Common Casting, HRDC: An analysis as a matching market

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Project report, SI 787, Winter 2006
University of Michigan, Ann Arbor

1 Introduction

Harvard-Radcliffe Dramatic Club (HRDC) is an arts club at Harvard University which presents plays, musicals and other theatrical works every semester in and around the campus. "Common Casting" is HRDC's audition week in which people can audition separately for nearly EVERY show for that semester that's happening around the Harvard campus. It is held at the beginning of every semester at some central location and anyone, whether affiliated with Harvard or not is eligible to audition.

1.1 Birth of "Common Casting (CC)"

Since its inception in 1908, Harvard presents various theatrical works every semester. Before the current system, auditions for each show were conducted independently throughout the semester. A major problem that this distributed audition system was facing was that actors used to drop out of one production to do another, which is clearly not good for the production groups. To overcome this problem CC was introduced in 1984 as a centralized audition system. The CC begins with a casual meet and greet at which directors advertise their shows. Then auditions take place throughout the week. Each show holds a number of three hour slots during this week in which it auditions actors. Callbacks occur on the weekend and cast lists are posted the following Monday. Actors have 24 hours to accept or reject roles.

As we can see from the description of the market above, the CC is a matching market with the "directors" and the "actors" forming the two sides of the market. In the article that describes the CC market, it has not been discussed as to how the HRDC actually goes about selecting actors for the shows but it is clear that some matching mechanism must be used at the end of the auditioning week for these assignments.

In this report we will analyze Common Casting as a matching market and will present some approaches to model the features of this market different from other studied matching markets. In sections 2.1 and 2.2 we compare the CC market with the well known matching markets we studied in the class followed by a discussion of the new features of this market. In sections 3.1 and 3.2 we discuss an approach to model the CC market and its new features mathematically. In section 3.3 we propose an algorithm that considers group effects present in the CC market. The conclusions are given in section 4.

2 Common Casting as a matching market

2.1 Comparison with the markets we studied in the class

2.1.1 Marriage model

The basic feature of the CC market is to match actors to the "characters" in different shows. Both sides of this market have some preference over the members of the other side; the actors have their preference over the roles in different shows, and the characters have preference over the actors based on their acting skills and personality. The latter preference is basically reflected through the judgement of the directors of respective shows. With the actors and characters forming the two sides of the market, this problem is a two-sided one-one matching problem and hence can be treated as a marriage market.

Since in the CC market it is the actors who go and audition for different shows, it should be viewed as actors proposing to the characters. The castlists can be viewed as the outcome of the match after the acceptance or rejection of the actors by the characters based on their preference. In section 3.1 I have described how we can set up the preferences of the two sides after completion of auditioning. Thus once the preferences of the two sides (they should be complete and transitive) are known, Gale-Shapley mechanism can be used as in a marriage market to yield a stable match for the current problem. With the CC market being viewed as an "actors proposing" market, we expect to observe the following if Gale-Shapley mechanism is used:

- The match will be actors-optimal and hence the dominant strategy of the actors will be to state their true preferences.
- Directors will have incentives to misrepresent their preferences with regard to the characters because there will be atleast one character which can make itself better off by misrepresenting the preference.
- The match will be stable with respect to the stated preferences.

An interesting difference from the marriage market can arise if there are two shows which are to held in two different months of the semester, say 1st and 3rd month, such that their rehearsal timings do not overlap. In such a case, the same actor can be assigned two different roles in these two shows. This cannot happen in the 1-1 marriage market. However such situations can be handled if we treat this actor as two different actors and match each of the two superficial actors to atmost one character.

The marriage model is good for the CC market if the actors are all independent and they don't care which other actors they end up working with in the shows. However in practice, in such group productions, some people prefer to work with their friends or acquaintances to derive more pleasure from their work. Infact this might have been one of the reasons why, before introduction of CC, people used to drop out of plays in between e.g. if somebody happens to find that some of his very good friends are working in another show whose audition/practices have just started, then it may urge him to ditch his current director in order to work with his friends.

So in order to model this aspect of the CC market, we will have to tie the characters with their shows/directors and consider them as a group while looking at the preferences of the actors. Being a one-one matching market, this aspect is absent from the marriage model. Therefore when group effects are present, it is more reasonable to compare the CC market with a two-sided many-one matching market like a Hospital-Intern (HI) market and model the grouping effect like marriage couple problem of the HI market. The comparison follows.

2.1.2 Hospital Intern (HI) match

In an HI market, the two sides of the market consist of the interns and the hospitals along with their intern programs. In case of the CC market we can make an analogy with the actors representing the interns and the directors representing the hospitals who have a fixed quota equal to the number of characters in their show which has to be filled by hiring actors. With this analogy, the CC market can be treated as an HI market and this again suggests to use the equivalent of Gale-Shapley mechanism proposed by Roth ('84) in which students propose to the hospitals. In such a case, we can predict the following from Roth's results:

- The outcome of the Gale-Shapley mechanism will be stable with respect to the stated preferences.
- Gale-Shapley mechanism will yield an actors-optimal match and will make it a dominant strategy for the actors to state their true preferences.
- No stable matching mechanism exists which makes it a dominant strategy for all directors to state their true preferences.

As we can see, the third prediction above differs from what we would expect if we use a "directors proposing" Gale-Shapley mechanism corresponding to the marriage model. This is because in modeling the CC market as an HI market, although we are able to identify the characters as a group, there are some important features of the CC market which are not captured as described below:

- In an HI market or other many-one matching markets like school choice, people don't state preferences for different departments within a hospital or for different class sections within a school. This is because all the positions which make a specific quota for a hospital/intern program are considered to be "equivalent".
- On the other hand in the CC market, different roles in a show create different interests for the people and so, people have different preferences for different characters within the same show. Thus these positions are not equivalent.

Furthermore, in the above model, we haven't yet captured the aspect the aspect that some people would prefer to work together. So even though the Gale-Shapley mechanism will yield a stable matching with respect to the individual preferences, the mechanism will suffer from inefficiency. In the HI market, this was reflected as lack of participation in the centralized clearinghouse during the 70's. In the CC case, it will be reflected by people swapping offers/dropping out of the shows after the castlists are announced. Therefore to prevent that, we must consider some improvement as proposed for the marriage couple problem for the HI market. The problem of grouping in the CC case is however more complicated and different from the marriage couple problem as explained below:

- The group size may not be fixed in the CC unlike the couple case which has a fixed group size of 2. In CC there may be groups of 2,3,5... friends and the size of the group is not known in advance. Furthermore, the participating group sizes can differ from year to year.
- The groups may not even be fixed for the same year. In the solution to marriage couple problem, the couples submit their preference lists consisting of ordered pairs of intern positions. What it means is that they will either accept the positions together or will not accept any position from the centralized match. In the case of CC, although people

prefer to work with friends, it may be the case that if they don't end up getting an offer with their group, they would still like to participate in some show either alone or say with 2 other members of their group instead of 5.

The above features require that we come up with new improvements for the CC market which extend the solution of the marriage couple problem for varying group size. Having compared the CC market with the well studied markets, we are now able to characterize the CC market by the features particular of this market.

2.2 Features of the CC market

We can broadly identify the following features persistent in the CC market.

- It is a one-one matching market with the actors and the characters forming the two sides of the market, however the preference of people to work in groups gives it some resemblance to the many-one matching market.
- The groups are of different sizes and the people belonging to a particular group may accept working with or without the group.
- One important feature that has not been discussed in the previous sections is "synergy". As we know, working in a theatrical work requires the whole group to work as a "team" and the team can achieve its maximum productivity only if there are no conflicts among its members. So in order to better capture the real essence of the market, synergy should be taken care of while considering the preferences of the directors i.e. their preference should be over the "set" of actors instead of individual actors.
- Another feature which draws a similarity from the "Law clerk market" is that actors might face an external pressure from the directors to work with them. Since the market is comparatively small, the actors cannot betray the directors because it is very likely that they will encounter these directors again and in the coming years, they might want to work with these directors too. We know that in the Law clerk market, coercion is a major hindrance to the success of the centralized-clearinghouse and so we can expect a similar loss of efficiency in the CC market too.

In the following sections, I discuss a mathematical model for the CC market that captures some of the above features of the CC market.

3 Mathematical modeling of the CC market

3.1 General setup

We can classify the players in the CC market as follows:

- Actors: $A := \{a_1, a_2, \ldots, a_m\}$
- Characters: $C := \{c_1, c_2, \ldots, c_n\}$
- Directors(Shows): $D := \{d_1, d_2, \ldots, d_p\}, \quad p \leq n$

If character c_i belongs to the director/show d_j, it will be denoted by $c_i(d_j)$.
The **preference lists** for the two sides can be generated as follows:

- When the directors advertise their shows, the actors get a chance to evaluate the characters and they make a preference list for the characters. The preference list of actor a_i can be represented as:

$$P(a_i) = c_2, c_5, c_1, \ldots, a_i \ldots$$

Actor a_i would audition for all the characters before a_i in the above list and submit his preference list to the center. Anything after a_i is unacceptable to him and he would not audition for those characters.

- When actors audition for the characters, the directors evaluate them and come up with a preference list for each character in his their show.

$$\begin{aligned} P(c_i) &= a_3, a_7, a_2, \ldots, c_i \ldots (\text{all characters who didn't audition} \\ &\quad + \text{those who auditioned but are not eligible for role } c_i) \end{aligned}$$

We assume that the preference list of a character consists of only those actors who audition for that character. Since the present case is like "actors proposing" there is no point considering those actors who didn't audition for this character as it means that this character is unacceptable to that actor.

3.2 Group preferences

We now describe a method to model group preferences and the way it will affect a matching algorithm. To keep the analysis simple, we make the following assumptions:

- The directors do not consider synergy in their preferences.
- There do not exist any external political pressures on the actors from the directors.
- An actor can belong to atmost one group.

As a point of departure, the first thing that should be done is that along with their preference lists, every actor also submits the group name/number to which he belongs. Since no two groups have a common member by above assumption, this identifies all distinct groups in the beginning. We denote these groups by $G_1, G_2, \ldots, G_K$. With these preidentified groups, one way to model the group effect would be to rank the allocations (matches) so that every member of a given group prefers the allocation in which he is accompanied with higher number of group members in his show compared to that in which there are lesser number of group members with him. One problem that can be immediately observed in such formulation is the following: In order to work with a group member, one may have to accept a role that is much lower in his individual preference than some other role that he may get if he works alone. Therefore our model should be such that it can capture the trade-off between working with a group member vs. working on a role one prefers. With this goal, a good idea in my opinion will be to introduce quantitative preferences i.e. associate a utility function with every player and a payoff corresponding to the outcome (matching) of the mechanism. To capture the aspects discussed above, such a utility function should satisfy the following properties:

- If an actor has other members from his group in his show, his utility for that match increases compared to being matched to the same character alone.
- The increase in the utility depends on the number of people of that group in the same show.

- If an actor say actor a_i prefers character c_1 to c_2 even if he gets to work with two other group members if he chooses c_2, then the incremental utility he gets by working with two members of his group should be lesser than the difference in the utilities he gets from c_1 and c_2 if he works alone.

To keep it simple, we assume in the following that one gets a fixed increment in the utility with every extra member of his group i.e. we don't discriminate among group members. However in practice, one's utility may increase significantly if he gets to work with his girlfriend whereas with some other friend it may increase very little. But we don't consider such scenario in our modeling.

We will now discuss through an elaborate example as to how the actors should set their preferences in light of the above properties of the utility functions.

- An actor assigns points, say on a scale of 100, to the characters he is auditioning for. Higher points will imply higher preference for that character. The point assigned to character c_j by actor a_i will be denoted by $U_{a_i}(c_j)$.
- Let the incremental utility an actor gets from every extra member of his group be 2 points.
- Now the actors have to be more thoughtful while submitting their preferences e.g. if a_i likes c_1 so much that if he is getting c_2 even with 4 other members of his group, he would not like to switch to c_2, then a_i should assign points to c_1 and c_2 in such a way that $U_{a_i}(c_1) > U_{a_i}(c_2) + 8$.

This point system goes with the intuition that,

(a) If a_i prefers to reject c_2 with 4 group members in favor of c_1, then a_i would also reject c_2 with 3 or lesser group members in favor of c_1.

(b) Similarly, if $U_{a_i}(c_1) = U_{a_i}(c_2) + 5$, it means that to accept c_2 over c_1, there should be atleast 3 other group members of a_i in c_2's show.

It should be noted here that in case (a), the same condition would carry over with respect to rejection, to all the characters below c_2 in a_i's preference list. In case (b), the same condition would carry over with respect to acceptance, to all the characters above c_1 in a_i's preference list. This raises the following question: Suppose c_1 and c_3 are achievable for a_i and c_2 is not. When a_i submits his preference list, he is not aware of it. If he wants to indicate that with 3 other group members, he will accept either c_2 or c_3 over c_1 but if offered alone, he would prefer c_2 to c_3, he cannot do that with the point system described above. To overcome this problem, we can define two different point scales,

Scale 1 This will be an absolute scale on which actors will assign points to different characters to indicate their "individual" preference.

Scale 2 This will be a relative scale on which actors will indicate their relative preferences in the presence of other group members. They will have to make a separate list, one corresponding to each character e.g if actor a_i makes this relative list for character c_1, he will assign points only to the characters below c_1 in his absolute scale list which will indicate how he would value those characters with respect to c_1 in the presence of other group members. This list can have multiple characters at the same point, and if there are ties, they will be broken according to the absolute scale list.

3.3 An algorithm that considers group effects

Having defined the preferences in terms of utilities, we can now generalize the deferred acceptance matching algorithm for the CC market:

Step 1) Every actor proposes to the first character in his preference list.

Step 2) Every character temporarily selects the actor highest in its preference list among those who have proposed to this character. Instead of rejecting the lower ranked actors who have proposed, the characters hold these actors too. We call the actors who have been selected (temporarily) in this step as forming the first "row" (This term makes sense if we write all the characters in a row, write the actors they select just below them in the "first row", and write the rest of the actors whom they hold in the column below the selected actor.). In steps 3 & 4, we will consider only the actors of the first row.

Step 3) Identify "groups" of actors in the first row. Count the number of members of a group in each show and increase the utility of all its members accordingly.

Step 4) If a change of character assignment is "realizable" i.e. it increases the payoff of the players involved, then the assignments will be changed. This change can occur in two ways. The first is that an actor of the first row swaps his character with some other actor of the first row in which case the actor with whom he swaps must also gain by the swap and the swap should be acceptable (it shouldn't decease the payoff of the character) to the characters involved. The second way is that an actor of the first row can propose to some other character to whom nobody has proposed till now and for whom this actor is acceptable.

Step 5) If a change occurs in the second way in Step (4), the character initially assigned to the actor who switches, will be left unassigned. So if there are other actors held by this character, the actor who is most preferred among those, will be assigned to this character and he will move to the first row. Thus the first row will be modified, and the algorithm will repeat from Step (3).

Step 6) If no more changes occur in Step (4), the actors which are still in the "held" queue are rejected by the corresponding characters.

Step 7) The rejected characters from Step (6) propose to the characters next in their preference lists and the algorithm repeats from Step (2).

A similar method can be used to model "synergy" in this case. Since synergy comes from the mutual cooperation of actors and the director, we can assume that when a director has more number of actors from the same group in his show, he gets a higher payoff than when he has all the actors from different groups. We can incorporate such incremental change in utilities of the directors in Step (3) above before we look for any possible change. In the same way, if we want to model the deterioration in the performance due to two anti groups in the same show, we can do that by incorporating appropriate decrements in the utilities in Step (3).

4 Conclusion

In this report, I studied Harvard-Radcliffe Dramatic Club's Common Casting as a matching market. I presented a comparison of the CC market with the known matching markets and discussed the conditions under which the known matching algorithms can be used for the CC

market. I discussed new features of the CC market and the challenges that exist to design an efficient matching algorithm for this market. I analyzed in detail the effects of varying group sizes that affect this market and proposed a way to model those effects by introduction of utility functions. Finally I proposed a matching algorithm to take into account the group effects.

The modeling approaches presented in this report are just a step forward to analyze some of the new features of the CC market. The proposed algorithm has not been studied for its stability or efficiency which leaves it as an interesting topic for further study. Some other aspects which remain to be explored are modeling synergy in the context of the CC market, the effects of external pressures from the directors on the final matches and matching actors to multiple shows which will be a one-many matching problem.