Modeling magnetospheric plasma processes and the resulting flows would be supersonic and that the collision of these streams would result in a pair of electrostatic shocks. Schulz and Koons [1972] developed a two-step mechanism for ion trapping: first, ions are trapped due to pitch-angle scattering as a result of wave-particle interactions; when the density becomes sufficiently large, Coulomb collisions are effective in thermalizing the plasma. Schulz and Koons [1972] also suggested that shock formation was unlikely, because the interaction of the individual plasma streams with the background plasma will not excite waves, which would dissipate the energy of the ion streams. They did not consider the interaction of the plasma streams themselves. More recent simulations by Singh et al. [1986a, b] showed that in a collisionless plasma, shocks would develop if \( T_e > 3 T_i \). Since \( T_e \) is the ionopause in the conjugate ionospheres, this would require preferential electron heating. Another potential mechanism for ion trapping is perpendicular ion heating.

The purpose of this paper is to present results which have been found using a time-dependent, one-stream hydrodynamic model with three species. It is an adaptation, for closed field lines, of a model for polar wind flows developed by Gombosi et al. [1985]. This model takes into account the effects of ionization, charge exchange, recombination, collisions, heat conduction, and allows for external heat sources; it includes O\(^+\) and H\(^+\) ions and electrons. Diffusive equilibrium assumptions are used and it includes an energy equation for each species, which means that shocks can be self-consistently modeled. In an earlier version of the model [Guitter and Gombosi, 1990] energy deposition was simulated by assuming a downward electron heat flux at the equator; now this is done using an equatorially confined heat source. The results found in the present case very closely match those found with the earlier version. A flux tube with \( L = 2 \) was used because then the density at the equator is large enough so that Coulomb collisions are effective in thermalizing and coupling the streams from the conjugate hemispheres. However, due to the one-stream nature of the model, shocks must form when the streams meet at the equator and so this work is not a proper test of the ion trapping mechanism proposed by Schulz and Koons [1972]. The boundaries of the flux tube are at 200 km altitude, which is low enough so that shocks are not reflected from the boundaries. Steady-state results were found and used as initial conditions in a subsequent simulation of the effect of a 5-fold flux tube density depletion above 2500 km altitude. Although \( L = 2 \) flux tubes are not usually depleted, this study should be useful for clarifying the stages of the refilling process at higher \( L \) values.

2. Model

The model includes the time-dependent coupled continuity, momentum, and energy equations for O\(^+\) and H\(^+\) ions and the energy equation for electrons. As a result of the strong geomagnetic field only motions along magnetic field lines are important; field, line curvature effects are neglected. Quasineutrality is assumed and no field-aligned electric currents are allowed. With these constraints, the governing equations become:
Fig. 1. Steady state solution for the northern (a) and southern (b) hemispheres. The six panels display flow velocity, hydrodynamic Mach number, particle flux (normalized to a reference altitude of 1000 km), number density, pressure, and temperature profiles between 200 km and 6500 km altitude.

\[
\frac{\partial}{\partial t}(\rho u_l) + \frac{\partial}{\partial z}(\rho u_l^2) = A S_l
\]
\[
\frac{\partial}{\partial t}(\rho u_l) + \frac{\partial}{\partial z}(\rho u_l u_l^2) + A \frac{\partial}{\partial z} = A \rho_l \left( \frac{e_m}{m_l} u_l - g \right) + A \frac{\delta M_l}{\delta t} + A u_l S_l
\]
\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho u_l^2 \right) + \frac{1}{\gamma_l - 1} A u_l p_l = A \frac{\delta M_l}{\delta t} + A Q_l + \frac{\partial}{\partial z} \left( A \frac{\delta T_l}{\delta t} \right)
\]
\[
+ A \rho_l u_l \left( \frac{e_m}{m_l} u_l - g \right) + A u_l \frac{\delta M_l}{\delta t} + \frac{1}{2} A u_l^2 S_l
\]
\[
\rho_e \left( \frac{\partial T_e}{\partial t} \right) = \frac{(\gamma_e - 1) m_e}{k \Lambda} \frac{\partial}{\partial t} \left( \Lambda \frac{T_e}{\partial t} \right)
\]
\[
- a_T^2 \frac{\partial T_e}{\partial r} - \tau_e \left[ S_e + \frac{(\gamma_e - 1)}{\Lambda} \frac{\partial (A u_e)}{\partial t} \right] +
\]
\[
+ \frac{m_e (\gamma_e - 1)}{k} \left[ Q_e + \frac{\delta E_e}{\delta t} \right]
\]
\[
E_l = - \frac{1}{e_n} A \left[ \frac{\partial}{\partial t} \left( p_e + u_e^2 p_e - \sum_{i=1}^{m} \frac{m_i}{m} (p_i + u_i^2 p_i) \right) - \right]
\]
\[
- \frac{\delta M_e}{\delta t} + \sum_{i=1}^{m} \frac{m_i}{m} \left( \frac{\delta M_i}{\delta t} \right) - \left( u_e - u_i \right) S_e
\]
\[
- \frac{1}{e_n} A \left[ u_e^2 p_e - \sum_{i=1}^{m} \frac{m_i}{m} u_i^2 p_i \right]
\]
In these equations the summations run over all species, and the neutral velocity is assumed to be zero; $v_{\|}$ is the momentum transfer collision frequency of species $s$ with species $t$. The velocity-dependent correction factors [cf. Schunk, 1977; Burgers, 1969; Tanenbaum, 1967] have been approximated by one. This approximation is not expected to influence the results significantly because at the altitudes used in the model the relative drifts and momentum transfer collision frequencies are small.

The neutral atmosphere model used was MSIS-86 [Hedin, 1987]; this includes N$_2$, O$_2$, O, and H. It is important to note that this model is not symmetric about the magnetic equator. Also, in the present set of calculations the neutral atmosphere was not allowed to vary with time. The O$^+$ ions are produced by photoionization, whereas the H$^+$ ions are created by charge transfer only. O$^+$ is removed by chemical reactions with N$_2$ and O$_2$; H$^+$ is removed by charge transfer with O. The ion-ion, ion-neutral, and ion-electron collision frequencies and the heat conductivities used in this model were taken from Raitt et al. [1975].

In the model, an L=2 flux tube connects two external reservoirs, each at an altitude of 200 km; these reservoirs represent photochemically controlled regions of the ionosphere. The ions in the stationary reservoirs...
are assumed to be in chemical and thermal equilibrium with the neutral atmosphere, while the electron temperature in the reservoirs is assumed to be 1000 K. Since the reservoirs are at 200 km altitude, the neutral species densities are high enough so that downward propagating shocks are absorbed at the boundaries rather than being reflected from them. An electron heat source of $6.3 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ confined to within ten degrees of the magnetic equator is used to simulate energy deposition from the magnetosphere. For this work the model was run assuming equinoctial conditions during solar minimum (Sept. 22, 1980) at noon local time and no external ion heating.

The coupled time-dependent partial differential equations are solved using a combined Godunov scheme/Crank-Nicholson method with dimensional splitting.

3. Summary of Results and Discussion

3.1 Steady State

Figures 1a and 1b show the steady-state altitude profiles for the northern and southern hemispheres. The six panels in each figure display flow velocity, hydrodynamic Mach number, particle flux, number density, pressure, and temperature profiles between -40 and +40 degrees magnetic latitude.

Fig. 2. Solution for $t = 6$ minutes in the 5-fold density depletion case. The six panels display flow velocity, hydrodynamic Mach number, particle flux (normalized to a reference altitude of 1000 km), number density, pressure, and temperature profiles between -40 and +40 degrees magnetic latitude.
The O* flows down to replace O* which is lost in reactions with O₂ and N₂ at lower altitudes.

The temperatures of the ions and electrons are comparable for most latitudes. Tₑ is greater than T(O*) for all points on the flux tube; T(H*) is greater than Tₑ for altitudes between 1000 km and 2500 km. In that altitude region the hydrogen ions are heated as a result of frictional interactions with the O* ions and the neutral species.

3.2 Density Depletions

Next, the effect of a 5-fold density depletion was modeled. In order to simulate such conditions, the steady-state densities for all species were reduced by a factor of five above 2500 km altitude; however, the temperatures and velocities were not changed. The density depletion was also assumed to be instantaneous. This type of initial profile was used in order to simulate refilling after a strong magnetic storm; because of it, shock structures clearly develop. These structures move upward because the ion and electron gases expand into the low density regions. The model was allowed to run for three hours of simulated time.

For this case, the result was similar to a blast wave. The blast wave is the result of a strong discontinuity in the density of the medium; the disturbance moves into the low density region with a speed which exceeds the local sound speed of the upstream medium. Figures 2 through 4 show results at t = 6, 11, and 14 minutes for this case. It should be noted that positive latitudes refer to the northern hemisphere. Shocks develop which move upward, as shown by the velocity profiles of Figure 2; this figure also shows that the H* shocks are outrunning the O* shocks. This occurs because the H* ions are sixteen times lighter. The propagation speed of the H* shock is about 12.5 km/s; the propagation speed of the O* shock is initially 1.8 km/s but by 30 minutes this has increased to 2.8 km/s. The O* shock speed increases because the O* velocity increases behind the shock, which implies that the shock velocity must increase because of mass flux conservation relationships. The H* shocks are supersonic but the O* shocks are subsonic, with respect to the ion-acoustic speed (~ 11.7 km/s). The H* flow velocities approach the sonic velocity at 1 minute and
then its Mach number decreases, but the O⁺ ion gas definitely becomes supersonic with its Mach number reaching a maximum of 1.5 at 17.5 minutes. The H⁺ shocks meet at 11 minutes, as shown in Figure 3. Note the H⁺ density and pressure enhancement at the equator. Also note the H⁺ temperature increase near the equator. This figure also shows that the O⁺ temperature decreases behind the O⁺ shocks, due to adiabatic cooling. After this, the shocks reflect and the H⁺ ions start refilling from the top, as can be seen in Figure 4. The O⁺ maximum flow velocity decreases after 17.5 minutes due to increased collisions with the refilling hydrogen ions.

Figures 5 and 6 show results at t = 45, and 60 minutes in the 5-fold density depletion case. The O⁺ shocks meet at 45 minutes as shown in Figure 5. This figure also shows a strong O⁺ density and pressure enhancement at the equator and an equatorial O⁺ temperature increase. The flux profiles in this figure also show counterstreaming of O⁺ and H⁺ ions between about 1100 km and 4900 km altitude. The O⁺ shocks reflect and after this the O⁺ ions start refilling from the top; this is shown in Figure 6. These reflected shocks move down and merge with reverse shocks which are moving up from the ionospheres. By 90 minutes these have almost completely merged and are moving up. These have met and reflected by 150 minutes; thereafter the O⁺ ions refill from the top. By 180 minutes the shocks have almost decayed and there is steady refilling; there are upward H⁺ fluxes in both hemispheres. These upward fluxes are expected since the equatorial H⁺ density is still lower than the steady-state value.

When the upwelling ion streams for a particular ion species collide the temperature of that ion increases sharply at the equator. This is due to heating caused by the collision of the upwelling streams. In addition, at that time the ion flow velocity at the equator becomes very small due to the one-stream nature of the model and, hence, the density must increase.

4. Conclusion

A time-dependent, one-stream hydrodynamic model for plasmaspheric flows has been used to investigate plasmaspheric refilling. The model
Fig. 5. Solution for $t = 45$ minutes in the 5-fold density depletion case. The six panels display flow velocity, hydrodynamic Mach number, particle flux (normalized to a reference altitude of 1000 km), number density, pressure, and temperature profiles between -40 and +40 degrees magnetic latitude.
Fig. 6. Solution for t = 60 minutes in the 5-fold density depletion case. The six panels display flow velocity, hydrodynamic Mach number, particle flux (normalized to a reference altitude of 1000 km), number density, pressure, and temperature profiles between -40 and +40 degrees magnetic latitude.
includes O+ and H+ ions and electrons. Energy deposition from the magnetosphere was simulated by including an electron heat source confined to within ten degrees of the magnetic equator. A steady-state solution was found and used in a simulation of the effect of a 5-fold density depletion. Since ion streams from conjugate hemispheres are not distinguished in the model a shock pair must form at the equator when the upwelling ion streams collide; however, since an L = 2 flux tube was used for this work the equatorial density should be high enough for Coulomb collisions to be effective in thermalizing the streams. The results found matched those found using an earlier version of the model in which a downward electron heat flux at the equator was used to simulate magnetospheric energy deposition.

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