring current region.
Most global MHD magnetosphere models use the so-called electrostatic ionosphere approximation. The MHD code has an inner boundary at a radius of $R_p$ (most codes use values of $R_p = 2.5-4.5 R_E$). At this inner boundary, the MHD model is coupled to the ionosphere model with the help of appropriate boundary conditions. In practice, either plasma velocities or corresponding electric fields are imposed at the boundary that are calculated in the ionosphere in a three-step process:

1. Field aligned currents are calculated in the magnetosphere from the curl of the magnetic field near $R_p$, and these Birkeland currents are mapped down to the ionosphere along unperturbed (intrinsic) magnetic field lines.

2. A height-integrated ionospheric conductance pattern is generated and the ionospheric potential is calculated from the equation:

$$j_x(R_i) = [\nabla_i \cdot (\Sigma \cdot \nabla_i \psi_i)]_{R=R_i}$$

This describes the relationship between the height integrated conductance tensor, $\Sigma$, the ionospheric electric potential, $\psi_i$, and the radial component of the current, $j_R$ (here $R_i$ is the radius of the ionosphere and the subscript $i$ denotes the two tangential components of a 3D vector along the spherical surface).

3. The electric potential is mapped along unperturbed field lines to the inner boundary at $R_p$ where electric fields and velocities are generated. The corotation velocity field is added to the ionosphere generated velocity field.

The details of this method were summarized by Goodman [1995] with some corrections by Amm [1996].

The electrostatic ionosphere approximation captures some of the fundamental features of the M-I coupling process, but it suffers from several shortcomings, including: (1) inconsistency between the Birkeland currents and the dynamic (non-intrinsic) component of the magnetic field, and (2) neglect of skin-effect currents in the ionosphere (which cannot be described by potential electric fields).

There are additional limitations to the electric potential description of the ionosphere: (1) near strong electric fields, such as auroral arcs, the electric field is not a potential field, so the potential description is incorrect. (2) For a space weather prediction model, the ground induced currents (GICs) are an important consideration. These GICs are driven by electric fields by strongly varying magnetic fields, which are related to varying magnetic fields by Faraday’s law:

$$\frac{\partial \mathbf{B}}{\partial t} = -[\nabla \times \mathbf{E}],$$

where $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field. If the electric field is described as a potential field, it has no curl. Therefore it cannot have a $\partial \mathbf{B}/\partial t$ associated with it. (3) The calculation of the potential solution is costly in terms of computational time, and typically do not spread out well over large numbers of processors. On massively parallel machines, this can slow the main MHD solver down significantly. Therefore, the coupling is done only every few seconds, instead of every MHD iteration, which is the way the coupling should be done.

Another methodology is to relate the electric field (and therefore ion velocity) directly to the magnetic field structure at the boundary. This method allows the coupling to take very little computation time, so it can be completed every iteration. In addition, it has the physical properties which can clearly describe the $dB/dt$ term, so it can be used for space weather applications. Furthermore, it will better physically model strong electric field sources, such as those that occur near auroral arcs.

In this paper we describe an electrodynamic ionosphere model that addresses these limitations.

MODEL ASSUMPTIONS AND EQUATIONS

Here we describe the main features of the proposed M-I coupling technique and compare it step by step with presently used methods.

**M-I Boundary Is at $R_b = R_i$**

In this model the interface between the ionosphere and the magnetosphere is placed at the top of the ionosphere. Therefore, there is no need for any mapping of either the field aligned currents or of the electric field potential between the ionosphere and the magnetosphere. The MHD equations are solved above the ionosphere ($R > R_b = R_i$).

It should be emphasized that in most global MHD models relatively large values were chosen for $R_b$ in order to exclude regions with very high Alfvén speeds (and consequently with very small explicit time-steps). However, moving the inner boundary away from the actual M-I interface necessitates mapping physical quantities between the ionosphere and the inner boundary of the MHD simulation region. This mapping process includes additional simplifying assumptions that are not well justified. For example, it is usually assumed that the total electric current density in the mapping region ($R_i < R < R_b$) is directed along the unperturbed magnetic field line, $\mathbf{B}_y$. The conservation of the total current density yields a mapping relation that couples the electric current density $j_p(R_i)$ flowing into the ionosphere (see Eq.(1)) to the Birkeland current at the magnetospheric inner boundary $R = R_b$:

$$j_R = \mathbf{n}_x \cdot \mathbf{j}_p(R_i)$$

(3)
where \( \mathbf{n}_R \) is the unit vector in the radial direction and the field aligned current at the inner boundary of the simulation is given by:

\[
j_{\phi}(R_{i1}) = \frac{B_\phi(R_{i1})}{\mu_0} \left[ \mathbf{B}_\phi(R_{i1}) \cdot [\nabla \times \mathbf{B}(R_{i1})] \right] \tag{4}
\]

Here a point on the sphere with radius of \( R_{i1} \) is connected by the magnetic field line \( \mathbf{B}_\phi \) to a corresponding point on the sphere with radius \( R_{i2} \). On the other hand, if the total electric current is not field aligned, then there is no conservation law for the field aligned component and Eq. (4) is no longer valid.

The condition for the total electric current to be field aligned (“force-free” magnetic configuration) appears to require that there is no motion and absolutely no pressure gradient in the region \( R_{i1} < R < R_{i2} \). Obviously, this condition is not satisfied in the real M-I system. For this reason it is advantageous not to impose any assumption on the orientation of the electric current and to derive it directly from the MHD numerical solution. We also think that the problem of very small explicit time steps when the inner boundary is in the region of high magnetic field \( (R_{i2} = R_{i1}) \) can and should be solved using available numerical technology, such as local or implicit time stepping [Hirsch, 1990] or a physics-based convergence acceleration [Gombosi et al., 2002], rather than by using assumptions that are not well justified.

**Arbitrarily Directed Coupling Current**

The boundary condition for coupling the solution of the MHD equations in the magnetosphere to the electric current density distribution in the ionosphere (see Eq. 1) is obtained from the radial component of the simplified Ampere’s law by neglecting the displacement current:

\[
\mu_0 \mathbf{j}_r = \nabla \times \mathbf{B} \tag{5}
\]

The radial component of this equation is:

\[
j_r(R_{i1}) = \frac{\mathbf{n}_R \cdot [\nabla \times \mathbf{B}] \cdot \nabla}{\mu_0} = \frac{\mathbf{n}_R \cdot [\mathbf{B}_\phi \times \mathbf{n}_R]}{\mu_0} \tag{6}
\]

The easiest way to prove the transformation from the radial component of \( \nabla \times \mathbf{B} \) to the two-dimensional divergence of a two-dimensional vector is to compare the appropriate expressions for the components of an arbitrary vector \( \mathbf{A} \) (see Appendix in Landau et al. [1985]):

\[
n_s \cdot [\nabla \times \mathbf{A}] = \frac{1}{R \sin \theta} \left[ \frac{\partial}{\partial \phi} (\sin \theta A_\phi) - \frac{\partial A_s}{\partial \phi} \right] \tag{7}
\]

\[
\nabla_t \cdot \mathbf{A} = \frac{1}{R \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta A_\phi) + \frac{\partial A_s}{\partial \theta} \right] \tag{8}
\]

and note that \( \mathbf{A} \times \mathbf{n}_R = (0, A_\phi - A_\theta, A_r) \) for \( \mathbf{A} = (A_r, A_\phi, A_\theta) \).

We note two important points. First, Eq. (6) is not equivalent to Eq. (4), even in the limit when \( R_{i2} \rightarrow R_{i1} \). In this limiting case Eq. (4) becomes identical to Eq. (6) only if the electric current near \( R_{i1} \) is field aligned. Second, equation Eq. (6) automatically ensures that the total radial current vanishes when integrated over the entire outer boundary of the ionosphere: \( R\int_{R_{i2}} dR = \int_{R_{i2}} \cdot [\mathbf{B}_\phi \times \mathbf{n}_R] \cdot dR = 0 \) as a complete integral of the divergence (here \( dR \) is a spherical surface element). This is significant because the corresponding integral for the field aligned currents in Eq. (4) does not have to vanish.

It should also be mentioned that the magnetic field in Eq. (6) is the magnetic field perturbation (the deviation from \( \mathbf{B}_0 \)). Although it is typically small compared to the unperturbed \( \mathbf{B}_0 \) field, it controls the electric current density \( (\mathbf{B}_0 \) is usually a potential field).

**Non-Potential Electric Fields in the Magnetosphere and Ionosphere**

In a good approximation the motional electric field near the boundary surface is

\[
\mathbf{E}(R_{i1}) = [\mathbf{B}_0 \times \mathbf{u}]_{R = R_{i1}} \tag{9}
\]

where \( \mathbf{u} \) is the plasma bulk velocity. Here the contribution from the magnetic field perturbation \( \mathbf{B} \) is neglected. The electric field is perpendicular to the magnetic field line \( (\mathbf{E} \cdot \mathbf{B}_0 = 0) \), so that its radial component can be expressed in terms of the tangential components:

\[
E_r(R_{i1}) = -\frac{\mathbf{E}_t(R_{i1}) \cdot \mathbf{B}_0}{B_{0r}} \tag{10}
\]

A consequence of Faraday’s law is that the tangential components of the electric fields are continuous at any surface. This means that the same tangential electric field \( \mathbf{E}(R_{i1}) \) should be used in Eq. (1) (the general electric field now replaces \( -\nabla \psi \)):

\[
j_r(R_{i1}) = -[\nabla_t \cdot (\mathbf{E} \cdot \nabla)]_{R = R_{i1}} \tag{11}
\]

In Eq. (11) it is not assumed that the electric field has a scalar potential, and consequently, the electric field is not necessarily curl-free, \( \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t \approx 0 \). We will discuss the magnetospheric and the ionospheric consequences of this point separately.

The present generation of global MHD models assume that magnetic field lines are equipotentials between the ionosphere and the inner boundary of the magnetospheric simulation domain ((\( \mathbf{B}_0 \cdot \nabla ) \psi = 0 \). We know that only curl-free electric fields can be characterized by scalar potentials,
therefore in the mapping region the $\nabla \times \mathbf{E} = 0$ condition must hold. Using Faraday's law (Eq. (2)) one can readily see that the assumption of a potential electric field is equivalent to the assumption of a time independent (steady-state) magnetic field in the region between the ionosphere and the magnetospheric inner boundary. Since we place the inner boundary at $R_B = R_i$ we do not need to assume that the magnetic field lines in the inner magnetosphere are time stationary equipotentials.

In the ionosphere the assumption of a potential electric field is more restrictive resulting in mathematical and physical inconsistencies. Non-potential ionospheric electric fields also lead to important physical and technological effects. Metallic tubes, power transmission lines, huge transformers, land, sea-water and oceans form monumental closed conducting contours, that are not connected to the ionosphere by pairs of wires. However, they all are inductively coupled to the non-potential electric fields in the ionosphere. The non-potential part of the electric field, $\mathbf{E}(R_i)$, induces the time variation of the radial magnetic field, $B_r$, which can be obtained as long as $B_r(R_i)$ is known (see Eq. 2):

$$\frac{\partial B_r}{\partial t} = -\nabla \times \mathbf{E}_t$$

(12)

The voltage induced in a closed contour can be obtained by integrating Eq. (12) over the area enclosed by the contour or alternatively, by integrating the ionospheric electric field along the contour:

$$U = -\int \frac{\partial B_r}{\partial t} dS = \oint \mathbf{E}_t(R_i) \cdot d\mathbf{l}$$

(13)

This interesting effect is completely ignored and cannot be included in a potential field model for the ionospheric electric field: it follows from the assumption of $\mathbf{E}_t(R_i) = -\nabla \psi$, that the value of Eq. (13) is exactly zero.

The use of the Eq. (1) with a potential electric field as a boundary condition for the MHD equations also results in a mathematical problem. Substituting a potential electric field for $\mathbf{E}_i$ in Eq. (12) one can find that this equation requires that $\partial B_r(R_i)/\partial t = 0$. Therefore, the potential boundary condition is applicable only if $B_r(R_i)$ is known and it is constant in time (steady state).

Without assuming a potential electric field, the ionospheric field $\mathbf{E}_i$ can be written as the sum of a potential and non-potential component:

$$\mathbf{E}_i = -\nabla \psi + \left[ \frac{n_r \times \mathbf{B}_t}{\mu_0} \right]$$

$$\nabla \cdot \mathbf{E}_i = -\nabla^2 \psi$$

$$\nabla \times \mathbf{E}_i = n_r \nabla^2 \Pi_e$$

(14)

Here $\Pi_e$ is a 2D scalar potential. Eqs. (6) and (11) can be combined to yield:

$$\nabla \cdot \left( \frac{n_r \times \mathbf{B}_t}{\mu_0} - \sum \mathbf{E}_t \right) = 0$$

(15)

Eqs. (14) and (15) are the key to the new M-I coupling method.

**Height Integrated Ionosphere With No Radial Electric Currents**

We treat the ionosphere as a thin conducting layer occupying a very narrow altitude region, $h_L \leq h \leq h_U$ (here $h_L$ and $h_U$ represent the lower and upper boundaries of this layer). In addition, we ignore any altitude dependence. The physical and chemical processes responsible for the ionospheric conductivity are parametrized in terms of the height integrated components of the 2 x 2 conductance tensor $\Sigma$. The well known way to construct this tensor is described in Appendix A.

In addition, Eq. (15) implies that all the radial electric current flowing into or out of the ionosphere is only due to the M-I coupling and ignores all other radial electric currents, just as in the TIEGCM model [Richmond et al, 1992]. Until these additional currents are properly incorporated into the model, it cannot provide a quantitative description of some important geophysical phenomena.

Under these assumptions we integrate Eq. (15) over altitude for the ionosphere:

$$\frac{n_r \times \mathbf{B}_t}{\mu_0} - \sum \mathbf{E}_t = \frac{n_r \times \mathbf{B}'_t}{\mu_0}$$

(16)

where $\mathbf{B}'_t$ is the tangential magnetic field below the ionosphere. From Eq. (15) it follows that $\mathbf{B}'_t$ satisfies the condition

$$\nabla \cdot \left[ n_r \times \mathbf{B}'_t \right] = -n_r \cdot \left[ \nabla \times \mathbf{B}'_t \right] = 0$$

(17)

i.e., the tangential magnetic field below the ionosphere is a 1D potential field:

$$\mathbf{B}'_t = [\nabla \Pi_{b,t}]_{R - R_L + h_L}$$

(18)

where $\Pi_{b,t}$ is a 2D scalar.

The boundary condition relates the ionospheric electric field to that of below the ionosphere. From the continuity of the radial electric field and from Eqs. (12 and 14) we see that the non-potential part of the electric field should be continuous through the lower boundary of the ionosphere:

$$\mathbf{B}'_r = \mathbf{B}_r; \quad [\nabla \times \mathbf{E}'_t] = \nabla^2 \Pi_e$$

(19)

The electrostatic potential part of the ionospheric electric field is strongly distorted by charge separation at the lower ionospheric boundary and it is mostly radial: $\mathbf{E}'_r = -\psi/h_L$.
We note that the potential electric field does not induce currents and it only produces the charge separation.

THE ELECTRODYNAMIC PROCESSES BELOW THE IONOSPHERE ARE DECOUPLED

So far the assumptions relating to the M-I coupling model have been formulated in terms of variables, parameters and equations associated with the magnetosphere and/or ionosphere. However, eq.(16) goes beyond this and it couples the ionospheric electric field to the tangential magnetic field below the ionosphere. This magnetic field, in turn, depends on physical processes below the ionosphere.

Generally, there is a two-way coupling in this region because the magnetic field below the ionosphere is a function of \( \nabla \times \mathbf{E} = \nabla^2 \mathbf{I}_g \) and can be thought of as a linear response of some very complicated electrodynamic system (the Earth plus everything conducting on it) to the rotational part of the electric field. Using Eqs. (18) and (19), the most general linear response function can be written in the following form:

\[
\mathbf{I}_m = \int \mathbf{F}(\mathbf{I}_g) \, dt \, d\mathbf{R},
\]

Investigating such current systems is very interesting but complicated and we intend to explore this subject in subsequent publications. Here we discuss some simple limiting cases, in which the M-I coupling problem can be more or less readily decoupled from the electrodynamics below the ionosphere. All the discussed models allow for non-potential electric fields that can induce currents (GIC).

Concentrated Ground Impedance

Assume that the magnetic field variations and the electric fields are shielded by Earth's conductivity in a thin skin layer of \( h_s \sim (t_v/\sigma_E \mu_0)^{1/2} \), where \( t_v \) is a time scale for magnetic variations and \( \sigma_E \) is the ground conductivity. Hence, at \( h \approx -h_s \), the magnetic field vanishes, \( B \to 0 \).

For this case the physical interpretation of the \( [\mathbf{n}_s \times \mathbf{B}'_t] \) term in Eq.(16) is as follows. Our formulation takes into account all the currents and electric fields in the ionosphere and allows for non-potential electric fields that can induce closed electric current loops in the ground, ocean and so on (see Eq. 13). These currents in turn can induce some additional magnetic fields.

An explicit and unambiguous expression for \( \mathbf{B}'_t \) can be directly obtained from Ampere's law (Eq. 5). The height integrated tangential components of Eq. (5) can be written as follows:

\[
[n_s \times B'_t] = \left[ n_s \times B_t \right]_{h \to -h_s} + \left[ \nabla_t \times \int_{-h_s}^{h_s} n_s B_n \, dh \right] = \mu_0 \int_{-h_s}^{0} \mathbf{j}_i \, dh
\]  

The induced magnetic field at \( h = -h_s \) is assumed to tend to zero, hence the second term vanishes in the left hand side of Eq. (21). Comparing Eqs. (18 and 21), one can represent Eq. (21) in the form of Eq. (16) with the following unambiguous expression for \( \mathbf{B}'_t = [\mathbf{n}_s \times \nabla_t \mathbf{I}_m] \):

\[
[n_s \times \nabla_t \left( \mathbf{I}_m - \int_{-h_s}^{h_s} B_n \, dh \right)] = \mu_0 \int_{-h_s}^{0} \mathbf{j}_i \, dh \]

Our simple model of a concentrated ground impedance assumes that the integral of \( B_n \) in the left hand side can be evaluated as \( (h_s + h)B_R \), while the height integrated ground induced current density \( \mathbf{I}_g = \int_{-h_s}^{0} \mathbf{j}_i \, dh \) that is a function of \( \partial B_R / \partial t = -\nabla \times \mathbf{E} \) [cf. Viljanen et al., 1999], is assumed to be linearly proportional to the rotational part of the electric field:

\[
\mathbf{I}_g = \Sigma_E [n_s \times \nabla_t \mathbf{I}_m]
\]

where the integrated surface conductivity is \( \Sigma_E \sim \sigma_E h_s \).

With these simplifications Eq.(22) becomes:

\[
\mathbf{I}_m = \mu_0 \Sigma_E \mathbf{I}_g + (h_s + h_L)B_R \frac{\partial B_R}{\partial t} = -\nabla^2 \mathbf{I}_m
\]

Together with these relationships, the rotational part of Eq.(16)

\[
\nabla^2 \mathbf{I}_m + \nabla \times \nabla^{-1} [n_s \times \nabla_t \mathbf{I}_m] / \mu_0
\]

forms a closed system of equations that allows us to find \( \mathbf{I}_f, \mathbf{I}_m \) and then the ionospheric electric field \( \mathbf{E}_t \) for any given \( \mathbf{B}_t \) above the ionosphere.

The total jump in the magnetic field, \( [\mathbf{n}_s \times \mathbf{B}_t] / \mu_0 \) throughout the Earth skin layer, atmosphere and ionosphere can be related to the total current using Eqs. (15, 16, 17 and 23) in the following form ([cf. Untiedt and Baumjohann, 1993]):

\[
\frac{\mathbf{n}_s \times \mathbf{B}_t}{\mu_0} = \mathbf{I}_{\text{iono}} + \mathbf{I}_g + \mathbf{I}_{\text{ind}}
\]

\[
\mathbf{I}_{\text{iono}} = \Sigma_E \mathbf{E}_t
\]

\[
\mathbf{I}_g = \Sigma_E [n_s \times \nabla_t \mathbf{I}_m]
\]

\[
\mathbf{I}_{\text{ind}} = \left[ n_s \times \nabla_t \left( h_L + h_s \right)B_R \right] / \mu_0
\]

\[
\nabla_t \left[ \frac{\mathbf{n}_s \times \mathbf{B}_t}{\mu_0} - \mathbf{I}_{\text{iono}} \right] = \nabla_t \cdot (\mathbf{I}_g + \mathbf{I}_{\text{ind}}) = 0
\]
Without neglecting the displacement current, the magnetic field \( B' \), would be equal to the magnetic field in the Earth wave-guide (see [Yoshikawa and Itonaga, 2000]). The magnetic field above the ionosphere is separated from the effects of the ionospheric current (that is proportional to the total ionospheric electric field), from the GIC (proportional to the rotational part of the ionospheric electric field) and from the small reactive impedance (proportional to the radial magnetic field). Usually the latter term is small as compared to the input from GIC, Appendix B discusses the opposite limiting case for the interested reader.

Potential Ionosphere

In two limiting cases the M-I coupling can be decoupled from the processes taking place below the ionosphere and it becomes independent of the poorly defined \( \Sigma_E \).

First let us assume an infinite Earth conductivity: \( \Sigma_E \gg |\Sigma| \). According to Eq.(25), such a conductivity completely eliminates the rotational part of the ionosphere electric field \( (\Pi_E = 0) \). In this case Eq. (1) describes the total electric field that becomes a purely potential field. From Eqs. (1, 16, 24 and 25) with \( \Pi_E = 0 \) one can find an expression describing the GIC for this model:

\[
I_{gr} = \frac{\mathbf{n} \times \mathbf{B}_t}{\mu_0} \mathbf{\nabla} \psi, \quad \Pi_E = 0, \quad B_s = 0.
\]

We note that according to Eqs. (15, 16 and 27) the resulting GIC are divergence-free.

In this approximation we obtained the traditional potential ionosphere and the GIC as the limiting case of the concentrated impedance model. Mathematically, the potential ionosphere model is not anyway easier than the more realistic concentrated impedance model, because Eqs. (24 and 25) are of the same type and of the same complexity as Eq. (1). At the same time the physical model is still oversimplified and inconsistent because it includes GIC, but does not include the non-potential electric field. To improve the model, more realistic non-potential electric fields and related GIC should be incorporated.

Non-Potential Ionosphere With No Ground Induced Currents

The processes below the ionosphere also can be decoupled in the opposite limiting case when \( \Sigma_E \ll |\Sigma| \). This assumption means that we neglect the GIC, or more precisely, we neglect the feedback of GIC on the ionospheric electric field through the magnetic field below the ionosphere. The \( D_M = 0 \) condition makes it possible to obtain an explicit expression for the ionospheric electric field from Eq. (25):

\[
\mathbf{E}_t = \frac{1}{\mu_0} \left[ \mathbf{n} \times \left( \mathbf{B}_t - (h_S + h_L) \mathbf{\nabla} \psi \right) \right].
\]

For brevity we denote \( \mathbf{B}_t - (h_S + h_L) \mathbf{\nabla} \psi \) as \( \mathbf{B}_S \). The non-potential part of the electric field can be recovered, if desired, from the \( \mathbf{E} \) expression for the ionospheric electric field from Eq. (25). One can also obtain the small GIC by using \( \Pi_M = \Sigma_E \Pi_E \).

This model is again oversimplified and one needs the incorporation of a more realistic GIC description and related non-potential electric fields. However, this GIC-free non-potential model is very simple and computationally beneficial because it does not require the solution of an elliptic equation like Eq. (1) or Eq. (25). We performed a large number of numerical tests (see below) and found that the potential ionosphere model and the non-potential GIC-free model give similar results for the overall M-I coupling, so we can take advantage of the mathematical and numerical simplicity and physical reasonability of the GIC-free model. We choose this model to be used in our M-I coupling model. We note that improving these models requires more mathematical and computational sophistication.

We thus chose a simple boundary condition for the magnetosphere relating the tangential components of the magnetic field perturbation to those of the electric field. It describes the M-I coupling in a very simple, local manner. Comparing Eq. (28) to Eqs. (87.1-6) of Landau et al. [1985] we see that our boundary condition is basically identical to the Leontovich boundary condition that was introduced for describing the interaction of electromagnetic waves with a thin conducting layer (ionosphere or, originally, the skin layer of metals).

RESISTIVE SLIP VELOCITY

No Mass Exchange Solution

Eqs. (9, 10 and 28) can be combined to express the plasma velocity at the M-I boundary. However, the field aligned component of the plasma velocity (the component along the intrinsic field, \( B_J \)) is undefined since the cross product has no contribution from the parallel component. This uncertainty had been noted by Goodman [1995] and means that additional assumptions need to be made about the field aligned plasma velocity component at the M-I interface.

In this paper we focus on the electrodynamic coupling between the ionosphere and magnetosphere. While we recognize the importance of ionospheric outflow to magnetospheric composition and dynamics, we focus on the simple case when the mass exchange between the ionosphere and
magnetosphere is neglected. We will generalize our approach in a subsequent publication.

For the sake of simplicity we choose the radial plasma velocity to be zero at the M-I boundary. By combining Eqs. (9, 10 and 28), we find the boundary condition coupling the velocity and the magnetic field perturbation:

$$B_{\text{r0}} [\mathbf{n}_r \times \mathbf{u}_\text{r}] = \frac{\Sigma_{\text{H0}}^{-1}}{\mu_0} \cdot [\mathbf{n}_r \times \mathbf{B}_\text{t}] \quad (29)$$

In addition, we have the $u_R = 0$ condition at the boundary, describing no mass exchange between the ionosphere and the magnetosphere.

**Boundary Conditions at the M-I Interface**

Combining Eqs. (29 and B8) we finally find the boundary condition at the magnetosphere - ionosphere interface in a simple vector form:

$$u_R = 0 \quad (30)$$

$$\mathbf{u}_\text{t} = \mathbf{v}_{\text{rot}} - \frac{\mathbf{n}_r \times \mathbf{B}_\text{t}}{\mu_0 e N_e} + \frac{\mathbf{B}_{\text{t}||}}{B_{\text{r0}} \mu_0 \Sigma_{\text{H0}}} + \frac{\mathbf{B}_{\text{t}||}}{B_{\text{r0}} \mu_0 \Sigma_{\text{H0}}} \quad (31)$$

The condition for the tangential velocity is surprisingly simple and has a transparent physical interpretation: the tangential velocity component of the nearly perfectly conducting layer and has a transparent physical interpretation: the tangential velocities but it is of the same physical nature), plus the current velocity (in the general case this term can differ from both the electron and ion current velocities but it is of the same physical nature), plus the “resistive slip” velocity:

$$u_{\text{slip}} = \frac{\mathbf{B}_{\text{t}||}}{B_{\text{r0}} \mu_0 \Sigma_{\text{H0}}} + \frac{\mathbf{B}_{\text{t}||}}{B_{\text{r0}} \mu_0 \Sigma_{\text{H0}}}$$

The “slip velocity” is due to finite ionospheric plasma resistance because the magnetic field lines can slip through the plasma as long as the magnetic field is not completely frozen in.

**NUMERICAL MODEL**

We use the BATSRUS code of the University of Michigan [Powell et al., 1999] to simulate the magnetosphere. The ideal MHD equations with full energy equation are solved using a conservative finite volume scheme with second order of accuracy. An adaptive block grid is used, the control volumes (“cells”) being rectangular boxes.

The values of the MHD variables are interpolated to the faces of the control volume using the van Leer /3-limiter [Hirsch, 1990] with $\beta = 1.2$, applied to the increments of the primitive variables (density, velocity, magnetic field B and pressure), as in the MUSCL scheme [cf. Toro, 1999].

The monotone numerical fluxes through the faces of the control volume are upwinded using the Artificial Wind scheme [Sokolov et al., 1999, 2002]. Second order time update is used as in the second order Runge-Kutta scheme.

The boundary conditions given by Eqs. (30 and 31) are used to construct the first order monotone boundary condition (via a Riemann solver) at the inner boundary for the MHD computational domain at $R = R_\text{e}$ that is interpreted as the interface between a moving perfectly conducting fluid (magnetosphere) and a rotating thin spherical shell of finite conductivity (ionosphere).

For comparison we also used the potential ionosphere model. The equation for the potential Eq. (1) with the same conductivity tensor as in the Eq. (B6) can be written in the following form:

$$\nabla \cdot \left( \left( \Sigma_\text{H} - \Sigma_\text{e} \right) \left( \nabla \psi \cdot \mathbf{B}_{\text{r0}} \right) \frac{\mathbf{B}_{\text{r0}}}{B_{\text{r0}}^2} + \Sigma_\text{H} \nabla \psi \right)$$

$$- \left[ \nabla \psi \cdot \mathbf{n}_r \right] \cdot \nabla \cdot \mathbf{E} = j_k \left( R_\text{e} \right)$$

Eq. (32) is numerically solved using GMRES algorithm.

**SIMULATION RESULTS**

The main purpose of the present simulation is to compare the newly introduced non-potential model for the M-I coupling with the existing potential model. That is why we choose a test problem for which the widely used potential model is applicable and should give a physically reasonable answer. That is we simulate the steady-state plasma flow around the Earth, the direction of the magnetic field dipole axis being perpendicular to the velocity of solar wind. For our non-potential model choosing a steady-state problem is not important, however, the potential model is strictly applicable only for a steady-state problem. In the simulations the solar wind parameters are: $n_e = 5$ cm$^{-3}$, velocity $= 500$ km/s, temperature $T = 1.5 \cdot 10^5$ K and southward IMF $= -5$ nT.

In the first test we consider the case of a single constant conductivity, $\Sigma_\perp = 4$ Ohm$^{-1}$, $\Sigma_\parallel = 0$. Again, for the non-potential model any conductivity gradients are unimportant, while for the potential model the results obtained with non-constant conductivity depend on a particular choice of the scheme for the conductivity gradients.

Plate 1 shows the field aligned current distribution calculated at the $R = 2R_\text{e}$ surface. This seems to be a representative characteristics for M-I coupling because, although calculated in the magnetosphere, it is known to be sensitive to any change of the ionosphere parameters [cf. Ridley et al., 2003].

In the top panel of Plate 1 we present the distribution of the ionosphere current obtained using non-potential M-I
Plate 1. Ionospheric radial current distribution obtained with the non-potential technique for M-I coupling (upper panel) and the same current density obtained with a potential model (lower panel).

Plate 2. Ionospheric radial current distribution obtained with the non-potential technique including Hall conductivity (top panel) and the same, obtained with a potential model (lower panel).
coupling model with the boundary condition in the form of Eq. (31). The result seems to be reasonable. In the bottom panel of Fig. 1 the same distribution is presented obtained from the potential model in solving the Eq. (32). The agreement between the results is very good.

In Plate 2 we present the results of the same simulations but with the Hall conductivity taken into account. In the simulations we used $\sigma_y dh = 1000 \text{ S m}^{-1}$ and $\sigma_x dh = \sigma_y dh = 4 \Omega^{-1}$ with the proper latitude dependence according to Eqs. (B2-B5). The difference in the results is again reasonably small, mainly due to the significant differences in the numerical algorithms we used in the potential and non-potential models.

**DISCUSSION AND CONCLUSION**

We demonstrated that the results obtained using the new non-potential model for M-I coupling are very similar to compared to the results obtained with the potential model.

The non-potential model seems to be physically more relevant when describing fast and abrupt processes in the M-I system, for which the essentially non-potential perturbations of the electric field are not applicable.

The non-potential model is of higher computational efficiency, since the electric potential is not needed for describing the M-I coupling, and consequently there is no need to solve Poisson’s equation Eq. (32).

The non-potential model allows us to find the non-potential electric field, calculate and predict the electromotive forces, which are induced in the natural and technological large-scale closed electric circuits at the Earth during fast geomagnetic processes.

**APPENDIX A: SOURCE SURFACE METHOD AND ACCELERATED DAMPING RATE FOR NON-POTENTIAL ELECTRIC FIELD**

Consider the limiting case when Earth is a perfect insulator (with zero conductivity) and there are no GICs. In this case, the magnetic field below the ionosphere is purely potential, hence, it can be unambiguously expressed in terms of its radial component ($B_R$) at a sphere, using the so-called source surface method. This method is widely used in solar physics, allowing to reconstruct 3D spatial distribution of the coronal magnetic field from measurements of the radial component of the field at the solar surface [Altschuler et al., 1977]. Recently our group applied this method to simulate the 3D solar wind driven by solar magnetogram observations [Roussev et al., 2003].

In the case of the terrestrial ionosphere with no ground conductivity, the normal component of the magnetic field (that is continuous across the ionosphere in the thin ionosphere approximation) can be expressed as a series of spherical functions, $Y_{nm}$:

$$B_{R} \mid_{R = R_{E} + h_{L}} = \sum_{n,m} a_{nm} Y_{nm} (\theta, \phi) \quad (A1)$$

Once the expansion coefficients, $a_{nm}$, are specified, the full magnetic field vector at the sphere can be expressed in the following form:

$$B' = \nabla \psi', \quad \psi' = (R_{E} + h_{L}) \sum_{n,m} \frac{a_{nm}}{n} Y_{nm} (\theta, \phi) \quad (A2)$$

Next we can substitute Eq. (A2) into Eq. (16) and express the tangential electric field in the ionosphere in terms of all the three components of the magnetic field:

$$\Sigma \cdot E_t = \frac{1}{\mu_o} [n_R \times B_t] - \frac{1}{\mu_o} (R_{E} + h_{L}) [n_R \times \nabla_t] \sum_{n,m} \frac{a_{nm}}{n} Y_{nm} (\theta, \phi) \quad (A3)$$

A physically instructive simplification of Eq. (A3) can be obtained by neglecting the factor $1/n$ in the series expansion, even though this approach overestimates the influence of the magnetic field $B^0$ below the ionosphere. Now one can recognize that the series expansion is the radial magnetic field component at the sphere and we can combine Eqs. (A1) and (A3) to obtain the following:

$$\mu_o \Sigma \cdot E_t = [n_R \times B_t] - (R_{E} + h_{L}) [n_R \times \nabla_t] B_R \quad (A4)$$

The physical interpretation of this equation becomes clear if we consider the case of a uniform and scalar conductivity $\Sigma_0$, and take the time derivative of Eq. (A4):

$$\frac{\partial E_t}{\partial t} = \frac{1}{\mu_o \Sigma_0} \left[ n_R \times \frac{\partial B_t}{\partial t} \right] - \frac{R_{E} + h_{L}}{\mu_o \Sigma_0} [n_R \times \nabla_t] \frac{\partial B_R}{\partial t} \quad (A5)$$

This equation can be further simplified using Eq. (12):

$$\frac{\partial E_t}{\partial t} = \frac{1}{\mu_o \Sigma_0} \left[ n_R + \frac{\partial B_t}{\partial t} \right] + \frac{R_{E} + h_{L}}{\mu_o \Sigma_0} [n_R \times \nabla_t] (\nabla_t \times E_t) \quad (A6)$$
Finally, we apply the $\nabla \times$ operator to Eq. (A6):

$$\frac{\partial}{\partial t} (\nabla \times \mathbf{E}_t) - \frac{R}{} + \frac{h_L}{\mu_0 \Sigma_\alpha} \nabla \tau \cdot (\nabla \times \mathbf{E}_t) = \frac{\partial}{\partial t} \nabla \tau \cdot \mathbf{B}_t \tag{A7}$$

The solutions of Eq. (A7) with vanishing right hand side would decrease in time, not slower than $\sim \exp(-\nu t)$, where the damping rate $\nu$ is:

$$\nu = \frac{1}{2(R + h_L) \mu_0 \Sigma_\alpha} \approx \frac{1}{4 \Sigma_\alpha [\text{mhos}]} \left[ \text{s}^{-1} \right] \tag{A8}$$

This means that the perturbation of the non-potential electric field caused by a variation of the magnetic field above the ionosphere (which comes from the magnetosphere) spreads over the ionosphere and decreases in time. Approximation (A4) overestimates the damping rate for spherical harmonics with larger $n$ values. However, Eq. (A8) is correct in any case, because it is derived for $n = 1$ spherical harmonics and the omission of $1/n$ factor is justified for it. The maximum damping rate approximation given by Eq. (A4) is useful both from the theoretical (it allows to treat non-potential time-varying electric fields, while it rapidly converges to the steady-state solution for purely potential electric field) and from the practical points of view (it results in a simple and fast converging numerical method).

On the other hand, the physical reasoning under this model is rather poor. The magnetic field is assumed to freely penetrate to the ground down to the Earth center due to infinitely small conductivity, while with realistic finite Earth conductivity $\sigma_\alpha$, which is especially high at larger depths, the time $\mu_0 R^2 \sigma_\alpha$ needed for such the penetration is enormously long and is a way greater than time scale $t_\tau$ for any geomagnetic variation: $t_\tau \gg \mu_0 R^2 \sigma_\alpha$.

**APPENDIX B: CONDUCTANCE TENSOR**

In this section we discuss some properties of the height integrated electric conductance tensor, $\Sigma$. Our observations are simple and we make them to avoid misunderstandings about the physical nature of the various terms. In the literature both the conductance and resistance tensors are used, and, in addition, in the M-I coupling problem the height integrated 2D conductance tensor is used, rather than the 3D conductivity tensor.

The generalized Ohm’s law in the ionosphere plasma is:

$$\mathbf{j} = (\sigma_{\parallel} - \sigma_p) \frac{(\mathbf{E}' \cdot \mathbf{B}_0) \mathbf{B}_0}{B_0^2} + \sigma_p \mathbf{E}' - \sigma_n \frac{[\mathbf{E}' \times \mathbf{B}_0]}{B_0} \tag{B1}$$

where $\sigma_p$ and $\sigma_n$ the Pedersen and Hall conductivity, $\sigma_{\parallel}$ is the conductivity along the magnetic field, $\mathbf{E}' = \mathbf{E} + [v_{rot} \times \mathbf{B}_0]$ is the electric field in the rotating frame of reference, $v_{rot}$ is the velocity of rotation (the ionosphere is assumed to rotate with the Earth). Eq. (B1) is discussed in textbooks [cf. Gombosi, 1998] and is well-known.

Next we consider the ionosphere as a thin conducting layer. This assumption allows us to neglect the radial component of the current density $j_r$ in Eq. (B1) that is small compared to the tangential components (see Amm [1996]). By finding the radial electric field $E'_r$ from the condition $j_r = 0$, one can reduce Eq. (B1) to an equation for the tangential current density

$$\mathbf{j}_t = (\sigma_{\parallel}' - \sigma_p') \frac{(\mathbf{E}' \cdot \mathbf{B}_0) \mathbf{B}_0}{B_0^2} + \sigma_p' \mathbf{E}' - \sigma_n' \frac{[\mathbf{E}' \times \mathbf{n}_n]}{B_0} \tag{B2}$$

where

$$\sigma_{\parallel}' = \frac{\sigma_{\parallel} \sigma_p}{\sigma_{\parallel} \cos^2 \alpha + \sigma_p \sin^2 \alpha}, \tag{B3}$$

$$\sigma_p' = \frac{\sigma_p \cos^2 \alpha + (\sigma_p^2 + \sigma_n^2) \sin^2 \alpha}{\sigma_p \cos^2 \alpha + \sigma_p \sin^2 \alpha}, \tag{B4}$$

$$\sigma_n' = \frac{\sigma_n}{\sigma_p \cos^2 \alpha + \sigma_p \sin^2 \alpha} \tag{B5}$$

and

$$\cos \alpha = \frac{B_{mT}}{B_0}. \tag{B6}$$

We note that the conductivity $\sigma_{\parallel}'$, which is usually denoted as $\sigma_{\parallel o}$ is ‘parallel’ only with respect to the (small) projection of the full magnetic field on the ionosphere layer $B_p$, but the physical processes under it are mostly the same as for the Pedersen conductivity, because the $B_{0R}$ component is dominant. Eqs. (B2–B5) are identical to the results obtained by Amm [1996]. Eq. (B2) can be height integrated assuming that the tangential electric field in the thin conducting layer does not depend on altitude:

$$\mathbf{j}_t = (\Sigma^\prime_{\parallel} - \Sigma^\prime_p) \frac{(\mathbf{E}' \cdot \mathbf{B}_0) \mathbf{B}_0}{B_0^2} + \Sigma^\prime_p \mathbf{E}' - \Sigma^\prime_n \frac{[\mathbf{E}' \times \mathbf{n}_n]}{B_0} \tag{B6}$$

$$\Sigma^\prime_{\parallel} = \int \sigma_{\parallel}' d\theta, \Sigma^\prime_p = \int \sigma_p' d\theta, \Sigma^\prime_n = \int \sigma_n' d\theta \tag{B7}$$

Note, that in the 2D Ohm’s law (Eq. B6) the difference between the “parallel” and “Pedersen” conductances is small, especially at high latitudes (at $\sin^2 \alpha \to 0$): $\Sigma_{\parallel} \approx \Sigma_p \approx$
\[ \int \sigma_p \, dh, \text{ although for actual 3D conductivity coefficients usually } \sigma_{||} \gg \sigma_p \text{ in the ionosphere}. \] This point is physically evident, because, due to the large radial magnetic field, the tangential current is not field aligned at all and is not much sensitive to the comparatively small tangential component of the magnetic field. Particularly, in our first numerical test (see above) we use \( \Sigma_p' = \Sigma_{||}' = 4 \text{ Ohm}^{-1} \).

Finally, we should consider the right hand side of Eq. (B6) as the product of the 2 x 2 matrix, \( \Sigma \), multiplied by the vector \( \Sigma_p' \), and invert this matrix (see Eq. (29)) by expressing the tangential electric field in terms of the tangential current:

\[ E'_t = \Sigma^{-1} J_t = \left( \begin{array}{c} \frac{J_{||}}{\Sigma_{||}} \right) + \left[ \frac{J_{||} \times n_x}{eN_e} B_{0r} \right] \frac{\Sigma_{\perp}}{\Sigma_{||}} \]  

where

\[ \Sigma_{||} = \frac{\Sigma_{||}' \Sigma_{||} + (\Sigma_{\perp}' \cos \alpha)^2}{\Sigma_{\perp}'}; \quad \Sigma_{\perp} = \frac{\Sigma_{\perp}' \Sigma_{||} + (\Sigma_{\perp}' \cos \alpha)^2}{\Sigma_{||}'}; \]

\[ N_e = \frac{B_{0r} \left( \Sigma_{||}' \Sigma_{\perp} + (\Sigma_{\perp}' \cos \alpha)^2 \right)}{e\Sigma_{||}'}; \]

where \( \Sigma_{||} \) and \( \Sigma_{\perp} \) are proper non-linear combinations of the height integrated conductances. Nevertheless, if only the electrons are responsible for the conductivity and their conductivity coefficients can be considered as height independent variables in the integration, then \( \Sigma_{||} = \Sigma_{\perp} = \int \sigma_{||} \, dh \) and they become the height integrated Lorentz conductivity and do not depend on the magnetic field. Analogously \( N_e \) is also some formally introduced non-linear combination of the height integrated conductances, which has nothing to do with the electron density in a general case, but in the same particular case it becomes \( N_e = \int n_e \, dh, \) \( n_e \) being the electron density. This simplification is not used in our numerical solutions, it is invalid for the Earth ionosphere, because the ions are mainly responsible for the Pedersen conductivity, but it is useful in analysing the physical interpretation of the boundary conditions described in the body of this paper.

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