COMETARY DUSTY GAS DYNAMICS

T.I. Gombosi and A. Körösmarty

Space Physics Research Laboratory, Department of Atmospheric, Oceanic and Space Sciences
The University of Michigan, Ann Arbor, MI 48109

Abstract. A well-developed dusty cometary atmosphere extends to distances over 4 orders of magnitude larger than the size of the nucleus. Similarly, the solar wind-dominated heliosphere extends to about $10^4$ solar radii. The first part of this review explores the similarities and differences between the solar and cometary winds. The paper also presents initial results of a new time-dependent, two-dimensional dusty comet wind calculation.

Introduction

Our present, post-encounter understanding of cometary nuclei is based on Whipple’s (1950) “dirty iceball” idea, which visualizes them as chunks of ice, rock, and dust with negligible surface gravity. Whipple’s hypothesis quickly replaced the century-long series of “sandbank” models, wherein the nucleus was thought of as a diffuse cloud of small particles traveling together. As comets approach the Sun, water vapor and other volatile gases sublimate from the surface layers generating a rapidly expanding dusty atmosphere. The sublimated gas molecules (often called parent molecules) undergo collisions and various fast photochemical processes in the near nucleus region, thus producing a whole chain of daughter atoms and molecules. There is growing evidence that dust grain photochemistry, as well as gas-dust chemical reactions, also contribute to the maintenance of the observed atmospheric composition.

In the vicinity of the nucleus the gas and dust flows are strongly coupled: frequent gas-dust collisions accelerate small grains to velocities up to several hundreds of meters per second and inject them into the extensive cometary exosphere, where the gas and dust are decoupled. The expanding gas converts most of its original internal energy to bulk motion, while it also loses momentum and energy to the dust flow. At the same time the nucleus surface and the accelerating dust grains are heated by the attenuated and multiply scattered solar radiation. Most of the thermal radiation of the solid components is emitted in the 1-20 μm wavelength range, where several rotational and vibrational transients exist for the highly dipolar water molecules which have very large resonance cross sections. The resonant radiation is continuously absorbed and reemitted by the water molecules; in other words, it is trapped by the gas (Marconi and Mendis, 1986). A large fraction of the rotational/vibrational excitation energy is transformed into translational motion via molecular collisions, thus increasing the gas temperature. The higher gas temperature represents an increased source of internal energy, which eventually results in higher terminal velocities due to adiabatic expansion.

One of the most important factors influencing cometary dynamics is the “retarded” nature of gas and dust production. The radiation reaching the surface and supplying energy for sublimation must first penetrate an extensive, absorbing dusty atmosphere. Any change in the gas and dust production alters the optical characteristics of the atmosphere, thus causing a delayed (or "retarded") effect on the production rates themselves. This "retardation" makes inner coma modeling efforts complicated and time consuming.

Dust Production

The chemical composition and physical structure of the surface layers of a cometary nucleus are very important factors affecting the mass, momentum, and energy of the outflowing gas-dust mixture, as well as the relative abundances of various gas molecules. When the comet approaches the Sun, it absorbs an increasingly larger flux of solar radiation, and the vaporization rate of volatile molecules at the surface increases. Gravitational forces are very small; therefore, the vaporized gases leave the surface and form an expanding atmosphere. In this process the gas drags away some of those dust grains which have already been evacuated of their ice component (at least partially), but others may remain on the surface (or may fall back). In his original presentation of the icy-conglomerate model Whipple (1950) predicted that an inert layer of large dust particles, evacuated of the volatile component, would form an insulating crust on the surface (mantle). The thickness of the mantle varies with time because the continuous vaporization increases the thickness of the evacuated layer, and the "erosion" due to the drag of the outflowing gas decreases it. The development and thermal structure of such a mantle has extensively been discussed in the literature (Shulman, 1972; Mendis and Brin, 1977; Brin and Mendis, 1979; Horányi et al.,1984; Fanale and Salvail, 1984; Houpis et al.,1985). These models were able to predict several different mantle evolution patterns (for a detailed review, see Gombosi et al., 1986). The pre-encounter view of the mantle evolution process assumed that active periodic comets were covered with friable surface dust layers, so that a repetitive cycle appeared as the comet orbited the Sun. The prevailing view was that apart from the first approach to the
vicinity of the Sun, the mantle thickness and the total gas production rate basically followed similar curves during subsequent revolutions. As a Halley-type comet approaches the Sun the mantle thickness increases up to a critical heliocentric distance, and then it starts to decrease. By the time the comet passes its perihelion most of the mantle is blown off and it keeps eroding further resulting in additional post-perihelion brightening. When the comet again leaves the vicinity of the Sun, a new mantle is developed; this new mantle is blown off during the next perihelion passage.

Elementary gas kinetic theory has been used to calculate the gas production rate since the early work of Delsemme and Swings (1952). It was widely assumed that the sublimated gas molecules leave the vicinity of the nucleus without collisions, so that the outflow velocity could be approximated reasonably well by the local sound velocity. At the same time the typical mean free path of the gas molecules near a cometary nucleus is on the order of 10 cm - 10 m; therefore, a hydrodynamic rather than a kinetic approach is needed to calculate gas outflow rates. To resolve this problem, Gombosi et al. (1985) introduced a “reservoir outflow” model for gas production from the nucleus where the sublimating surface was replaced by a gas reservoir containing a stationary perfect gas. The surface of the reservoir was assumed to be covered by a thin layer of friable dust so that the gas could flow through it and which also “loaded” the discharging gas flow with dust grains. It was also assumed that the gas slowly diffused through the porous mantle and at every point was heated to the local mantle temperature. In this model the practically stationary gas at the top of the nucleus had a pressure which was the same as the sublimation pressure. Gombosi et al. (1985) have considered the time-dependent dusty gas outflow from such a nucleus, assuming a realistic surface and sublimating temperatures.

The Comet Halley images revealed that most of the dust production (and supposedly the gas production, too) was concentrated on several active areas on the sunlit side, which covered only about 10% of the cometary surface. Careful analyses of ground-based observations also helped to identify active spots and line sources, which turn on and off fairly randomly. All this evidence points toward a much more complex picture of cometary gas and dust production than described by our present models; it seems to be increasingly probable that thermal stresses and other effects can cause dust to rapidly developing openings (cracks) in the surface layers, which later slowly ”heal” as a new dust mantle develops. Presently there are only a few initial attempts to model such localized, random active areas (cf. Kührt et al. 1986).

Coma Dynamics

It was recognized as early as the mid-1930's that gas outflow plays an important role in cometary dust production. In early treatments of the gas-dust interaction it was assumed that the gas drag coefficient was independent of the gas parameters and that the gas velocity was constant in the dust acceleration region. In the late 1960's this very naive picture was replaced by a two-component approach, which used free molecular approximation to describe the rarefied gas flow and neglected the random motion of the dust particles. In Probstin's (1968) dusty gas dynamic treatment (which later became the prototype of such calculations), the traditional gas energy conservation equation was replaced by a combined dust-gas energy integral. This approach was later considerably refined by a series of authors (Shulman, 1972; Hellmich and Keller, 1980; Gombosi et al., 1983, 1985; Marconi and Mendis, 1982, 1983, 1984, 1986; Kitamura, 1986), but it still represents the main method of dusty gas dynamics calculations.

Modeling efforts have shown that the spatial extent of the gas acceleration region (where dust particles accelerate to about 80% of their terminal velocity) is less than about 30 cometary radii. Gas particles typically spend less than 100 s in this region, which is not long enough for any significant change in the gross chemical composition of the gas. In a first approximation a single-fluid dusty gas hydrodynamical technique seems to be adequate for describing the overall dynamics of the gas-dust interaction.

In the early dusty gas dynamic calculations, attenuation of the solar radiation field by dust grains and gas particles was neglected. Based on a spherically symmetric dust density distribution produced by an isothermal nucleus, Hellmich (1979) developed a model to calculate the transfer of multiply scattered radiation in the inner coma and to determine the energy input to the nucleus. Surprisingly, Hellmich (1979) and later Weissman and Kieffer (1981) found that the net effect of the dust on the sublimation somewhat enhances the gas and dust production because the radiative flux scattered within the inner dust coma is partially trapped and this effect overcompensates the attenuation of direct solar radiation. Marconi and Mendis (1983) published an alternative model, which considered mainly the transfer of the solar UV radiation responsible for the major photolytic processes and treated the longer wavelength diffuse radiation field only superficially. It was also pointed out by Marconi and Mendis (1986) that several rotational and vibrational transitions exist for the highly dipolar water molecular in the 1-20 μm wavelength range, where most of the dust thermal radiation is emitted. In a collisionless gas the resonant radiation is continuously absorbed and reemitted by the water molecules; in other words, it is trapped by the gas. On the other hand, collisions play an important role in the inner coma and a large fraction of the rotational/vibrational excitation energy of water molecules can be transformed via collisions into translational energy, thus increasing the gas temperature. Marconi and Mendis (1986) have included a new approximate gas heating rate describing the effects of the dust thermal radiation and obtained a significant increase in the gas temperature and velocity profiles. The details of this complicated interaction process are not adequately understood, and the application of generalized transport equations may help us to have a better insight into this significant effect.

As the vaporized gases leave the surface they drag away some of those dust grains which have already been evacuated of their ice component. The gas drag force accelerates the dust particles to terminal velocities comparable with the gas flow velocity. The mass, momentum, and energy conservation equations of the single-fluid neutral gas are the following:

\[ \frac{Dp}{Dt} + \rho \nabla \cdot \mathbf{u} = 0 \]  
(1)

\[ \rho \frac{Du}{Dt} + \mathbf{v} \cdot \rho \mathbf{g} = \mathbf{F}_{gd} \]  
(2)

\[ \frac{1}{\gamma - 1} \frac{Dp}{Dt} + \frac{\gamma}{\gamma - 1} \rho \nabla \cdot \mathbf{u} = Q_{ext} - Q_{gd} \]  
(3)
where

\[ \frac{\partial \mathbf{D}}{\partial t} = \frac{\partial \mathbf{J}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{D} \]  

(4)

is the convective derivative, \( \rho = \) mass density, \( u = \) gas velocity, \( s = \) acceleration due to the gravitational attraction of the comet, \( F_{\text{gd}} = \) gas to dust momentum transfer rate, \( Q_{\text{gd}} = \) gas to dust energy transfer rate, \( Q_{\text{ext}} = \) external heating rate. In the innermost coma where most of the gas-dust interaction takes place the radiation pressure effect can be neglected and the equation of motion of an individual dust grain becomes

\[ \frac{\partial \mathbf{V}_a}{\partial t} = \frac{3}{4\rho_a} \mathbf{p}_C \mathbf{V}_a + \mathbf{g}_c \]  

(5)

where \( \mathbf{V}_a = \) dust particle velocity. The dimensionless gas-dust relative velocity, \( s_a \), and the modified free molecular drag coefficient, \( C_D \), are (Probstein, 1968)

\[ s_a = \frac{u - \mathbf{V}_a}{\sqrt{\frac{2kT}{m}}} \]  

(6)

\[ C_D = \frac{2\sqrt{\pi}}{3} \frac{T_a}{T} + \left( \frac{2s_a^2 + 1}{s_a^2 \sqrt{T_a}} \right) e^{-\frac{4s_a^2 + 4s_a^2 - 1}{2s_a^3}} \text{erf}(s_a) \]  

(7)

where \( s_a = \) magnitude of the normalized gas-dust relative velocity vector, while \( T \) and \( T_a \) are the gas and dust temperatures, respectively. In the presence of an external radiation field the energy balance equation for a single dust particle is (Probstein, 1968)

\[ \frac{D T_a}{D t} = \frac{3}{4\rho_a} \left[ \frac{pC_H}{\sqrt{T}} + \frac{1 - A_v}{4} \rho_H \rho_a C_v \frac{1 - (A_{\text{IR}}) \sigma T_a}{T} \right] \]  

(8)

where \( C_v = \) dust specific heat, \( A_v \) and \( A_{\text{IR}} \) are the visible and infrared dust albedos, respectively, while

\[ C_H = \frac{\Gamma_a}{\gamma - 1} \sqrt{\frac{2k}{m}} \left[ 2\gamma + (\gamma - 1) \frac{s_a^2 - (\gamma - 1) \frac{T_a}{T}}{s_a^2 \Gamma_a} \right] \]  

(9)

where

\[ \Gamma_a = \sqrt{\pi} s_a^2 + \left( \frac{1}{2s_a} + s_a \right) \text{erf}(s_a) \]  

(10)

Finally, it is assumed that the dust particles do not undergo any further sublimation or fragmentation in the coma (there is recent indication that this assumption is probably violated to some extent); consequently, the dust size distribution function, \( f_a \), must obey the following continuity equation:

\[ \frac{DF_a}{D t} + f_a \nabla \cdot \mathbf{V}_a = 0 \]  

(11)

The gas to dust momentum and energy transfer rates can be obtained by integrating over all dust sizes

\[ F_{\text{gd}} = 2\pi \rho \int_0^{a_{\text{max}}} a \mathbf{C}_D a f_a s_a \]  

(12)

\[ Q_{\text{gd}} = 2\pi \rho \int_0^{a_{\text{max}}} a \mathbf{C}_D \mathbf{V}_a s_a + 4C_H \sqrt{T} \]  

(13)

It should be noted that these integrals are dominated by the momentum and energy transfer to small particles. External gas heating is mainly caused by photochemical and radiative heating/cooling processes (cf. Gombosi et al., 1986), \( Q_{\text{ext}} = Q_{\text{phc}} + Q_{\text{IR}} \). The main contribution to the photochemical heating rate comes from the photodissociation of water molecules (cf. Mendis et al., 1985)

\[ Q_{\text{phc}} = Q_0 \rho \frac{n}{d} e^{-\frac{T}{T_w}} \]  

(14)

where \( n = \rho/m \), \( Q_0 = 2.8 \times 10^{-17} \text{erg cm}^{-3} \text{s}^{-1} \), \( d = \) heliocentric distance (AU), \( \tau_{\text{UV}} = \) ultraviolet optical depth. The two main processes contributing to the infrared radiative heating/cooling terms are the infrared radiation from the H\(_2\)O molecules (Shimizu, 1976; Crovisier, 1984) and the radiative trapping of the dust thermal radiation Marconi and Mendis (1986). The combined effect of these processes can be approximated as (cf. Marconi and Mendis, 1986; Gombosi et al., 1986):

\[ Q_{\text{IR}} = Q_{\text{emiss}} n e^{-\tau_{\text{UV}}} + q_{\text{abs}} h_{\text{IR}} \sigma (1 - e^{-\tau_{\text{IR}}}) \int_0^{a_{\text{max}}} a^2 f_a T_a^4 \]  

(15)

where \( \sigma = \) Stefan-Boltzmann constant, \( \tau_{\text{IR}} = \) infrared optical depth, \( h_{\text{IR}} = \) transforming efficiency of internal to translational energy (\( h_{\text{IR}} \approx 0.5 \)), \( q_{\text{abs}} \) = relative width of absorbing bands with respect to the infrared spectrum emitted by dust (\( q_{\text{abs}} \approx 0.001 \), and

\[ Q_{\text{emiss}} = \begin{cases} 4.4 \times 10^{-22} T^{3.35} & \text{T} < 52 K \\ 2.0 \times 10^{-20} T^{2.47} & \text{T} \geq 52 K \end{cases} \]  

(16)

It can be seen that for \( \tau_{\text{IR}} \ll 1 \) \( Q_{\text{IR}} \) gives back the well known Shimizu cooling, while in a dense coma (\( \tau_{\text{IR}} > 1 \)) it describes a strong heat source.

Spherically symmetric steady state approximation of the dusty gas mixture system yields a solar-wind type equation, which describes a transonic flow in the immediate vicinity of the nucleus

\[ \frac{du}{dr} = - \frac{u}{1 - M^2} \left( \frac{2}{r} \cdot \frac{F_{\text{gd}}}{p} + \frac{(\gamma - 1)(Q_{\text{gd}} - Q_{\text{ex}})}{\gamma p u} \right) \]  

(17)

Here \( r \) denotes cometocentric distance, while \( M \) is the flow Mach number. The physical solution describes a reservoir outflow to a low pressure external medium (the external
pressure is at least 10 orders of magnitude smaller than the pressure at the nucleus. This means that at large cometocentric distances the gas pressure must vanish while the flow velocity remains finite, i.e.,

$$\lim_{r \to \infty} M = \infty$$  \hspace{1cm} (18)

At the surface the dust loading acts as a momentum sink, consequently the outflow is subsonic (in reservoir outflow problems unobstructed flows leave the reservoir with the local sound velocity; mass loaded flows always have subsolar outflow velocities)

$$M(r=R_n) < 1$$  \hspace{1cm} (19)

Equations (18) and (19) mean that the only physical solution to equation (17) is a transonic flow.

It is interesting to note that equation (17) is quite analogous to the classic solar wind equation. In the cometary case the gas to dust momentum transfer replaces the effect of solar gravity, otherwise the mathematical form of the equation is essentially unchanged. Figure 1 shows the various types of mathematically possible solutions, which are the same as the solutions of the solar wind equation.

The physical solution of equation (17) has a 0/0 type singularity at the sonic point, causing many numerical complications. Earlier models assumed a supersonic gas flow at large cometocentric distances and a subsonic flow close to the nucleus; in this case, there is one and only one transonic solution which passes smoothly through the sonic point. Transonic solutions cannot, in principle, be obtained numerically without additional assumptions. Generally, it has been assumed that the gas velocity (or Mach number) and its first derivative behaved continuously at the sonic point.

Following Proctor's original work (1968), practically all early transonic coma calculations used a type of "shooting method;" the initial flow velocity (or Mach number) value was "fine tuned" until a transonic solution was reached (Heilmich, 1979; Marconi and Mendis, 1982, 1983, 1984; Gombosi et al., 1983). In order to avoid the numerical difficulties of the earlier treatments and also to make it possible to describe dynamic phenomena in the coma, the first time-dependent (but still spherically symmetric) dusty gas dynamic model of the gas-dust interaction region was developed by Gombosi et al. (1985). The method was later combined with a three-dimensional, kinetic dust treatment in the outer coma (Gombosi and Horányi, 1986). In this model the gas dynamic equations are solved using a modified version of Godunov's first scheme which can naturally handle shocks and discontinuity surfaces. The model was able to describe such phenomena as dust halo formation in the inner coma, temporal evolution of dust and gas parameters following a comet outburst, etc. However, the numerical model can still use further improvements. In its first published form the model assumed a rather simple photochemistry, gas collisional cooling, neglected infrared radiation trapping (Marconi and Mendis, 1986), and assumed that the total radiative energy flux was approximately constant everywhere in the coma (an assumption that is more or less justified on the basis of earlier radiative transfer calculations (Keller, 1983; Marconi and Mendis, 1984)).

Time-Dependent, Axisymmetric Dusty Jet Models

The Comet Halley imaging experiments showed that cometary activity is concentrated to limited areas on the sunlit side of the nucleus, with most of the dust ejection coming from fairly localized jets. Spherically symmetric, steady state models were proven to be totally inadequate to describe cometary inner regions. Recently Kitamura (1986, 1987) has developed the first time-dependent, two-dimensional dusty gas dynamic code using one characteristic dust size, simple energetics, and a highly simplified chemistry. Kitamura's (1986, 1987) model has serious problems with the time-dependent treatment of the gas outflow, therefore he published only steady state results. Nevertheless, his first two-dimensional calculation represents an important step forward in describing dusty jets.

In this section we present preliminary results of a new, time-dependent, two-dimensional (axisymmetric) dusty jet model. We believe that these calculations will eventually help us to delineate the dominant physical processes governing the evolution of dusty cometary jets.

The model solves the coupled, time-dependent continuity, momentum, and energy equations for a dust-gas mixture (equations (1) through (11)). It is assumed that initially (t = 0) the near nucleus region is dust free and filled with low density gas. At t = 0 a localized dusty jet is generated on the nucleus surface ($R_n = 6 \text{km}$). The adopted dust surface density distribution was similar to that of used by Kitamura (1986)

$$\rho(\Theta) = \rho_0 \left[ (\alpha-1) \exp\left(-\frac{\Theta^2}{\Theta_0^2}\right) + 1 \right]$$  \hspace{1cm} (20)

where $\alpha = 10$, $\Theta_0 = 10^6$, $\rho_0 = 2.71 \times 10^{-11} \text{ g cm}^{-3}$. The surface dust temperature was 350 K. For the sake of

![Fig. 1. Various types of solutions to equation (17). The only physical solution is the cometary wind, which starts subsonically at the surface, goes through a sonic point, and monotonously increases in the supersonic region.](image-url)
simplicity only one dust size is considered in the present model \((a = 0.65 \, \mu m)\). The dust to gas mass production ratio is 0.5, close to the observed Comet Halley value. A very low pressure external "vacuum cleaner" was placed at a distance of 100 km, which helped to ensure a supersonic flow in most of the integration region (the sonic point was located at about 100 m from the surface).

A 40 x 30 grid structure was employed in the integration region. There were 40 linearly spaced azimuthal and 30 logarithmically spaced radial grids extending from 6 to 100 km. The time-dependent, coupled multidimensional partial differential system was solved with a second order upwind biased Godunov-type numerical scheme developed at the University of Michigan.

Figures 2 and 3 show snapshots of the gas and dust densities. The snapshots present two-dimensional equidensity curves at \(t = 20s, 40s, 60s, 100s, 200s\) and \(300s\) after onset. Inspection of Figures 2 and 3 reveals a very interesting feature of the localized axisymmetric jet. Initially the newly ejected gas and dust expands radially with relatively little horizontal broadening. The main reason for this almost entirely radial outflow pattern is that due to the \(-10^6\) spatial extent of the source region at the nucleus (the active "spot" on the surface has a radius of about 1 km) the surface radial pressure gradient is much larger than the azimuthal gradient. As the gas leaves the nucleus-coma interface region the azimuthal pressure gradient becomes comparable to the radial one and the gas flow starts to expand in the azimuthal direction as well. This effect can be observed at \(t = 40s\). In this phase the gas expansion goes faster than \(r^2\) (especially near the outer edge of the jet); therefore, two interesting things happen. First of all the jet cone expands (at \(t = 60s\) the jet cone is about 45° wide), and second the dust grains also attain a significant azimuthal velocity component. A natural consequence of this azimuthal velocity component is that a large fraction of the dust particles is "swept" away from the region near the axis of symmetry, thus resulting in a dust density depletion above the active area.

By about \(t = 100s\) the azimuthal gas expansion is stopped by the background gas. It has already been demonstrated by Kitamura's calculations (1986, 1987) that this effect is quite sensitive to the background gas production rate (gas production from the inactive part of the nucleus). This means that beyond about 50 km or so there is no significant azimuthal gas velocity and consequently the dust particles also lose most of their azimuthal velocity component. The late time (few hundred seconds) distributions, which are in a good qualitative agreement with Kitamura's steady state results, show that most of the dust is concentrated on a region surrounding the surface of a cone with a half-opening angle of about 50°. The dust density is quite low inside this cone.

Our conclusion is that the structure of a dusty jet resulting from an axisymmetric active region is cone-like. The opening angle of the dust cone is largely determined by the ratio of the gas pressures inside the active surface area and the inactive surface background. This jet structure is significantly different from the ones predicted by earlier spherically symmetric calculations (cf. Mendis et al., 1985; Gombosi et al., 1986);

---

**Fig. 2.** Snapshots of gas isodensity contours at \(t = 20s, 40s, 60s, 100s, 200s\) and \(300s\). In each panel the axis of symmetry is a horizontal line going through the center.
therefore, it is clear that our earlier picture of the dust acceleration region has to be considerably revised.

Acknowledgments. The authors are indebted to Bram van Leer for his advice in developing the second-order upwind biased Godunov-type numerical scheme to solve the coupled, time-dependent multidimensional partial differential equation system. This work was supported by NSF grant AST-8605994 and NASA grant NGR 23-005-015. Acknowledgments are also made to the University of Michigan and to the National Center for Atmospheric Research sponsored by NSF, for the computing time used in this research.

References


