Thermodynamic effect of the ion sound instability in the ionosphere

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Abstract. During geomagnetic disturbances when the ring current interacts intensely with the plasmasphere, the plasma of this region undergoes a strong heating due to an ion cyclotron instability. This is followed by the transfer of heat along geomagnetic field lines from the heating region to the ionosphere. One of the results of this process is the formation of a non isothermal region (in which \( T_e > 3.4 T_i \) at ionospheric heights) caused by a rapid cooling the \( \text{H}^+ \) ions due to their resonant charge exchange with neutral hydrogen. Heat transfer from the top of the flux tube to the ionosphere is investigated using a hydrodynamic model for the ionosphere-plasmasphere coupling. Field-aligned currents, present in the topside ionosphere, are often accompanied by ion sound turbulence. The turbulence scatters electrons, increasing the total electron collision frequency through wave-particle effects. The influence of wave-particle interactions introduces an anomalous component to the total collision frequency, which modifies substantially the heat conduction coefficient of the plasma. As a result, the plasma is heated more intensely above than below this region of ion sound turbulence.

1. Introduction

Until recently, the "photoelectron" model was the model of choice for simulating the thermal regime of the plasmasphere. Photoelectron models assume that the plasma temperature in the plasmasphere is totally controlled by photoelectrons produced through solar radiation in the ionosphere. In this case the highest plasma temperature in the plasmasphere is determined by the power of the photoelectron source of heating, which depends mainly on the intensity of solar radiation. For normal conditions it's value does not exceed 0.5-0.6 eV [Krinberg and Tashchilin, 1984]. Moreover, the electron temperature typically exceeds the ion temperature since the photoelectrons lose their energy more efficiently to the electrons through Coulomb collisions than to the ions [Khazanov, 1979]. According to the photoelectron model, heat transfer processes are characterized by heat conduction coefficients [Khazanov, 1979; Krinberg and Tashchilin, 1984; Oraevskii et al., 1985] which contain no "anomalous" phenomena (i.e., the exclusion of wave-particle effects).

During the past several years, however, our understanding of the processes occurring in the plasmasphere have changed drastically. This is attributable, first of all, to the discovery of the "hot zone" in the topside plasmaspheric region [Gringauz and Berzukh, 1976] and to subsequent measurements of the electron and ion temperatures by satellites GEOS [Decreau et al., 1978, 1982; Horwitz and Chappell, 1979; Comfort and Horwitz, 1981; Oils et al., 1987; Farrayia et al., 1989] and DE 2 [Kozya et al., 1986; Brace et al., 1987, 1988]. According to the experimental data, the electron and ion temperatures of the cold plasma in the plasmaspheric region, adjacent to the plasmapause, can reach several electron volts with the ion temperature greater than the electron temperature.

The ring current is thought of as the source of free energy producing this observed increase in temperature, where the mechanism for energy transfer, from the ring current to the plasmasphere, is wave-particle interactions [Kenne and Pettit, 1966; Cornwall et al., 1970, 1971; Galeev, 1975]. The thermal regime of the plasmasphere was simulated, with the inclusion of the effect of a ring current by Konikov et al. [1989] and Gorbachev et al. [1992]. Gorbachev et al. [1992] solved the hydrodynamic equations for \( \text{O}^+ \) and \( \text{H}^+ \) ions and electrons and showed good agreement between theory and observations (equatorial and ionospheric plasma temperatures).

Figure 1 (taken from Gorbachev et al. [1992]) shows the height profiles of the electron and ion plasma temperatures at times corresponding to disturbed magnetospheric conditions. The calculations employed the following values of spectral energy density for the ion cyclotron waves, which determine the power source of plasmaspheric electron and ion heating by the ring current: Alfvén waves \( W_A = 0.1 \gamma^2/\text{Hz} \) and fast magnetosound waves \( W_{FMS} = 0.01 \gamma^2/\text{Hz} \). Estimates of these parameters, for disturbed magnetospheric conditions, were made by Gorbachev et al. [1992] on the basis of the experimental data reported by Young et al., [1981] and Perraut [1982]. The dashed curve in Figure 1 corresponds to the calculations carried out in this paper and will be analyzed below.

Figure 1 shows a region near 300 km where the electron temperature is significantly larger than the ion temperature. Galeev [1975] and Gorbachev et al. [1992] have shown that the ion component of the thermal plasma during the interaction of the ring current with the plasmasphere is heated more effectively than the electron component. As a consequence, the ion temperature exceeds the electron temperature at equatorial heights, \( T_i > T_e \) (see Figure 1). However, with increasing distance from the equator,
The spatial distributions of electron and ion temperatures (continuous curves) at the end of action of disturbed ring current for \( L = 5 \) [Gorbachev et al., 1992]. The dashed curve correspond to spatial distribution of electron temperature in the presence of ion-sound plug.

The electron temperature decreases far more slowly than the ion temperature; therefore at topside ionospheric heights the electron temperature is larger, \( T_e > T_i \) (at \( h < 800 \) km). The reversal of this temperature inequality, for the equatorial \( (T_i > T_e) \) and ionospheric \( (T_e > T_i) \) regions, is associated with the different velocities at which heat escapes from the region of intense plasma heating (i.e., the equatorial zone of the plasmasphere [Kennel and Petchek, 1966; Cornwall et al., 1970, 1971] where the most intense interaction occurs between the ring current and plasmasphere). Moreover, at heights between 400 and 500 km, an abrupt decrease in the ion temperature occurs, which is associated with the resonant charge exchange of ion \( \text{H}^+ \) with neutral hydrogen. As a consequence of resonant charge exchange, the ion temperature becomes equal to the neutral gas temperature (see Figure 1).

The total electron collision frequency with charged and neutral particles determines the shape of the electron temperature profile. This electron temperature profile (see Figure 1) is generally smooth, showing no sharp gradients throughout the entire length of the geomagnetic field line. As was discussed above, the electron temperature can exceed the ion temperature by factors of 3 to 7 in an altitude range between 300 to 400 km. In this case the damping of the ion sound waves reduces and relatively small field-aligned currents will be the source of the energy of such waves. They will become unstable, leading to changes in the effective collision frequency through wave-particle effects. The "anomalous" component increases the total collision frequency causing a decrease in the electron energy flux, that drains off the ring current region to the lower ionosphere. In other words, an ion sound "plug" can develop, in the height range where \( T_e > T_i \), preventing a rapid heat sink from the ring current-plasmasphere interaction region.

We consider only instability due to currents carried by the cold ionospheric electrons, ignoring any effects due to a hot component of magnetospheric origin. As was pointed out by Kindel and Kennel [1971], the hot component will not appreciably affect the cold plasma instabilities, because of the low density and high thermal speed of hot component.

The thermal conduction effects outlined in this paper are dependent on field-aligned currents. This process, for example, plays a very important role in the modeling of the thermal structure of the polar ionosphere [Schunk et al., 1987; Ganguli and Palmadesso, 1987; Gombosi and Nagy, 1989]. We did not include this process in the calculations of the electron thermal flux, because it requires additional assumptions about the spatial distribution of the field-aligned currents along geomagnetic field lines.

2. The Ion Sound Plug

The presence of a non-isothermal region in the ionosphere and field-aligned currents can lead to the existence of ion sound waves. The dielectric permittivity for low-frequency oscillations, with \( v_i \leq \omega / k \ll v_e \) for the phase velocity, has the following form [Mikhailovsky, 1975]:

\[
e_{ii} = 1 + \frac{\omega_{pe}^2}{k^2 v_e^2} \left[ 1 - J_0 \left( \frac{\omega}{kv_e} \right) \right] + \frac{\omega_{pi}^2}{k^2 v_i^2} \left[ 1 - J_0 \left( \frac{\omega}{kv_i} \right) \right]
\]

where the Kramp function has the following asymptotic representation:

\[
J_0 \left( \frac{\omega}{kv_e} \right) = -i \sqrt{\pi / 2} \frac{\omega}{kv_e}
\]

\[
J_0 \left( \frac{\omega}{kv_i} \right) = \sqrt{\frac{k^2 v_i^2}{\omega^2 + \omega_i^2}} - \frac{3 k^2 v_i^4}{2 \omega_i^5} ...
\]

for the electrons and ions where \( v_i \) and \( v_e \) are the electron and ion thermal velocities, and \( \omega_{pe} \) and \( \omega_{pi} \) are the electron and ion plasma frequencies, respectively. We will consider ion sound waves with wave vectors parallel to the magnetic field (\( k \times B = 0 \)). The threshold for the production of ion sound waves is determined from the dispersion equation \( \text{Re} \; \epsilon_{ii} = 0 \), and from the electron and ion temperature ratio, \( T_e/T_i \). The threshold condition on the temperature ratio,

\[
T_e / T_i > 3.4
\]

occurs for ionospheric heights where the oxygen ion is the main ion component.

The frequency of ion sound waves is determined from (1) and has the form

\[
\omega_s = kv_e
\]

where

\[
v_e^2 = \frac{T_e}{m_i} \left( 1 + \frac{T_i}{T_e} \right), \quad T_e = m_e v_e^2, \quad T_i = m_i v_i^2
\]

and where \( m_e \) and \( m_i \) are the electron and ion masses, respectively.

The damping decrement is defined by the imaginary part of the dispersion relation

\[
\gamma_e = -\sqrt{\frac{\pi}{8}} \omega_s \left[ \frac{m_e}{m_i} \right]^{1/2} \left[ \frac{T_e}{T_i} \right]^{3/2} \exp \left( -\frac{3}{2} \frac{T_e}{2 T_i} \right)
\]

with wave numbers \((k)\) for the ion sound waves between 0 and \( k_d = 1/r_D \) where \( r_D \) is the Debye radius \((r_D = v_e/\omega_{pe})\).

The ring current that flows across equatorial geomagnetic field lines is partly completed through the current-conducting ionosphere by means of field-aligned currents [Liperovsky and Pudovkin, 1983]. Thus part of the ring current energy is...
dissipated at ionospheric heights. The field-aligned currents, closing within the ionospheric E layer, may act as a source of free energy for ion sound waves even for relatively small electron to ion temperature ratios, \( T_e/T_i = 3 - 6 \). For an ion sound plasma instability, in a weakly non-isothermal ionospheric region, the electron parallel velocity (\( u \)) must exceed a critical value of \( u_c \) [Mikhailovsky, 1975]. The ion sound instabilities occur for values of \( u \) lying within \( v_e (m_e/m_i)^{1/2} \leq u < v_e \). The growth rate for these ion-sound waves can be determined from (1) provided the substitution \( \omega \to \omega - k_\parallel u \) is performed in the electron part of dielectric permittivity. Hence the growth rate is found to be

\[
\gamma = \sqrt{\frac{\pi}{8}} \frac{\omega}{v_e} (u - u_c),
\]

(7)

where

\[
u_c = v_e \left( \frac{T_e}{T_i} \right)^{3/2} \exp \left( -\frac{3}{2} \frac{T_e}{2T_i} \right)
\]

(8)

For values of \( u \) and \( T_e/T_i \), taken from experiments and numerical calculations, the linear growth rate for the ion sound plasma instability can be determined; however, the values of the growth rate only determine the presence of an instability (when \( \gamma > 0 \)) but not the level of plasma turbulence.

The level of plasma turbulence, however, can be determined through quasi-linear theory. Assuming that the electron distribution is shifted in velocity with respect to the ion distribution by \( u \) (Figure 2), we suppose that a relaxation of the nonequilibrium distribution of the plasma particles proceeds in a quasi-linear manner. Quasi-linear relaxation will destroy the instability source (\( \gamma \to 0 \)). In this case the source of free energy for the instability is the directed velocity of the thermal electrons. The result of quasi-linear relaxation therefore would be to decrease the electron flow speed to values equal to or less than the critical velocity \( u_c \). Some part of the energy of the directed motion of the electrons will transform into the energy of ion sound waves.

The energy which is transferred from the resonant electrons (the shaded area in Figure 2) to the electrostatic energy of ion sound plasma oscillations is easily determined if assuming the electrons and ions are Maxwellian,

\[
W = \int \frac{m_e n}{2} f(v) d^3v
\]

\[
= \frac{m_e n}{\sqrt{2\pi} v_e^2} \left( \frac{v_e^2}{2v_e} \right)^{3/2} \exp \left( -\frac{(v_e-u)^2}{2v_e^2} \right) \int_0^\infty \exp \left( -\frac{v_e^2}{2v_e^2} \right) v_e dv_e
\]

(9)

Since \( u \geq u_0 \) and \( u << v_e \), one can obtain an estimate for the energy of the waves:

\[
W = \frac{u^3 - u_0^3}{6\sqrt{2\pi} v_e^2}
\]

(10)

The fraction of energy attributed to only ion sound waves \( W_e = |E|^2 / 8\pi \) (\( E \) being the electric field amplitude of the wave) is determined:

\[
W_e = \frac{W}{\omega R_e c_i} = \frac{\omega^2}{2\omega_p^2} W = \frac{k^2 v_e^2}{2\omega_p^2} W,
\]

(11)

where

\[
R_{ei} = 1 + \frac{1}{k^2 r_d^2} \frac{\omega_p^2}{\omega^2}
\]

(12)

for \( \omega_e \leq \omega_p \). From (11) it is evident that the main part of the energy of the ion sound waves is distributed on wave numbers near \( k \) (\( k_0 \)).

Ion sound waves, in this non-isothermal ionospheric region, are able to modify substantially the ionospheric electric and heat conduction properties. Electrons that transfer heat from the ring current region to the ionosphere, will undergo an additional scattering due to ion-sound plasmons. If the plasmon is represented as a heavy charged particle, then the electron will undergo a Coulomb scattering from the plasmon with an effective collision frequency \( \nu_p \) [Artmanovich and Sagdeev, 1979].

Since plasma electrons excite waves as a consequence of the instability, they must lose their momentum anomalously. In order to determine the effective collision frequency \( \nu_w \) of electrons with ion sound waves, we will use conservation of momentum in the "electron-wave" system. The mean loss of momentum by electrons per unit time is

\[
\frac{dP}{dt} = v_w m_e n \bar{u}
\]

(13)

where the momentum is transferred to the waves with energy density

\[
\frac{dP}{dt} = \int \gamma \frac{W_k}{v_{ke}/k (2\pi)^3} \frac{d\bar{k}}{v_{ke}/k (2\pi)^3} = \int W_k \bar{d}k = \frac{\gamma}{v_e} W_s \bar{c}_k,
\]

(14)

where \( \bar{d}k = \pi d\bar{k}^2 d\bar{k}_i \), \( \bar{c}_k = \bar{c}_k k \), and

\[
W_s = \frac{1}{(2\pi)^3} \int W_k d\bar{k}
\]

(15)

By setting both expression equal and substituting for the growth rate (equation (7)), its maximum estimated value of \( \gamma = \omega_p \nu_e / k \) for \( k = 1/\lambda_e \) one can obtain a relationship for \( \nu_w \) when \( v_{ke} >> \omega_p / k \):

\[
\nu_w = \frac{\omega_p}{\omega_c} \frac{W_s}{nT_e}
\]

(16)

From (10), (11), and (16), one can obtain an estimate (for a maximum) for the scattering frequency of the plasma electrons with ion sound waves:

\[
\nu_w = \frac{\omega_p}{\omega_c} \frac{u^3 - u_0^3}{12\sqrt{2\pi} v_e^2}
\]

(17)
In (17), the parameter $u$ is specified from experimental measurements of field-aligned currents, and $u_c$ is determined from (8) using the electron to ion temperature ratios calculated in the next section.

3. Results and Discussion

The calculations presented in this paper are based on a model for the ionosphere-plasmasphere coupling that includes the effects of a ring current on the plasmasphere. The model is based on a numerical solution of the system of hydrodynamic equations for electrons and $O^+$ and $H^+$ ions. The hydrodynamic equations, which are solved numerically for the "ionosphere-plasmasphere" system, are the continuity, momentum and temperature equations along with equations describing the transfer of thermal fluxes for electrons and ions. The right-hand sides of the equations, apart from standard expressions describe the decay and birth of particles, friction forces, electron-ion Coulomb interaction, interaction of electrons with neutral particles, and the heating of thermal electrons by photoelectrons, include the ring current interaction with the plasmasphere. We only mention the model briefly and refer the reader to a detailed description of the model and equations by Gorbachev et al., [1992]. The model represents the properties of the thermal regime of the ionosphere and plasmasphere associated with the effects of a ring current during magnetically active periods. In this case, as was previously mentioned above, a nonthermal region occurs for an altitude range near 300 to 400 km where $T_e/T_i = 3-7$.

In order to obtain the space-time distributions of $T_e$ and $T_i$ under the influence of a disturbed ring current, they must first be calculate in the absence of the ring current-plasmasphere interaction. The boundary conditions needed to solve the hydrodynamic equations, are specified at a height of 100 km, based on the fact that transport processes at these heights can be neglected [Gorbachev et al., 1991]. The temporal integration is performed until a steady state periodic solution with a period of 24 hours is obtained. The effects of the ring current on the plasmasphere were modeled by incorporating, into the right-hand sides of the hydrodynamic equations, terms describing the interaction between plasmaspheric particles and the ring current hot protons. This interaction was "turned on" by means of a step function pulse during the evening time (1800 - 0100 LT). Thus an abrupt increase in the ring current intensity during the main phase of a geomagnetic storm, and its rapid decay during the recovery phase were modeled.

The influence of ion sound waves, in the thermal regime of the electron plasma component, is modeled by including an effective collision frequency (equation (17)) in the electron flux transfer equation which can be written as [Gorbachev et al., 1990]:

$$\frac{dS}{dt} + S = \frac{16}{5} \frac{\partial V_e}{\partial s} + \frac{7}{5} \frac{V_e}{\sigma} \frac{1}{\partial s} + \frac{5 N_e T_e}{m_e} \frac{\partial T_e}{\partial s} + \frac{4}{5} v_{\text{eff}} S,$$

where $dS/dt = \partial \Theta /\partial s$, $\sigma = B/B_0$, is the cross section of the geomagnetic flux tube equal to 1 cm$^2$ at its base ($B = B_0$), $m$ is the mass, $N$ is the density, $V$ is the hydrodynamic velocity along $B$, $T$ the temperature, $S$ is the heat flux, and $v_{\text{eff}}$ representing the total electron collision frequency:

$$v_{\text{eff}} = v_{se} + 13/8 v_{si} + v_{\text{eff}} + v_W,$$

where $v_{se}$ and $v_{si}$ are the Coulomb collision frequencies for electron with electrons and ions, and $v_W$ is the total collision frequency for electrons with neutral particles such as hydrogen, oxygen and nitrogen, and where $v_W$ is the effective collision frequency determined from (17).

Kindel and Kennel [1971] were the first to point to the possibility that collective phenomena arise in the ionosphere due to current instabilities. They also assumed that field-aligned currents form part of the Birkeland current system and can reach rather large values during geomagnetic disturbances [Arnoldy, 1974]. From Cloutier et al., [1970], the electron flux, at topside ionospheric heights, takes on values between $10^9$ and $10^{10}$ electrons cm$^{-2}$ s$^{-1}$, corresponding to a directed velocity for the electrons of $2 \times 10^8$ and $2 \times 10^6$ cm s$^{-1}$, respectively. Using this data and taking into consideration that the directed velocity decreases with increasing plasma density, we have made estimates of the effective collision frequency in accordance with (17). For maximum values of the electron fluxes the effective collision frequency $v_W$ is comparable to the total electron collision frequency (electron-electron, electron-ion, and electron-neutral).

The effective collision frequency, for this case, took on typical values on the order of $10^2$ to $10^3$ s$^{-1}$ with a turbulence level of $W_s / \pi T - 10^4$ to $10^5$.

Plasma temperatures were calculated assuming an electron flux at height $h = 1000$ km of $10^{10}$ electrons cm$^{-2}$ s$^{-1}$ with a corresponding directed velocity for the electrons (see Figure 3). For smaller values of the electron flux, the ion sound instability does not develop since $u$ is less than $u_c$ in (7) (threshold condition). Figure 3 shows the height profiles for the electron temperature at different times, starting from the time at which the source was turned on (1800 LT). In the calculations we used the same parameters as Gorbachev et al., [1992] for the ion cyclotron waves which are the power source for the thermal plasma heating. Curve 1 represents the distribution of $T_e$ in the absence of the ring current interaction with the plasmasphere. Curve 2 represents the $T_e$ distribution, corresponding to the when the threshold for the appearance of an ion sound instability, for an oxygen plasma, is satisfied (equation (4)). Thus curve 2 represents the $T_e$ distribution that includes the plasmasphere heating by the ring current but does not take into account the existence of an ion sound plug in the ionosphere. Curves 3 and 4 correspond to the temperature profiles for 2400 and 0045 LT, at which the ion sound...
waves reach a maximum amplitude. In order to visualize the outcome of the effect of the ion sound plug on the height distribution of the electron temperature, curve 4 from Figure 3 is drawn by dashes in Figure 1.

From the figures it is evident that the presence of the ion sound plug at heights \( h = 300\text{-}400 \text{ km} \) modifies abruptly the height profile of the electron temperature as compared with the corresponding curves in Figure 1. This change arises due to a significant decrease in the amount of heat that drains off the top of the field line down to the ionosphere located above the plug. As a consequence of the smaller electron heat flux, the energy of the electrons is distributed more uniformly along that part of the geomagnetic field line which lies above the plug. In other words, the presence of the ion sound plug increases significantly the value of equatorial electron temperature. Note also that in the region where the ion sound plug exists, there appears a large electron temperature gradient as a consequence of the abrupt increase in the total electron collision frequency.

An increase in the electron flux (that is, an increase in the directed velocity of the electrons) leads to an increase in the effective collision frequency of electrons with ion sound waves and hence to a more effective resistance of the electron heat flux in the height range \( h = 300\text{-}400 \text{ km} \). This is accompanied by an increase of the equatorial value of electron temperature and by a decrease of \( T_e \) gradient along the field line. Moreover, there is an increase in the electron temperature gradient in the region of the ion sound plug. It should be noted that any qualitative differences in the height variation of \( T_e \) in this case, are not observed. As the calculations have shown, with a further increase in the directed velocity of the electrons, the height profile of electron temperature differs little from that presented in Figure 3. This is attributed to the fact that, with a certain level of ion sound turbulence in the ionosphere, an equilibrium is reached between the amount of energy transferred to the plasma by the ring current, and the amount of energy spent for the excitation of electron oscillations and the transformation of energy into ion sound waves.

The calculations have demonstrated that the presence of the ion sound plug in the ionosphere does not affect the height distributions for the ion temperature: this is due to the following reasons. First, the ion sound plug does not influence directly the ion component of plasma. Second, the timescale for the formation of the ion sound plug is relatively small. Third, ion heating proceeds through an energy exchange with the electrons and is a slow process due to the large mass difference between the electrons and ions.

4. Conclusions

During geomagnetic disturbances when the ring current interacts intensely with the plasmasphere, the plasma of this region undergoes a strong heating due to an ioncyclotron instability. This is followed by the transfer of heat along geomagnetic field lines from the heating region to the ionosphere. One of the results of this process is the formation of a nonisothermal region (in which \( T_e > 3.4 \bar{T}_e \)) at ionospheric heights caused by a rapid cooling the \( H^+ \) ions due to their resonant charge exchange with neutral hydrogen. Heat transfer from the top of the flux tube to the ionosphere has been investigated using a hydrodynamic model for the ionosphere-plasmasphere coupling. Field-aligned currents, present in the topside ionosphere, are often accompanied by ion sound turbulence. The turbulence scatters electrons, increasing the total electron collision frequency through wave-particle effects. We have determined the conditions for the development of an ion sound instability in the ionospheric plasma, and have estimated the effective collision frequency of the electrons with ion sound plasmons as well as taking into account the process of the interaction of the electrons with the ion sound waves in a model for the thermal regime of the plasmasphere. Estimates of the level of ion sound turbulence have been made. It has been shown that the presence of the ion sound plug prevents the heat from draining off the tops of the field line down to the lower-lying ionosphere. The calculations performed in this paper suggest the conclusion that a close interrelationship between the ring current and the processes occurring in the Earth's ionosphere exists.

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