On the processes in the terrestrial magnetosheath

1. Scheme development

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Abstract. We propose a new method to study the structure of the magnetosheath and thereby determine the underlying processes that create this structure. This method provides a systematic means of separating perturbations due to the solar wind variations from those generated within the magnetosheath. As a result, we are able to study the magnetosheath processes as well as the dynamic solar wind-magnetopause interaction. We use the solar wind measurements from an upstream monitor as the input to the gasdynamic convected field model and then compare the model output with the in situ magnetosheath observations. We introduce three parameters to scale the model prediction to match the timings of the magnetopause crossing, bow shock crossing, and upstream variations. With this procedure the relationship between the upstream measurements and the magnetosheath observations and the location of the magnetosheath satellite relative to the magnetopause and bow shock boundaries are highly constrained. We then introduce a series of normalization procedures that provide the means to remove the effects of the solar wind variations. The systematic differences between the model prediction and observation indicate physical processes that are not included in the gasdynamic model. An application of this approach is presented in a companion paper.

1. Introduction

One of the reasons why the magnetosheath is important in magnetospheric physics is that it plays a crucial role in the solar wind-magnetopause interaction. It is the magnetosheath field and plasma that interact with the magnetopause and magnetosphere, not the unshocked solar wind. Early models describing the solar wind-magnetosphere interaction without incorporation of the bow shock and magnetosheath have been useful but provide only a zeroth order approximation to the interaction. Examples of such models include calculation of the magnetopause location and shape using the solar wind pressure [e.g., Mead and Beard, 1964] and the model of antiparallel merging at the magnetopause using the interplanetary magnetic field (IMF) direction directly [Nishida, 1978; Crooker, 1979]. As we now know, the location and the shape of the magnetopause depend on both pressure and the presence of reconnection [e.g., Aubry et al., 1970; Fairfield, 1971; Petrinec et al., 1991; Roelof and Sibeck, 1993; Shue et al., 1997, 1998]. The draping of the magnetic field due to the bow shock and the magnetosheath processes in turn modifies the locations where the fields on the two sides of the magnetopause are antiparallel [Lührmann et al., 1984]. Moreover, the reconnection processes are sensitive to the plasma conditions, such as field shear [Paschmann et al., 1979; Sonnerup et al., 1981], the flow pattern and plasma beta [Paschmann et al., 1986]. Therefore the magnetosheath is an essential element in any realistic solar wind-magnetosphere interaction model and in particular for a space weather forecast model. Furthermore, the prediction of the sheath properties near the magnetopause in response to the changes in the solar wind is important in understanding the dynamic interaction between the solar wind and the magnetopause.

The formation of the magnetosheath can be understood in terms of the magnetosphere interacting with the solar wind through waves. These waves carry the information to warn the incoming solar wind of the existence of the magnetosphere, an obstacle to the flow. The waves take various forms, such as standing waves, discontinuities, oscillations, and infinitesimal increments. A gradual change can be considered to be a series of small incremental changes. As is illustrated in Figure 1, these waves reconfigure the solar wind flow and its frozen-in IMF from the solar wind state to the state specified by the magnetopause boundary condition. Changes in the upstream solar wind or in the downstream boundary condition at the magnetopause produce additional temporal variations in the magnetosheath. Furthermore, waves are generated by the processes of the solar wind-bow shock interaction, such as the foreshock and upstream waves [e.g., Lührmann et al., 1984; Fairfield et al., 1990; Engebretson et al., 1991; Le and Russell, 1992]. Because the variations associated with these two additional sources are so strong, they have for a long time hindered the study of the processes in the magnetosheath. In this series of papers we present a new approach to understanding the physical processes that take place in the magnetosheath. In this paper we establish the physical foundation of our approach. In the companion paper [Song et al., this issue], we apply this approach to case studies and discuss the technical details and the physical implication of the results. A statistical study will be reported in a later paper of this series.

To understand the fundamental physical processes in the magnetosheath is important not only to magnetospheric physics but also to general fluid dynamics, because this setting is a classical problem of supersonic flow around a blunt body. The difference from what is commonly discussed in
textbooks is that this flow is not an ordinary flow but a "magnetized highly electrically conducting flow." The understanding of the processes in this system can be applicable to many problems in astrophysics, solar physics, space physics, and plasma physics.

1.1 Theoretical Magnetosheath Models

The effects of the magnetic field on the sheath flow were recognized soon after the magnetosheath was introduced [Midgley and Davis, 1963; Lees, 1964]. Zwan and Wolf [1976] (herein after referred to as ZW) provided the first formulation and numerical solution of the magnetosheath. Their model is essentially one-dimensional (1-D) with three-dimensional (3-D) extensions. It follows a 1-D magnetic flux tube moving from the bow shock to the magnetopause while satisfying the conservation laws along the flux tube. The temporal evolution of this flux tube as it moves toward the magnetopause provides a description of the second dimension, along the trajectory of the flux tube. The third dimension, normal to the plane containing the magnetic field and the flow, is incorporated according to the electric potential difference across such a flux tube based on the frozen-in condition. ZW identified two mechanisms that lead to draining and depletion of the plasma inside of the flux tube. Hence the model is widely known as the plasma depletion model. The first depletion mechanism is the diversion of the flow at the bow shock along the magnetic field direction and away from the stagnation streamline. It is important to point out that the diversion may not necessarily lead to a decrease in the density because while the flux tube moves toward the magnetopause, it also gets compressed, which tends to increase the density. A net density decrease could occur in a region where the deceleration of the flow is not efficient and the diversion is dominant.

The second depletion mechanism is the so-called squeezing effect. This effect occurs near the magnetopause where the magnetic flux tubes pile up. As the magnetic pressure increases, to maintain the total pressure balance normal to the magnetopause, the plasma pressure has to decrease, and hence the plasma is squeezed out. This mechanism has been compared with squeezing toothpaste out of the tube. Although this analogy has some pedagogical value, an important issue is what processes act as the "hand" squeezing the plasma. Note here that every flux tube is completely enveloped in other flux tubes. ZW derived a mathematical solution describing a smooth transition between these two mechanisms. It is important to point out that a discontinuity may exist in the region where the two mechanisms are in transition. In other words, if each mechanism were solved starting from one of the boundaries, they would not necessarily match in the middle, and a jump condition between the two solutions may be required. Nevertheless, the solution of the ZW model joins the two mechanisms smoothly and predicts a monotonous density decrease from the bow shock to the magnetopause with a layer near the magnetopause where the density drops rapidly while the field strength increases. This layer is referred to as the plasma depletion layer. ZW correctly pointed out that the plasma depletion layer is physically a slow mode rarefaction wave.

Wu [1997] has calculated the magnetosheath profile using a 3-D magnetohydrodynamic (MHD) numerical simulation. An important difference from ZW is that the density along the Sun-Earth line first increases rather than decreases as the flow proceeds toward the magnetopause. According to Wu's simulation, the diversion of the flow initially is not sufficient to compensate the compression of the flux tube. Figure 2 shows a comparison of the density from the MHD simulation with the depletion model. A weak depletion layer near the magnetopause is confirmed in Wu's results. Most other MHD simulations show similar profiles [e.g., Lee et al., 1991; Lyon, 1994; Yan and Lee, 1994; Berchem et al., 1995].

Southwood and Kivelson [1992, 1995] revisited the ZW formulation and made a significant contribution to its physical understanding. They provided a comprehensive discussion on the assumptions made in the ZW model and the physical consequences of these assumptions. The assumption that allows ZW to follow and calculate an isolated flux tube in the magnetosheath is the so-called thin flux tube approximation. With this approximation, the forces acting on the isolated flux tube are provided by its surrounding medium in the form of external forces. These external forces vary according to the location of the flux tube in the sheath and are different from the external forces given by the boundary conditions if the flux tube is away from the boundary. In the ZW model the force is specified using the condition at the magnetopause boundary that introduces a kinematic element into the model because a

Figure 2. Comparison of the density profiles along the Sun-Earth line predicted by models. Wu's [1992] MHD calculation (solid line), the Zwan and Wolf [1976] depletion model (dashed line), and Southwood and Kivelson's [1995] new model and Song et al.'s [1992] observations (dot-dashed line).
flux tube that is not near the magnetopause boundary will not experience an external force as strong as that at the boundary. Most importantly, Southwood and Kivelson [1995] identified the following paradox in the ZW formulation. We first recall that in isotropic plasmas the only force that can make the plasma move along the field is the plasma pressure gradient force because the Lorentz force is zero along the flux tube, and the total pressure balance required is normal to the flux tube. When flux tubes pile up near the stagnation region, the magnetic pressure increases. The resulting lower thermal pressure will act against further squeezing. If the first depletion mechanism, the diversion, results in a lower density along the stagnation streamline, according to the ZW model, there will be little plasma available to produce the higher thermal pressure needed for the second depletion process. One possible solution, as proposed by Southwood and Kivelson [1995], is to add a slow mode compressional front to the end of the diversion-dominant region, as shown in Figure 2. This compressional front provides the required higher thermal pressure for the second depletion process. The model described above describes only steady state processes. When the upstream conditions are not in a steady state, the interaction of the solar wind variations with the bow shock will further complicate the picture [Wu et al., 1993; Yan and Lee, 1994; Lin et al., 1996].

In theory the collision of a solar wind discontinuity with the bow shock creates seven discontinuities: a pair of shocks for each of fast, intermediate, and slow modes, plus a contact discontinuity. This is the so-called Riemann problem, which has been comprehensively reviewed by Lin and Lee [1994]. Each pair consists of a forward moving and a backward moving shock relative to the contact discontinuity. The backward moving fast shock forms the new bow shock front as required by the conditions post solar wind discontinuity. In the Earth’s frame all other six discontinuities move in the antisunward direction. Because of the differences in the phase velocities along the discontinuities, they will spatially spread as they propagate. A realistic magnetosheath model has to describe not only a steady state magnetosheath but also these temporal changes.

In practice, the magnetopause is not perfectly impermeable because of reconnection and possible diffusion, and it is dynamically active, with many temporal variations such as FTEs and surface waves. The coupling of the outer magnetosphere and boundary layers with the ionosphere plays an important role in regulating these activities, (see the review by Lotko and Sonnerup [1993]). Since the energy involved in these processes is small compared with the solar wind flow energy, their effects on the sheath flow are expected to be important in the region near the magnetopause boundary.

1.2 Previous Observations

Signatures of the plasma depletion layer have been observed on occasion [Paschmann et al., 1978; Russell and Elphic, 1978; Crooker et al., 1979; Song et al., 1993]. Phan et al. [1994] reported based on a statistical study that the depletion layer is more likely to occur during northward interplanetary magnetic field (IMF) but not during southward IMF. The overall density profile of the magnetosheath had not been studied until Song et al., [1990]. Song et al. surveyed the satellite passes near the Sun-Earth line and reported some then unexpected profiles. The density remains fairly constant and similar to expectations in the outer part of the magnetosheath and undergoes some relatively abrupt enhancements in front of the magnetopause. These density enhancements can be simplified as a compressional front followed by a depletion when the magnetopause is approached, as indicated in Figure 2, plus oscillations. They interpreted this compressional front as a slow mode (shock) front. Similar density enhancements in front of the magnetopause can be found in observations from AMPTE/IRM [Hill et al., 1995], ISEE [Siebeck and Gosling, 1996], and WIND [Phan et al., 1996], and the Jovian magnetosheath [Phillips et al., 1993]. Petrinec et al. [1997] reported some very interesting phenomenon. Near the flank magnetopause, the magnetosheath flow perturbations appear to be Alfvénic even when the upstream solar wind does not change. What is even more interesting is that these Alfvénic perturbations are limited to a finite region near the magnetopause. The perturbations farther in the magnetosheath do not have the Alfvénic characteristics. This is a piece of indirect evidence that the Alfvénic perturbations launched from the magnetopause propagate into a limited region in the magnetosheath.

Because several numerical calculations [Wu, 1992; Lyon, 1994; Yan and Lee, 1994; Berchem et al., 1995] and observations from the Venus magnetosheath [Luhmann, 1995] failed to confirm the slow compressional front, some skepticism about the existence of the slow shock front has arisen. Skepticism further arose because of various theoretical concerns, e.g., Landau damping of the slow mode and the possibility that the density enhancements observed in the magnetosheath could come from solar wind irregularities. The latter issue is particularly difficult to resolve: a successful test of the shock jump conditions across a front in the magnetosheath [Song et al., 1992] does not exclude the possibility of a density enhancement from the solar wind because a jump condition test does not provide information about the source of a discontinuity. Moreover, the solar wind plasma and magnetic field, which provide the input boundary conditions, vary in time, sometimes suddenly, and the size of the obstacle varies in response to the changing dynamic pressure of the solar wind flow. The position of a satellite relative to the two boundaries alters in response to these solar wind changes. The temporal variations in the in situ observations cannot be simply interpreted as spectral. To isolate the magnetosheath processes from these dynamic upstream variations is extremely challenging but becomes a crucial issue if one is to conclusively identify a new shock front in the magnetosheath. If such a standing shock does exist, our understanding of how a magnetized collision-free fluid changes its properties as it flows around an obstacle will be altered significantly.

1.3 Wave Modes

There has been some debate regarding wave modes in recent years. Because we will frequently use "wave modes" to discuss physical processes in this and the companion papers, we clarify the terminology we use. We use a "mode" to describe the characteristic relationship among different quantities of a perturbation. The perturbation is not necessarily periodic and can be incremental. The relationship describes the property and function of the perturbation. We concentrate on underlying physical functions of perturbations instead of terminology. The definition of modes is different in homogeneous and inhomogeneous plasma theories and different in MHD and kinetic theory. We adopt the definition of homogeneous MHD plasma theory. The homogeneity may be in general valid locally. The linear dispersion relations of MHD modes are the same for incremental perturbations and for oscillations [Kantrowitz and Petschek, 1966]. In particular, for incremental slow mode perturbations, Landau damping does not occur because the Landau damping arises in periodic perturbations. We refer to a mode of a perturbation according to the diagnostic perturbation relations of the MHD mode. Namely, an Alfvén mode perturbation has no variation in the field strength and density, and a fast (slow) mode perturbation has the density and field strength varying in phase
(antiphase). We do not require that the phase velocity obey accurately the linear theory. With this definition, we can decompose, at least for the purpose of organizing the line of thought, the processes in the magnetosheath that accomplish the complicated field and flow reconfigurations as indicated in Figure 1. For example, as Song et al., [1992] pointed out, the fast mode is most efficient for transmitting pressure variations perpendicular to the field, the Alfvén mode can easily bend the field without compressing it, and the slow mode can convert the pressure variations from perpendicular to the field to parallel to it. Here we want to further point out that the MHD description is equivalent to the phase space integration of kinetic theory and is required for macroscopic force balance. To describe a general field perturbation (which is not necessarily periodic), all three MHD modes, which complete a set of three orthogonal perturbations, are required. The notion that because of the strong Landau damping of the (periodic) slow mode in the kinetic theory, the slow mode does not exist, is unlikely because without the slow mode, some required perturbations cannot be described. Although all three modes are expected to act throughout the magnetosheath, in each region in the magnetosheath a single mode may be dominant. We notice that in our definition, some intermediate shocks will be classified as the slow mode. In this series of papers, we focus on incremental perturbations and not periodic waves, which have been debated over the last few years [Song et al., 1994; Anderson et al., 1994; Denton et al., 1995]. We believe that after we understand the major large-scale physical processes in the magnetosheath, the periodic waves can be discussed and placed in a proper context. Then the differences in the debate can be resolved.

The present work describes a new method to study the magnetosheath. It provides a systematic means of separating the perturbations due to the solar wind variations from those generated within the magnetosheath. As a result, we will be able to study the magnetosheath processes as well as the dynamic solar wind-magnetopause interaction. In the following, in section 2 we first briefly describe the Gasdynamic Convected Field Model (GDCF M) upon which our new approach is based, and provide some discussion of the approximations made in deriving the model and on their consequences. These consequent processes become important when we try to understand what causes the differences between the model prediction and observation. Technical details of numerical implementation of the model can be found elsewhere [Spreiter and Stahara, 1980, 1985]. In section 3 we discuss the physical reasoning for the implementation of our method to compare the model prediction with observation. We then, in section 4, describe the normalization procedures that we have developed in order to compare the model prediction and observation and to isolate the magnetosheath processes from the solar wind variations. Finally, in section 5 we discuss where and why we expect to see when observation and model prediction are systematically different. Several test cases are presented in detail in a companion paper [Song et al., this issue]. A statistical study will be reported later.

2. Gasdynamic Convected Field Model

Although the ideal model with which to compare observations is a time-dependent global simulation, computer simulations have been limited by computing capability. To date, none of the various isotropic MHD models is able to provide full three-dimensional solutions with adequate temporal and spatial resolution for our problem. The GDCF M, on the other hand, provides a simplified description of the magnetosheath with high spatial and temporal resolution and a much shorter computing time.

2.1 Basic Equations

The GDCFM treats the solar wind as an ideal gas flow and the IMF frozen in the flow. The basic equations solved in the GDCF M are

Ideal gas

\[ p = NkT \]  

(1a)

Mass conservation

\[ \nabla \cdot \rho v = 0 \]  

(1b)

Momentum equation

\[ \rho (v \cdot \nabla) v = -\nabla p + \rho \mu_0 \nabla \times B \]  

(1c)

Energy conservation

\[ \nabla \cdot \left[ \rho \left( \frac{v^2}{2} + \frac{\epsilon}{\rho} + \frac{E}{\rho} \right) \right] = 0 \]  

(1d)

Maxwell equations

\[ \nabla \cdot \mathbf{B} = 0 \]  

(1e)

\[ \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \]  

(1f)

where \( \rho, v, p, k, T, N, \epsilon, q, \) and \( \mathbf{B} \) are the mass density, velocity, thermal pressure, Boltzmann constant, temperature, number density, internal energy (\(-C_vT\)), heat flux, and magnetic field, respectively.

The plasma parameters, \( \rho, v, p \) and \( T \), can be solved from equations (1a) to (1d) when \( q = 0 \). The magnetic field \( \mathbf{B} \) can then be solved from equations (1e) and (1f) when \( v \) is known. In fact, combining equations (1a),(1e), and (1f) yields

\[ \frac{\mathbf{B} - \mathbf{B}_0}{\rho_0 \Delta s} = \nabla \times (\mathbf{v} \times \mathbf{B}) \]  

(1g)

Equation (1g) is equivalent to [Landau and Lifshitz, 1960]

\[ \frac{\mathbf{B}}{\mathbf{B}_0} = \frac{\rho}{\rho_0} \frac{\Delta s}{\Delta s_0} \]  

(1g')

where \( \Delta s \) is the length of an element of a flux tube. One then can follow the element using the density and velocity to derive the magnetic field.

In comparison, standard isotropic ideal MHD equations are

Ideal gas

\[ p = NkT \]  

(2a)

Mass conservation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho v = 0 \]  

(2b)

Momentum equation

\[ \rho \frac{\partial v}{\partial t} + \rho (v \cdot \nabla) v = -\nabla p + \frac{\nabla B^2}{2\mu_0} + \frac{1}{\mu_0} \nabla \times (\mathbf{v} \times \mathbf{B}) \]  

(2c)

Energy equation

\[ \frac{\partial}{\partial t} \left( \rho \frac{v^2}{2} + \rho \epsilon + \frac{E}{\rho} \right) + \nabla \cdot \left[ \rho (v \cdot \nabla) v \right] + \nabla \cdot \left( \rho E + \frac{v^2}{2} + \frac{E}{\rho} \right) + q = 0 \]  

(2d)

Maxwell equations

\[ \nabla \cdot \mathbf{B} = 0 \]  

(2e)

\[ \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \]  

(2f)
\[ \mathbf{V} \cdot \mathbf{B} = 0 \quad (2e) \]
\[ \mathbf{V} \times (\mathbf{v} \times \mathbf{B}) = \frac{\partial \mathbf{B}}{\partial t} \quad (2f) \]

where \( \mu_0 \) and \( S = (B^2/2) + (F_\text{sh})/\mu_0 \) are permeability in vacuum and the Poynting flux. Comparing the basic equations of the GDCF model with the ideal MHD equations, the first approximation made in the GDCF model is the gasdynamic approximation that neglects the effects of the magnetic field in the momentum and energy equations. This requires \( C_\text{m} < C_\text{s} \) and \( \kappa = C_\text{m}/C_\text{s} \) where \( C_\text{s} = \sqrt{B/(\mu_0 \rho)} \) and \( C_\text{m} = \left( \frac{P}{\rho \gamma} \right)^{1/2} \) are the Alfvén speed and sound speed, respectively, and \( \gamma \) is the ratio of specific heats. Thus both the plasma beta \( (2\mu_0 \rho B^2)^{1/2} \) and Alfvén Mach number \( (\mathbf{V}/C_\text{m}) \) need to be much greater than 1.

Most of the time at 1 AU, the solar wind has a plasma beta of order unity, and moves relative to the earth at a speed equivalent to a sonic Mach number 7, for which the plasma heating at the bow shock is much greater than the thermal compression. Downwash of the subauroral bow shock, the plasma beta is typically much greater than 1 and the flow velocity is slightly below the magnetosonic velocity and hence significantly greater than the Alfvén speed. Therefore the gasdynamic approximation is in general valid in this region. Away from the Sun-Earth line, the plasma heating may not be as strong as near the nose. Depending on the solar wind conditions, the gasdynamic approximation may or may not be good. Close to the magnetopause near the stagnation region, the flow velocity is usually small, and the plasma beta is of the order of 1 as the result of the plasma depletion and field piling up. The conditions for the gasdynamic approximation deteriorate.

In order to improve the model application, we combine the magnetic forces in equation (2c) with the thermal pressure force and let the sum of the two forces be the pressure force in equation (1c). In doing this, we replace the sonic Mach number in the model with the fast mode Mach number \( M_\text{f} = \mathbf{V}/C_\text{m} \) where \( C_\text{m} \) is the magnetosonic fast mode velocity, \( 2C_\text{m}^2 = (C_\text{s}^2 + C_\text{d}^2) + [(C_\text{s}^2 + C_\text{d}^2) - 4C_\text{s} C_\text{d} \cos \omega B]/2 \), and \( \cos \omega B = B_z/B \) is the cone angle of the field. In 1-D and in high beta plasmas or for nearly perpendicular propagation, the equivalent pressure is \( p_{\text{GDCF}} = p + (B^2 \sin \theta_0)/2 \mu_0 \). Therefore using the fast mode Mach number to replace the sonic Mach number in the GDCF model, the time-independent terms in equations (1c) and (2c) are identical in the regions of high plasma beta or where the field is nearly perpendicular to the propagation direction. The GDCF model should then adequately describe a much larger parameter range and a large spatial region. For example, when \( B_z \) is zero, the magnetosonic fast mode velocity equals the fast mode velocity for perpendicular propagation, which is the square root of \( (C_\text{s}^2 + C_\text{d}^2) \), and the curvature force, the last term in equation (2c), goes to zero.

The second approximation in the GDCF model is the steady state assumption. With this approximation, variations on timescales much shorter than the intermediate of \( L/C_\text{m}, L/v \), and \( L/C_\text{d} \), where \( L \) is the spatial scale of interest and \( C_\text{m} \) is the magnetosonic speed \( (C_\text{s}^2 + C_\text{d}^2)^{1/2} \), are not included. Noting that the three characteristic velocities change throughout the magnetosheath, the affected temporal scale varies. Typically the three velocities in the dayside magnetosheath are about 50 to 200 km/s. Given the thickness of the magnetosheath to be about 3\( R_E \), the flow time and Alfvénic transient time are about 1 min, and the magnetosonic transient time is significantly shorter. Therefore temporal variations longer than 5 min should be adequately represented by the model.

A further approximation is the adiabatic assumption, or \( q=0 \). Equation (1d) becomes

\[ p = \alpha p' \quad (3) \]

where \( \alpha \) is constant but different in the solar wind and magnetosheath. The change of \( \alpha \) at the bow shock is determined according to the shock jump conditions. Here we recall that although it is dissipative at the bow shock, the heat flux can often be treated as zero which is used to derive the Rankine-Hugoniot relations. When we use the fast mode speed to replace the sound speed in the GDCF, a non-self-consistency is introduced. The non-self-consistency becomes most significant in the state and energy equations. The physical quantity affected most by the non-self-consistency is the temperature.

The comparison above is based on ideal single fluid models. Viscosity and electrical resistivity may become important near the magnetopause boundary. For an electron-proton two-fluid, the plasma temperature should be defined as the sum of the electron and proton temperatures. When the solar wind electron temperature is not available, we use the observed median electron temperature, \( 1.4 \times 10^3 \text{K} \), as the electron temperature in the model, since the observed electron temperature at the Earth’s orbit is not correlated with the ion temperature [Newbury et al., 1995]. The solar wind helium concentration is important because it increases the effective mass. A content of 5% \( \text{He}^+ \), the observed average \( \text{He}^+ \) concentration [Fieldman et al., 1977], increases the effective mass by 15%, which is used in the standard GDCF calculation although this value is adjustable or can be replaced with the simultaneous observed value when available. We assume that the \( \text{He}^+ \) temperature is 5 times the proton temperature based on observation [Fieldman, 1977]. The 5% of \( \text{He}^+ \) particles translates to 20% in the total ion temperature. In the magnetosheath, the electron temperature is often one-tenth to one-eighth the ion temperature and is neglected when comparing with observations.

The deviation of the plasma distribution functions from Maxwellian may result in phenomena that are not described by fluid models. These phenomena usually have shorter timescales than those appropriate to the GDCF because the model is quasi steady state.

### 2.2 Upstream Conditions

The upstream boundary is taken as the plane normal to the solar wind flow and containing a solar wind monitor based on which a prediction is made. The solar wind flow direction is aberrated from the Sun-Earth line by \( \lambda = \tan^{-1}(V_\phi/V_\rho) \) where \( V_\rho \) (\( \sim 30 \text{ km/s} \)) is the Earth’s orbital velocity. The upstream boundary conditions are uniform and specified according to the measurements from the solar wind monitor. As the solar wind changes, the boundary conditions change. The temporal variations at the upstream boundary seemingly introduce a time dependence of the model output. However, the fundamental difference between this time dependence and that solved from temporally varying basic equations is that the output given by the GDCF model has no history relating to the previous solution and has no effect on the following solution. The solution of a quantity \( t_{\text{GDCF}} \) depends solely on the upstream boundary condition at \( t_{\text{sw}} \) and

\[ t_{\text{GDCF}} = t_{\text{sw}} + \int_{s v}^{t_{\text{sw}}} \frac{dt}{v} \]

where \( dt \) is a small segment along the streamline on which the point of magnetosheath prediction is located, and the integration is along the streamline from the upstream boundary to the prediction point. The model contains the information
about the solar wind variation but not its evolution and dynamical interaction with the bow shock and magnetopause.

2.3 Magnetopause Boundary Conditions

The magnetopause is considered to be an impermeable solid body. Therefore the flow velocity is tangential to the surface at the magnetopause. The location of the subsolar magnetopause $R_{nose}$ is determined by

$$K\rho_m v_w^2 = \frac{B_{nose}^2}{2\mu_0}$$

where $B_{nose} = 2B_E/R_{nose}^3$ and $B_E$ is the Earth’s magnetic field at the equatorial surface. The constant $K$ reflects the pressure reduction at the stagnation point from the solar wind value due to the deceleration and diversion process and is 0.88 for $\gamma = 5/3$ and for moderate to high Mach numbers [Spreiter et al., 1966].

The shape of the magnetopause is taken to balance the pressure of the form of $K\rho_m v_w^2 \cos^2 \psi$, where $\psi$ is the angle between the solar wind direction and the normal to the magnetopause surface. This form assumes that the magnetopause or the pressure of the magnetosphere is axisymmetric with respect to the Sun-Earth line. In reality, we know that the magnetosphere field varies in latitude [Petrinec and Russell 1995]. Since the comparison to be presented in this series of papers is at low latitudes, the problem of the asymmetry is not a concern. The $\cos^2 \psi$ dependence can be derived from the momentum balance of a cold beam incident upon a reflecting surface. Since the solar wind is significantly heated at the bow shock, one may question the validity of the cold beam description. Nevertheless, this form is consistent with gasdynamic semi-empirical expression for a supersonic flow [e.g., Spreiter and Stahara, 1985]. Petrinec and Russell [1996] have recently investigated the pressure distribution along the magnetopause surface and Shue et al. [1997] introduced a new functional form to better model the shape of the magnetopause. Their results may be used in our future studies.

In the GDCF model, the location and shape of the magnetopause change in unison as the upstream pressure varies. The model magnetopause oscillates but is not in wave motion. Also note the assumption that the magnetopause changes instantaneously as the solar wind strikes. In reality, magnetospheric response time may regulate the magnetopause motion, in particular in the regions away from the subsolar point.

2.4 Bow Shock Jump Conditions

The GDCF uses an implicit Euler equation solver to solve the asymptotic solutions of the equations with a time-marching procedure. The remainder of the flow field is determined by a shock-capturing marching procedure that spatially advances the solution downstream as far as is required by solving the steady Euler equations [Spreiter and Stahara, 1980].

3. Scheme Implementation

3.1 Comparison Scheme and Time Shift

The scheme used to compare the observation with the GDCF is illustrated in Figure 3. In this scheme we use the time series of the solar wind/IMF measurements to specify the upstream boundary conditions of the GDCF model. The model calculates $\rho, v, p, B$ and $B$ as they convect in time and space. In principle, one should compare a observed quantity at the prediction time $Q_{os}[r_{ic}(t_{GDCF}), t_{GDCF}]$ with the prediction at the spacecraft location $Q_{GDCF}[r_{sc}(t_{GDCF}), t_{GDCF}]$. To obtain $t_{GDCF}[r_{sc}(t_{GDCF})]$ needs reiteration and is time consuming. Since the spacecraft moves slowly in the magnetosheath, $r_{sc}$ is fixed for each case. The error in the prediction time due to the fixed $r_{sc}$ is $\Delta t = \Delta l/\nu$, where $\Delta l$ is the difference in the lengths of the streamlines at two comparing sheath locations. If the magnetosheath flow velocity is 200 km/s and the spacecraft travels $3 R_E$ in the x direction during a pass, the error in $t_{GDCF}$ due to the fixed $r_{sc}$ is less than $\pm 1$ min.

The time shift changes in a systematic manner with the change in solar wind velocity.

In the GDCF model, the upstream boundary is approximated as uniform in the plane normal to the solar wind flow. In fact solar wind discontinuities, for example, are often oriented at an angle to this plane. On one hand, the magnetosheath plasma is formed by the solar wind within $15 R_E$ from the solar wind-Earth line. On the other hand, as seen from the Earth, solar wind monitors are often located more than 10° away from the Sun-Earth line to avoid solar radio noise in order that the

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**Figure 3.** The scheme using the GDCF to study the steady state magnetosheath processes. Solar wind data and the location of the magnetosheath satellite are input to the GDCF. Its output prediction is compared against the magnetosheath observations. The differences between the two provide a measure of the effects of the magnetic force.

**Adjustable Parameters:**
- Convection Time
- Size of the Magnetosphere
- SW Temperature
ground stations receive the radio transmissions. Therefore the arrival time of a discontinuity could be different from that predicted by equation (4) if the solar wind monitor is located 200R_e upstream. In practice, we add a free parameter in equation (4) to account for this effect. For each pass, this parameter is unchanged. This assumption is equivalent to assuming that the solar wind discontinuities during the 2 to 3 hours of the comparison are parallel. If several solar wind discontinuities having completely different orientations occurred on the same pass, the method for the time shift may not work. Fortunately, we have found that this is not usually the case. For a pass, one is able to find a most prominent change to align the prediction with observation. If other changes appear to have slightly different time shifts, we interpret these changes as due to propagating of the discontinuities with respect to the most prominent one based on which the time shift for the pass is determined.

3.2 Magnetopause Location

The magnetopause location determined from equation (5) and the dependence of the shape on cos²ψ do not take into account the effects of the IMF (both magnitude and direction), magnetospheric thermal pressure, and the source of plasma, field-aligned current, ring current, and tail current. The real magnetopause may be either farther from or closer to the Earth than the prediction. In practice, we introduce a factor, ~1, multiplied by R_m in order to align up the predicted and observed magnetopause. The systematic behavior of this factor can provide important information about the size of the magnetosphere as functions of other parameters (other than the ram pressure), such as IMF orientation. Note that this adjustment changes the size of the magnetosphere but does not significantly change the shape of the magnetopause. When a magnetopause crossing occurs in the flank, the adjustment represents the combination of the location and shape differences. Finally, it is worth mentioning that because the GDCFM assumes axisymmetry along the stagnation streamline, the effects associated with the dipole tilt, seasonal variations are not included.

In the presence of active reconnection near where the satellite crosses the magnetopause, the flow has a nonzero normal velocity, and the approximation of an impermeable boundary may not be valid. Under this situation, one may argue that the effective obstacle is located somewhere earthward of the real magnetopause. However reconnection does not take place uniformly over the magnetopause, so the effective shape and size will both change. Thus it is not trivial to take into account the effect of reconnection on the structures of the magnetosheath.

3.3 Bow Shock Location

After the time shift and magnetopause location are determined, in principle, there is no freedom to adjust the predicted bow shock location in the GDCFM. In gasdynamics, the thickness of the sheath is controlled by the upstream Mach number. The sheath thickness is inversely related to the Mach number [Spreiter et al., 1966]. Even when using the fast mode Mach number to replace the sonic Mach number, a replacement that lowers the Mach number and thickens the predicted sheath, the predicted sheath is often slightly thinner than that observed. Moving the bow shock outward with a lower Mach number in turn weakens the compression at the bow shock. This leads to lower predicted values for the sheath density, field strength, and temperature. As we discussed in the last section, due to the non-self-consistency in the energy equation, the temperature prediction is affected most. Also as we will see in the case study [Song et al., this issue], the predicted temperature is often lower than the observed temperature. To adjust further, we multiply the solar wind temperature by a factor so that the predicted temperature and thickness increase at the same time.

An alternative way to adjust the bow shock location is to use different γ values by arguing that the process has only two degrees of freedom (for γ = 2) or the heat flux is significantly present (for a smaller γ). In general, a greater γ results in a thicker sheath, a weaker density compression, and greater heating. This possibility will be investigated in a later study.

3.4 Field Strength Limiter

The stagnation point is a mathematical singularity [Spreiter and Alkane, 1969; Sonnerup, 1980]. This problem can be seen clearly from equation (1g'). For the flux tube that drapes over through the stagnation point, the portion of the flux tube at the stagnation region cannot move, and the two ends in the solar wind continuously stretch, in principle, to infinity. Thus the field strength can become infinitely strong unless the density goes to zero. In practice, we add a limiter that allows the field strength to equal the predicted magnetospheric value. The operation improves the overall look of the prediction, but it is not physically well based.

3.5 Intercalibration.

The accuracy of the GDCFM prediction critically depends on the accuracy of the solar wind measurements. The differences appearing in a comparison can also be caused by the errors in the calibration of the sheath measurements. Therefore a comparison requires high accuracy of every instrument involved. Intercalibration among the measurements using other techniques is important. Systematic differences shown in comparisons using different satellites and instruments may also provide some information about the calibration of an instrument.

4. Normalizations

A major objective that motivates the development of the new scheme is to find a systematic way to remove the effects of the solar wind/IMF temporal variations as discussed in the introduction. We have developed the following normalization procedures to accomplish this goal.

4.1 Normalized Distance

A satellite moves relatively slowly across the magnetosheath. A typical day-side magnetosheath crossing lasts about 2.5-5 hours. During this period, the upstream conditions may experience many changes. Variations in the solar wind dynamic pressure and the north-south component of the IMF will move the magnetopause location, and a varying Mach number, due to a change either in the velocity or in the temperature, will lead to the motion of the bow shock. When the solar wind changes its direction, the orientation of the magnetosphere may also tilt accordingly. Therefore during a traverse, the satellite's location relative to the two boundaries may vary suddenly. We define the normalized distance of a satellite relative to the magnetopause as Δ/D, where Δ is the distance of the satellite from the magnetopause as predicted by the GDCFM and D is the thickness of the magnetosheath predicted by the model. Both the distance and the thickness are defined along the radial direction from the
Earth at the solar zenith angle of the satellite. There are two issues to be noted here. First, the radial distance is not an ideal quantity to represent the thickness. A more accurate definition should be the distance normal to the boundaries. However, as the magnetopause and the bow shock have different shapes, it is difficult to operationally derive such a quantity. Second, since a satellite orbit is not exactly radial to the Earth, during a pass, the satellite may go through a finite range of the solar zenith angle. Our calculation includes the change in the solar zenith angle. The normalized distance is zero at the magnetopause and one at the bow shock if the model prediction is perfect. Otherwise, the observed bow shock may be located at either >1 (when the predicted sheath is too thin) or <1 (when the predicted sheath is too thick).

4.2 Normalized Density

The magnetosheath density depends on the upstream density, upstream Mach number (which determines the level of compression), and location relative to the boundaries. We define the normalized density as the ratio of the observed to predicted densities, \( N_{obs}/N_{GDCF} \). Noting that the density prediction is based purely on the gasdynamics, the deviation of the normalized density from 1 provides a measure of the effects that are not included in the GDCF model. In particular, since gasdynamics does not predict much postshock density compression in the dayside (less than 10% within 30° solar zenith angle, see Figure 4), the normalized density can be used directly to compare with (less than 10% error) with the density profiles along the stagnation streamline from other theoretical models as shown in Figure 2.

4.3 Normalized IMF Polar Coordinate System

As we will see in the companion [Song et al., this issue], the direction of the sheath magnetic field is among the best predicted quantities. The asymmetry (from the axisymmetry about the Sun-Earth line) of the sheath processes is most likely to be best organized according to the stagnation sheath field [e.g., Sonnerup, 1980]. In a steady state it is possible for the sheath quantities to be symmetric with respect to the IMF direction. When the IMF rotates, a satellite along its trajectory samples different regions with respect to the field direction. We define an IMF polar coordinate system in which the \( X' \) is the same as the aberrated \( \Delta X_{GSE} \) direction, the \( Z' \) is along the IMF direction projected on the YZ GSE plane, and the \( Y' \) completes the right-hand system. The satellite location in this new coordinate system is

\[
\begin{pmatrix}
X' \\
Y' \\
Z'
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
Y_{GSE} \\
Z_{GSE}
\end{pmatrix}
\] (6)

where \( \phi = \tan^{-1}(B_y/B_x) \) is the clock angle of the IMF. The polar angle of the satellite with respect to the IMF is \( \chi = \tan^{-1}(Y'/Z') \). The coordinate system provides the normalization of the satellite's location according to the IMF direction. With this normalization, we will be able to discern the effects of the IMF direction.

4.4 Length of Flux Tube

The field strength is one of the most important parameters in our comparison. However, as we discussed in the last section, the field strength cannot be adequately handled near the magnetopause boundary. Neither an extremely large value nor an artificially limited value is useful to gain any physical insight. Therefore we will not provide the normalized field strength. Among the most important concepts of the frozen-in field is that the length of the magnetic field, or the length of a flux tube, is proportional to \( B/p \) as described by equation (1g'). Therefore quantity \( B/p \) provides a measure of the change in the length of flux tubes. On the one hand, the idea of the length of a flux tube is derived following the motion of a flux tube, i.e., along a streamline. On the other hand, a satellite's orbit is often not along a streamline. Therefore the observed variations in \( B/p \) cannot be interpreted directly as stretching/shortening of the same flux tube. Instead, it compares the length of flux tubes which are of the same length in the upstream solar wind if the solar wind conditions remain unchanged and are uniform at the upstream boundary. For a single flux tube, it is possible that a portion of it is stretching while another portion is shortening. In particular, because the information propagates along the flux tube with a finite speed, segmentation of the flux tube may occur at places where the flow speed is in transition with respect to a characteristic speed. A flux tube may have undergone several stretching and shortening processes before it is observed and the comparison is made about the length when and where it is observed. Its history can be traced back where another measurement is made (at a different path, for example). To remove the effects of solar wind changes, we define the normalized length of flux tube as \( (B/p)(B_{IMF}/p_{IMF}) \).

4.5 Normalized Flow Direction

The flow direction can be defined as

\[
\theta = \tan^{-1}(V_X/V_Y)
\] (7)

where \( V_X \) is the component perpendicular to the Sun-Earth line. The normalized flow direction can be defined as

\[
\Delta \theta = |\theta_{obs} - \theta_{GDCF}|
\] (8)

when \( \Delta \theta \) is greater (smaller) than zero, the flow diversion is more (less) than predicted.

4.6 Temperature Normalization

There exist some significant differences between the observed and model predicted temperature because of the non-self-consistency in the energy equation in our method. We will defer its discussion to later studies.

5. Expectations

Systematic differences between the observations and GDCF predictions are most likely due to the physical processes that are not included in the GDCF model. Figure 4 shows the solution of the GDCF model under a steady solar wind. The sheath plasma is compressed and decelerated along the normal near the nose, within 30° solar zenith angle, and expanded and accelerated tangentially toward the flanks. The direction of the flow is essentially axially symmetric and diverted away from the Sun-Earth line. The geometry of the magnetic field is more complicated.

5.1 Wave Modes

What wave modes are neglected by the gasdynamic approximation? Let us examine equation (2e) for the moment. Roughly, the balance between the second term on the left and
the first term on the right is provided by the sonic mode. The second term on the left and the third term on the right describe the Alfvén (or intermediate) mode. The first and second terms on the right have two possible relationships. They vary in phase or they vary out of phase. The in-phase case, combined with the second term on the left, is the fast mode; and the out-of-phase case, combined with the last term on the right, is the slow mode. Since the physics of the fast mode is similar (by using a different definition of the Mach number as we discussed in section 2) to that of the sonic mode, the omission of the magnetic force effectively neglects the Alfvén and slow modes. Therefore we expect that any significant systematic differences between the GDCFM prediction and magnetosheath in situ observations should be associated with the Alfvénic and slow mode processes neglected by the GDCFM. The slow mode process is particularly important near the stagnation region in which the second term on the left is extremely small and the balance has to be reached by the right-hand-side terms. In the terrestrial magnetosheath, the plasma beta is significantly higher than 1. Under this condition, the speeds of the Alfvénic and slow modes are similar. If both exist, the Alfvénic and slow standing fronts may not be spatially distinguishable.

As we discussed in section 2, the steady state approximation of the GDCFM neglects nonstanding waves. Combining with the discussion above, we conclude that the GDCFM does not include the standing Alfvén and slow fronts, and time dependent processes faster than 0.01 Hz in the sheath. The fast time dependent processes include oscillatory waves, such as mirror-wave and slow-mode oscillations, processes associated with the foreshock and upstream waves, and transient phenomena originated at the magnetopause such as the flux transfer events [Russell and Elphic, 1978]. The timescale of these variations is less than 5 min.

5.2 Flux Tube Stretching/Shortening

The change in the length of a flux tube, from equation (1g'), is

$$\frac{d\lambda}{\lambda} - \frac{d\phi B}{B} - \frac{dp}{p}$$

Among different MHD modes, the Alfvén mode does not stretch/shorten a flux tube because it does not change either $B$ or $\rho$. The fast mode is less efficient in changing the length of a flux tube because the right-hand-side terms of equation (9) act against each other when the two variations are in phase. One can easily show that for perpendicular propagation the fast mode cannot stretch/shorten a flux tube at all. For parallel propagating fast modes, a flux tube is shortened when it is compressed (density increases), since the density fluctuation is always greater than the field strength fluctuation. The slow mode is most efficient in stretching or shortening flux tubes. A slow mode compression front, where the density increases and the field strength decreases, indicates a shortening process, and a slow mode expansion fan or depletion layer indicates a stretching process.

Stretching and shortening are associated with changes in the velocity along a flux tube. On the dayside, the slowdown of the sheath flow toward the magnetopause boundary squashes flux tubes but does not necessarily stretch them (the component along the flow is shortened). The diversion of the flow away from the stagnation streamline, on the other hand, stretches flux tubes while it bends them, as illustrated in Figure 1. On the nightside, the stretched portions of a flux tube will shorten while straightening. It is possible that a portion of a flux tube is stretching while another portion of the same flux tube is shortening. The shortening (stretching) occurs where two neighboring points on the same flux tube converge (diverge).

In the GDCFM, the sheath flow axisymmetrically diverges with respect to the Sun-Earth line. On the dayside, this model includes a monotonic stretching process although the stretching is not performed in a self-consistent manner. The density change is determined by the gasdynamics and is small. Therefore the stretching leads to a large increase in the field strength which eventually causes the failure of the model near the subsolar magnetopause. The reason for which the GDCFM fails near the stagnation point is that the flux tube that hangs on the nose can be stretched limitlessly. The velocity along the flux tube diverging from the subsolar point increases as the distance increases from the subsolar point, because the flow velocity is predetermined without taking into account of the magnetic force. When the magnetic field is considered, it will oppose such a rapid stretching. Therefore one expects that in practice the flow is anisotropic and not axisymmetric even in the absence of reconnection. Near the magnetopause the velocity at locations parallel or antiparallel to the IMF in the IMF polar coordinates should be less than that perpendicular.
to the IMF, on the dayside at the same distance from the subsolar point.

For comparison, the flux tube in the ZW mode is stretched as the flow diverges from the Sun-Earth line and moves along the flux tube. The model does not allow bending and shortening of the flux tube. Therefore, it does not describe the Alfvén mode and contraction. It describes a continuous, slow mode stretching process. In two-dimensional MHD simulations [e.g. Yan and Lee, 1994], a flux tube cannot be stretched significantly because it cannot be stretched in the third dimension, the direction of symmetry, as required by the imposed uniformity along Z. The Y component, which is perpendicular to both the upstream flow and symmetry axis, of the field has to be zero upstream in order to avoid the piling up of the flux against the obstacle. A flux tube can only be shortened in the X direction near the stagnation line because of the slowdown of the flow. Thus a 2-D model effectively eliminates the possibility of steady state slow mode processes if the magnetopause is impermeable. This explains why the slow mode compressional front does not appear in the steady state solution of 2-D models. Without a prestretching, the shortening associated with the slow front cannot take place Therefore, 2-D simulations cannot describe some of the most important processes in the steady state magnetosheath.

6. Conclusions

We have proposed a new method to study the properties and processes in the magnetosheath. We use the solar wind/IMF measurements as the input to the GDCFM, and compare the output from the model with in situ observations in the magnetosheath. On the basis of the GDCFM prediction, we introduce three parameters to the model output that adjust the time of the arrival of the solar wind perturbations, the size of the magnetosphere, and the thickness of the magnetosheath. Because they provide the references with respect to the GDCFM, each of these three parameters has its own physical meanings and can be studied systematically. With these three parameters, the timings of the magnetopause crossing, bow shock crossing, and the solar wind perturbations in the GDCFM model prediction can match the observed ones. The location of the magnetosheath satellite at each moment is then determined with respect to the simultaneous magnetopause and bow shock. Because of the accurate handling of the timing, the magnetosheath properties can be appropriately correlated to the corresponding upstream values. The effects of the upstream variations can be removed systematically by normalization procedures. We expect that the systematic differences between the GDCFM prediction and observation, or the deviation of the normalized quantities from one, point to the physical processes that are not included in the gasdynamic description of the magnetosheath. A major missing piece of physics in the gasdynamic models is a slow mode compressional front that stands nct far from the dayside magnetopause. The application of the method is presented in the companion paper [Song et al., this issue].

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