Intuitive Understanding of Throughput-Delay Tradeoff in Ad hoc Wireless Networks

Term Project: EECS 557

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by

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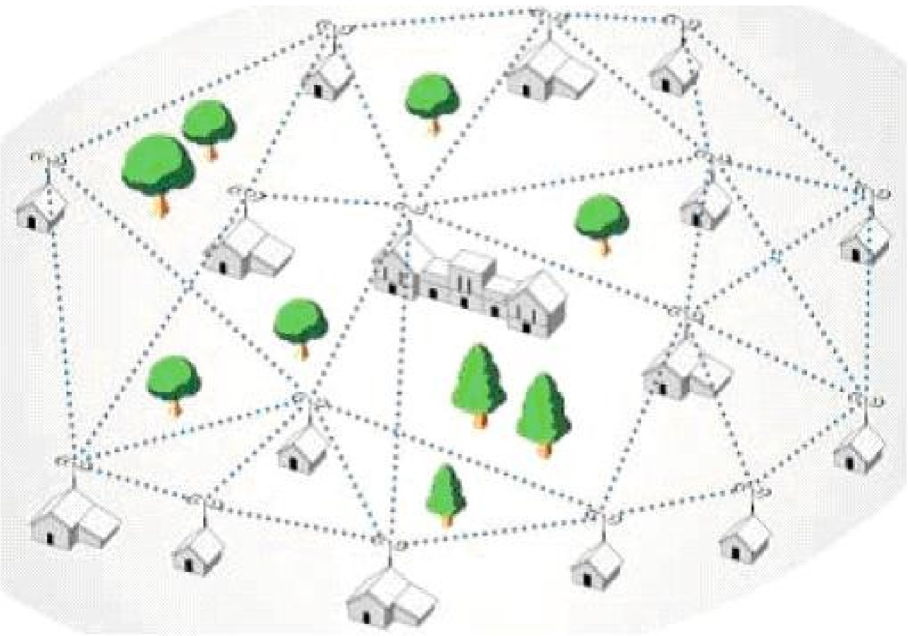
Outline

- Introduction
- Modeling of ad hoc networks
- Throughput-Delay tradeoff
  - Fixed networks
  - Mobile networks with one relay
  - A scheme achieving optimal tradeoff

Introduction

Ad hoc Network

- Each node can be an S, D and R simultaneously



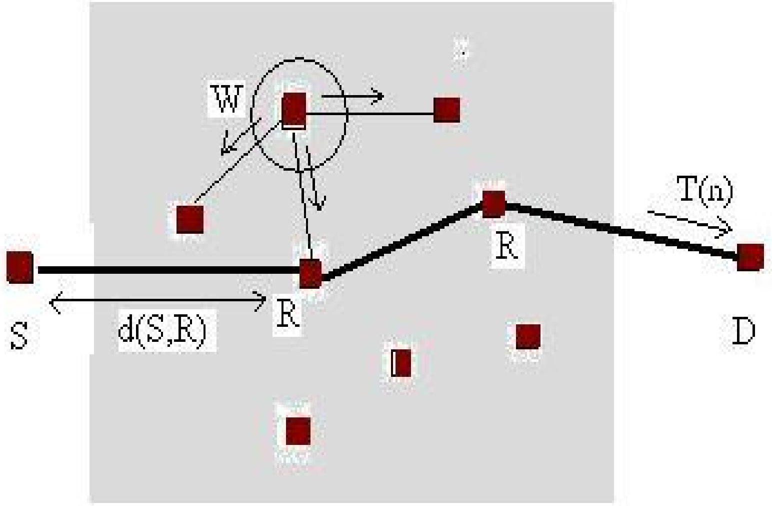
- Challenges
  - Interference
  - Topology change
- Analysis
  - Random configuration
  - Steady state
  - Orders are more important

Definitions

- Delay : $D(n)$
- Throughput : λ
  - Maximum delay constrained throughput : $T(n)$
- whp (With High Probability) : w.p. $\geq 1-1/n$
- Transmission rate : W

Factors affecting throughput and delay

- Fixed networks
  - Transmission range
  - Number of hops
- Mobile networks
  - Node velocity



Models for successful reception

- *Relaxed Protocol* model

$$d(k, j) \geq (1+\Delta) d(i, j)$$

- *Physical* model

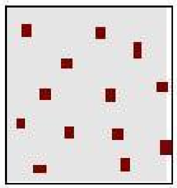
$$\text{SNIR} = \frac{\dfrac{P_i}{|d(i, j)|^{\alpha}}}{N + \sum_{\substack{k \in I \\ k \neq i}} \dfrac{P_k}{|d(k, j)|^{\alpha}}} \geq \beta$$

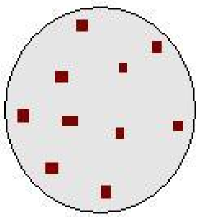
for $\alpha > 2$, and $P_i = P \quad \forall i \in P$

physical model is equivalent to Protocol model

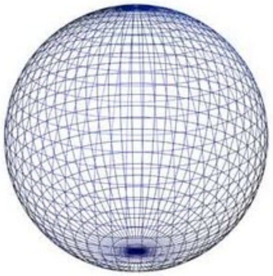
Network Topologies

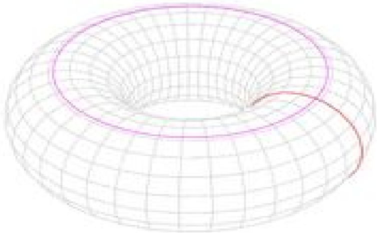
- 2-Dimensional
  - Square
  - Circular disk

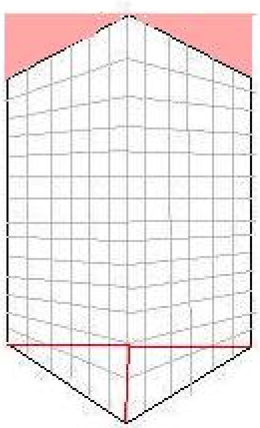




- 3-Dimensional
  - Sphere
  - Torus







- ➢ Avoid edge effects
- ➢ Need to take into account curvature of *sphere*
- ➢ For *torus,* the analysis can be done by considering equivalent *square*

Throughput-Delay Trade-off for Fixed Networks

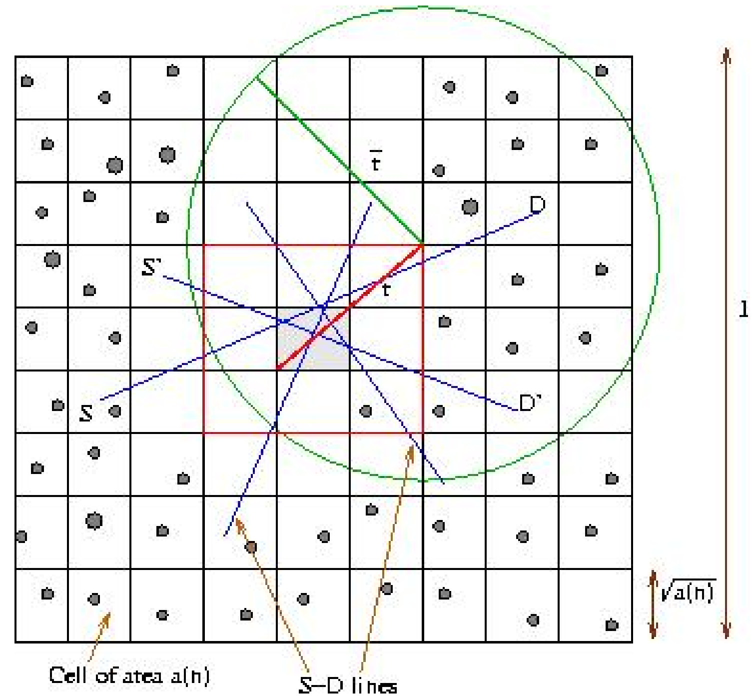
Assumptions

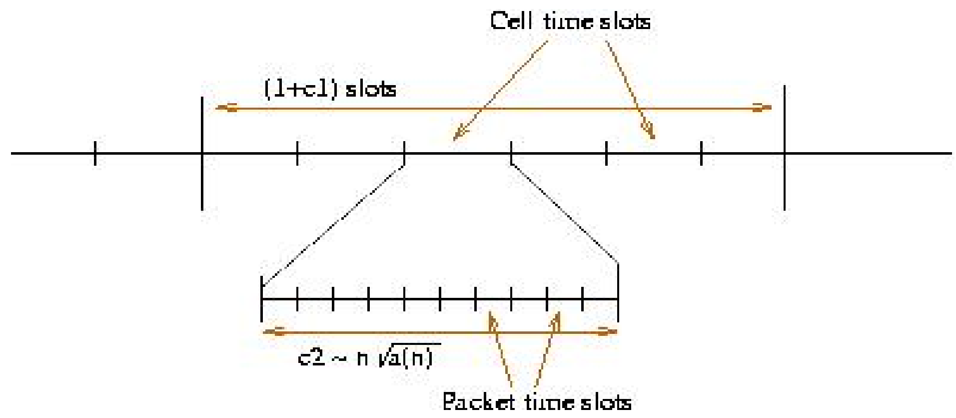
- Unit torus
- Slotted time
- Relaxed protocol model

$$T(n) = \Theta\left(\frac{1}{n\sqrt{a(n)}}\right)$$

$$D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right)$$

$$T(n) = \Theta\left(\frac{D(n)}{n}\right)$$





Throughput

- $a(n) \geq 2\log n / n$ ensures at least one node in each cell
- Maximum transmission range $\leq r = \sqrt{8a(n)}$
- Successful transmission requires – no other transmission in range $\bar{r} = (1+\Delta)r$
- # of interfering cells $= c_1 = \Theta\left(\frac{\pi r^2}{a(n)}\right) = \Theta\left(\frac{\left(\sqrt{a(n)}\right)^2}{a(n)}\right) = \Theta(1)$
- # of SD lines passing through any cell $= c_2 = \Theta\left(n\sqrt{a(n)}\right)$
- Fraction of time an SD pair's packet transmitted through a cell:

$$T(n) = \frac{1}{c_1 c_2} = \Theta\left(\frac{1}{1.n\sqrt{a(n)}}\right)$$

Delay

- Average distance of SD line $L = \Theta(1)$
- Average length of a hop $h = \Theta\left(\sqrt{a(n)}\right)$
- Delay per packet $D(n) = \Theta\left(\frac{L}{h}\right) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right)$
- Hence $T(n) = \Theta\left(\frac{D(n)}{n}\right)$

Throughput Delay Tradeoff for Mobile Networks

Assumptions

- **Unit torus with *n* cells, each area 1/*n***
- ***v*(*n*) scales down as a function of *n***

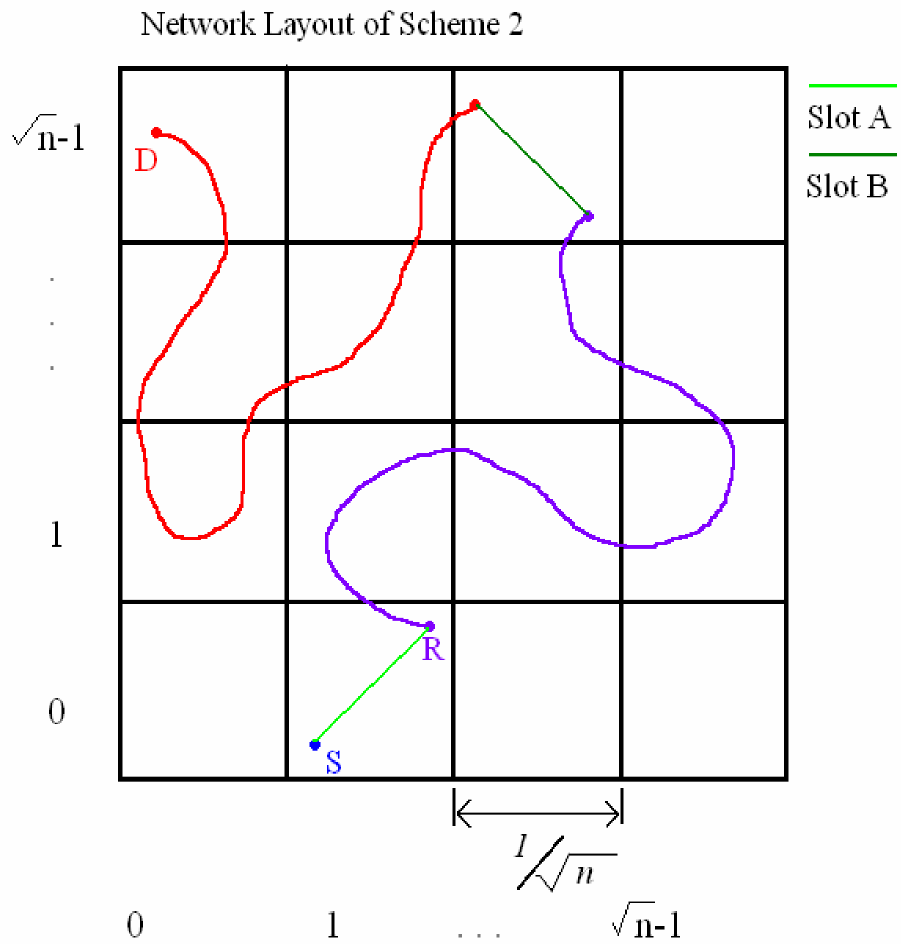
$$v(n) = \theta(\frac{1}{\sqrt{n}})$$

- **Slotted Time**

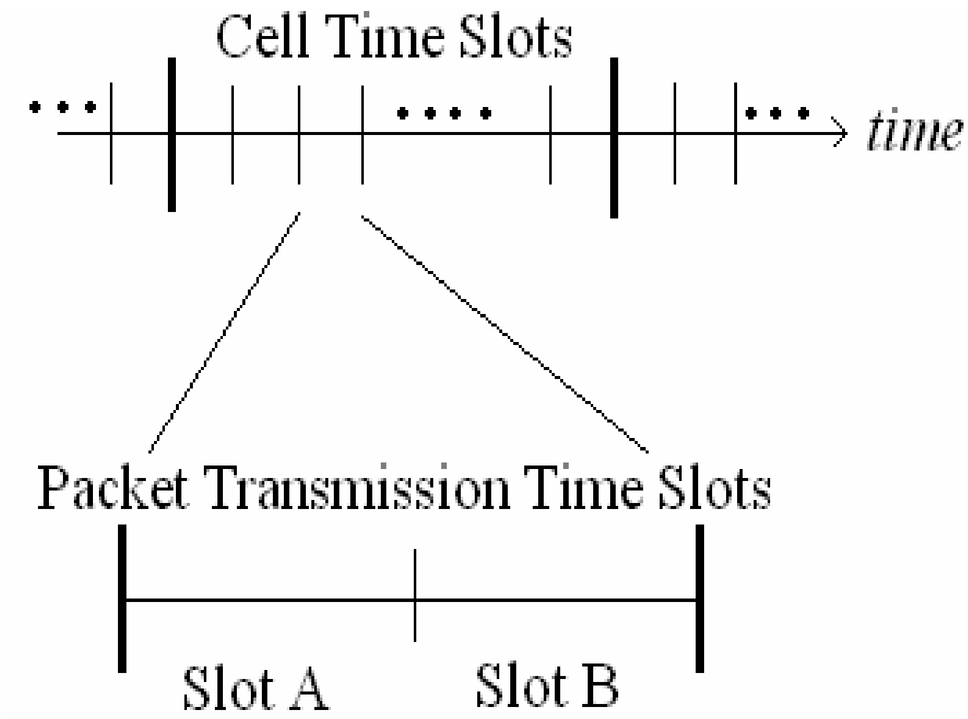
Results on throughput and delay

$$D(n) = O(\frac{\sqrt{n}}{v(n)})$$

$$T(n) = \theta(1)$$



TDMA for mobile networks



Packet transmission is broken up into slot A and B

Slot A

- Source transmits to destination if in same cell OR source transmits to a relay in the cell

Slot B

- the relay randomly chooses a node in the same cell and transmits a packet that is destined to it

Throughput

- # of cell time slots to avoid interference $= (1 + c_1)$

- For $a(n) = 1/n$ probability of at least two nodes per cell $= 1 - 1/2e = 0.26$

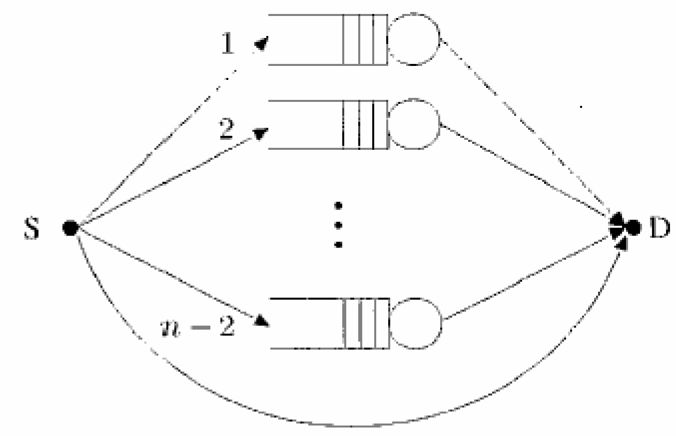
- Probability that a node has a packet for other node =
fraction of time for which a node has a packet for other node = c_2

- Probability that a transmission takes place within a cell $= 0.26\, c_2/(1 + c_1)$
$\Rightarrow$ throughput of each cell $= \Theta(1)$

- Combined throughput of all cells $= \Theta(n)$
- Throughput per SD pair

$$T(n) = \Theta(\frac{1}{n} n) = \Theta(1)$$

Delay



Direct transmission

- If $E[A] = \mu; \qquad E[S] = (1-\varepsilon)\mu$

  $Var[A] = \sigma_a^{\ 2}; \qquad Var[S] = \sigma_s^{\ 2}$

  then average delay,

  $$E[D] \le \max\left\{\mu, \frac{\sigma_a^{\ 2} + \sigma_s^{\ 2}}{2\mu\varepsilon}\right\}$$

- Time in which a node crosses a cell $t(n) = \Theta\left(\frac{1/\sqrt{n}}{v(n)}\right)$

- A GI-GI-1-FCFS queue can be defined with time step $t(n)$ and states $\left(X_1^{\ ij}(t), X_2^{\ ij}(t)\right)$

Delay

- The M.C. $\left(X_1^{ij}(t), X_2^{ij}(t)\right)$ spends equal time in all n states therefore,

  Avg. inter-meeting time of S&R = $E[A] = n$

- From the M.C. model, $Var[A^2] = \Theta\left(n^2\right)$

  hence, $E[D] = \Theta\left(\max\left\{n, \frac{n^2}{n}\right\}\right) = \Theta(n)$

- Actual delay $D(n) = \Theta\left(nt(n)\right) = \Theta\left(\frac{n}{\sqrt{n}v(n)}\right) = \Theta\left(\frac{\sqrt{n}}{1/\sqrt{n}}\right) = \Theta(n)$

- Hence $T(n) = \Theta\left(\frac{D(n)}{n}\right)$

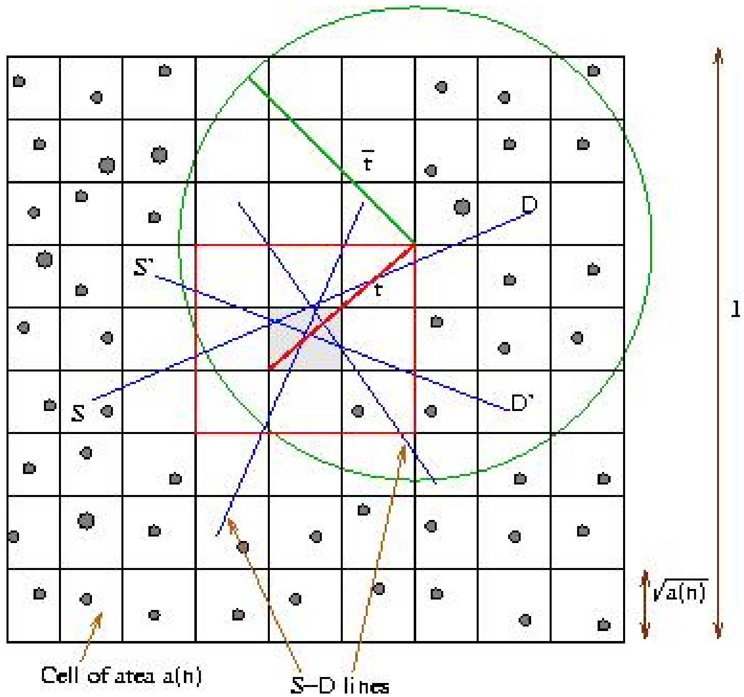
Optimal Throughput-Delay Trade-off for Mobile Networks

Scheme 3(a)

- For $v(n) = o\left(\sqrt{\log n / n}\right)$

$$T(n) = O\left(\frac{1}{n\sqrt{a(n)}}\right)$$

$$T(n) = \Theta\left(\frac{D(n)}{n}\right)$$



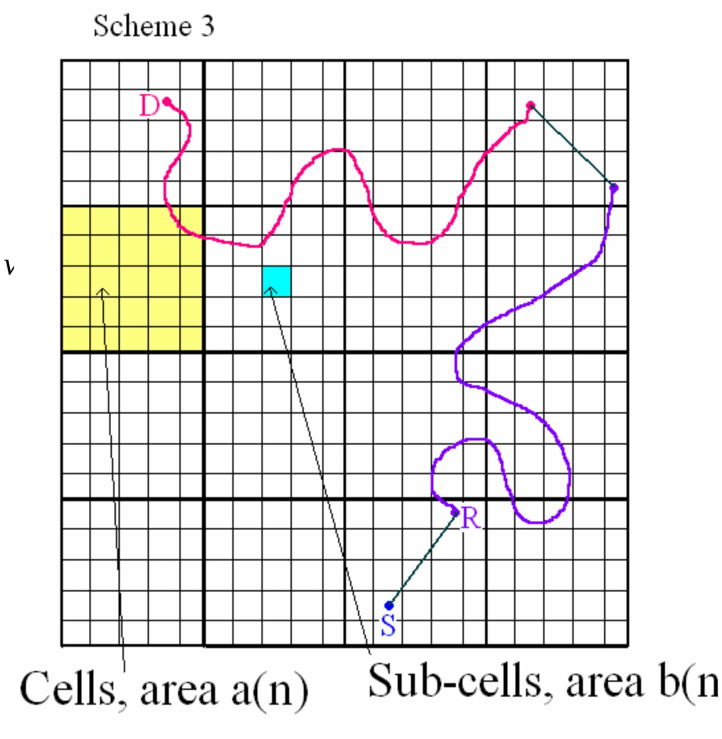
Justification for node velocity

- If initial distance between S&D = d
- In a single hop, the distance will increase by at least $\sqrt{a(n)}$ and will decrease by at most $(1+c_1)v(n)$
- After l hops, the distance will be less than $d - l\left(\sqrt{a(n)} - (1+c_1)v(n)\right)$
- For a packet to eventually reach the destination $v(n) = o\left(\sqrt{a(n)}\right) = o\left(\sqrt{\log n / n}\right)$
- Since velocity is very small compared to the edge length of the cell, Scheme 3(a) is essentially equivalent to that of Fixed Node Networks

Scheme 3(b)

Throughput

- For eventual delivery of a packet between SR and RD, $v(n) = o\left(\sqrt{b(n)}\right) = o\left(\sqrt{\log n / n}\right)$



- # of sub-cells $d(n) = a(n) / b(n)$

- # of nodes in a sub-cell $m = \Theta(na(n))$

- Throughput between SR & RD and hence SD

$$T(n) = \Theta\left(\frac{1}{m}\sqrt{d(n)}\right) = \Theta\left(\frac{1}{na(n)}\sqrt{\frac{a(n)}{\log n / n}}\right) = \Theta\left(\frac{1}{\sqrt{na(n)\log(n)}}\right)$$

$$m = \Theta(na(n))$$

Scheme 3(b)

Delay

Two kinds of delay, hop delay, and mobile delay

- Because of the constraint on velocity, mobile delay dominates hop delay
- The derivation of delay is identical to scheme two, except the random walk is on an area of size $\sqrt{1/a(n)}$ by $\sqrt{1/a(n)}$, instead of $\sqrt{n}$ by $\sqrt{n}$
- The time for a node to move out of a cell is $t(n) = \Theta\left(\frac{\sqrt{a(n)}}{v(n)}\right)$
- The delay is hence $D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}v(n)}\right)$

Comments

- When a torus is unfolded to make a square, the minimum distance between the nodes changes
- Torus has been used to avoid edge effects but in the derivations, the factor corresponding to edge effects was included
- In the M.C. model (Scheme 2), the states $\left(X_1^{ij}(t), X_2^{ij}(t)\right)$ should be in $\left(0,1,\ldots,\sqrt{n}/2-1\right)\times\left(0,1,\ldots,\sqrt{n}/2-1\right)$ instead of $\left(0,1,\ldots,\sqrt{n}-1\right)\times\left(0,1,\ldots,\sqrt{n}-1\right)$
- In the random walk model,

$$X_k^{ij}(t+1) = \left(X_k^{ij}(t) + K\right)\mathrm{mod}\,\sqrt{n}, \qquad \mathit{where} \qquad K \in \{-1,+1\}$$

but with random mobile nodes, it should be

$$X_k^{ij}(t+1) = \left(X_k^{ij}(t) + K\right)\mathrm{mod}(\sqrt{n}/2), \qquad \mathit{where} \qquad K \in \{-2,-1,0,+1,+2\}$$

- In Scheme 3(b), hop delay should be $\Theta\left(\sqrt{\frac{a(n)}{b(n)}}\right)$ instead of $\Theta\left(\frac{a(n)}{b(n)}\right)$

Comments

- The packet size was scaled to $T(n)$ in the analysis, so the actual throughput in terms of bits will be of the order

  $$\Theta\left(T(n) \times T(n)\right)$$

- Orders do not give a clear picture

  $n = 1{,}000$ and $n = 20{,}000$ make a huge difference!

Thank you!

Questions ?