An Externality Based Decentralized Optimal Power Allocation Scheme For Wireless Mesh Networks

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Outline

Power allocation: A new perspective

Future wireless communication

- High demand $\Rightarrow$ high competition
- Quality of Service (QoS) satisfaction
- Interference control
- More flexibility at users' ends

Decentralized power control

- In mesh networks centrally operated control adds on infrastructure and latency
- Decentralized power control draws interest
- Decentralized resource allocation - well studied in Mathematical Economics

Power allocation: A new perspective

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Mathematical Economics - to study power allocation

Literature survey

Classification of power allocation problem

Configuration of the network

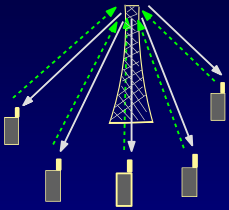
- Cellular uplink/downlink network
- Mesh/Ad-hoc network

Application

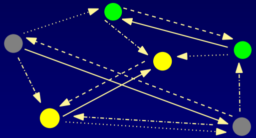
- Voice vs. data network
- Fixed rate vs. elastic data rate network

Interference control

- Interference temperature constraint (ITC)



Cellular uplink Cellular downlink



Mesh network

Contribution of our work

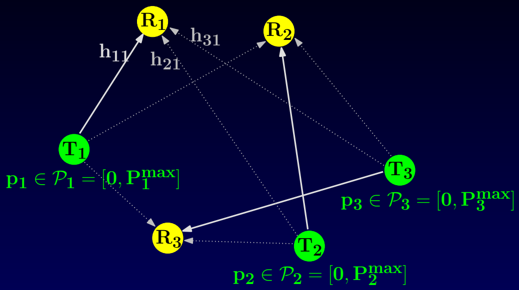
Main Contribution

- Formulation of power allocation problem for wireless mesh networks as a resource allocation problem with externalities

Characteristics of formulation

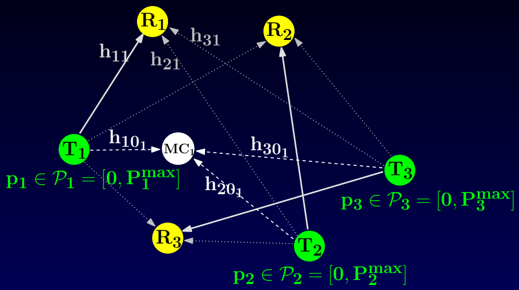
- Philosophically similar to Laffont and St. Pierre's formulation of Economies with externalities
- Allows us to appropriately modify an algorithm of Lions & Temam for static decentralized optimization so as to obtain optimal power allocation

The model



- *M* transmitter receiver pairs (*Users*), $\mathcal{M} := \{1, 2, \ldots, M\}$
- Transmissions of a user create interference to other users
  Interference depends on the transmission powers
- Performance determined by *utilities*: $U_i(p) = U_i(p_1, p_2, \ldots, p_M), i \in \mathcal{M}$

The model (cont')
Interference Temperature Constraint (ITC)



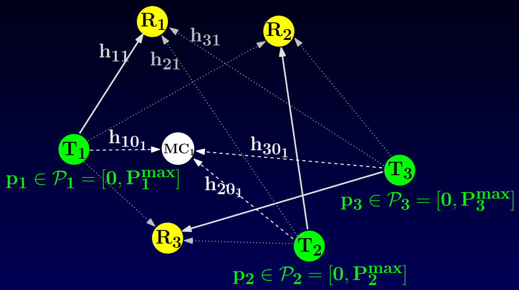
Interference Temperature (IT): Net radio frequency (RF) power measured at a receiving antenna per unit bandwidth

ITC: A measure to keep the RF noise floor below a safe threshold

$$\sum_{i=1}^{M} p_i h_{i0_1} \leq P_1$$

- Multiple ITCs: K measurement centers (MCs)/ Users $0_{\mathcal{K}} := \{0_1, 0_2, \ldots, 0_K\}$

Assumptions
Information available to the users



User $i \in \mathcal{M}$

- $\mathcal{P}_i = [0, P_i^{max}]$
- Utility U_i

User $0_k, k \in \mathcal{K}$ (MC_k)

- Channel gains $h_{j0_k}, j \in \mathcal{M}$

Common knowledge

- $\mathcal{P} = [0, P^{max}] \supset \cup_{i \in \mathcal{M}} \mathcal{P}_i$
- # of active users M
  M remains constant

Assumptions (cont')

Assumption on utility functions

- For all $i \in \mathcal{M}$, $U_i(p)$ from R^M into R is a non-negative, strictly concave, continuous function of p.
- $U_i(p : p_i = 0) = 0, \ \forall \, i \in \mathcal{M}$
- The utilities of the MCs are zero i.e. $U_{0_k}(p) = 0, \ k \in \mathcal{K}$.
- The utilities remain fixed throughout the power allocation period.

The optimization problem

Objective

Allocate transmission powers to the users to maximize the ***social welfare function*** for the wireless mesh network considered.

Optimization Problem (**A**)

$$\max_p \sum_{i \in \mathcal{M} \cup 0_{\mathcal{K}}} U_i(p) = \max_p U(p) \quad (1)$$

subject to:

$$p \in S := \{p \mid \sum_{i=1}^{M} p_i h_{i0_k} \leq P_k, \ \ k \in \mathcal{K}, \ \ p_i \in \mathcal{P}_i \ \forall\, i \in \mathcal{M}\} \quad (2)$$

- ∗ For each $i \in \mathcal{M}$ the utility function $U_i(\cdot)$ is known only to user i. (3)
- ∗ $\mathcal{P}_i$ is known only to user i. (4)
- ∗ A set $\mathcal{P} = [0, P^{max}] \supset \cup_{i \in \mathcal{M}} \mathcal{P}_i$ is common knowledge. (5)
- ∗ For each $k \in \mathcal{K}$, MC_k knows the channel gains h_{j0_k}, $j \in \mathcal{M}$. (6)
- ∗ $\forall\, i \in \mathcal{M}$, $U_i(p) : \ R^M \rightarrow R$ is a non-negative, strictly concave, continuous function of p and $U_i(p : p_i = 0) = 0$ (7)
- ∗ $U_{0_k}(p) = 0, \ \ \forall\, k \in \mathcal{K}$ (8)

Corresponding centralized problem

Centralized Problem ($\mathbf{A_C}$)

$$\max_p \sum_{i \in \mathcal{M} \cup 0_{\mathcal{K}}} U_i(p) \quad = \quad \max_p U(p) \tag{9}$$

subject to:

$$p \in S := \{p \mid \sum_{i=1}^{M} p_i h_{i0_k} \leq P_k, \quad k \in \mathcal{K}, \quad p_i \in \mathcal{P}_i \ \forall\, i \in \mathcal{M}\} \tag{10}$$

$*$ $\forall\, i \in \mathcal{M}$, $U_i(p) : \ R^M \to R$ is a non-negative, strictly concave, continuous function of p and $U_i(p : p_i = 0) = 0$ (11)

$*$ $U_{0_k}(p) = 0, \ \ \forall\, k \in \mathcal{K}$ (12)

Note: Problem ($\mathbf{A_C}$) has a unique optimum.

Corresponding centralized problem

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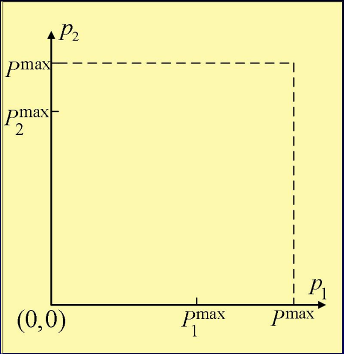
Note: Problem ($\mathbf{A_C}$) has a unique optimum.

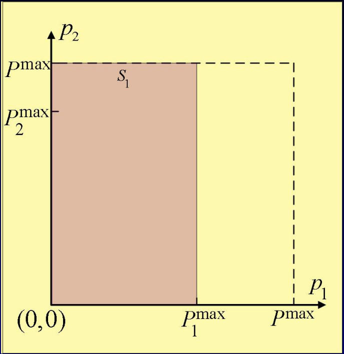
Objective

- To solve Problem (**A**) and obtain, if possible, the optimal solution of Problem ($\mathbf{A_C}$)
- Satisfy the constraints imposed by the decentralized system

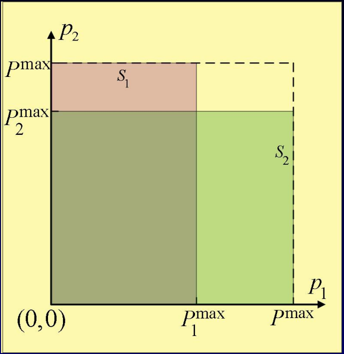
Formulation as an externality problem
The ingredients



Formulation as an externality problem
The ingredients (cont')



- Technically possible power profiles $S_i := \{p \mid p_i \in \mathcal{P}_i,\ \ p_{-i} \in \mathcal{P}^{M-1}\},\ i \in \mathcal{M}$



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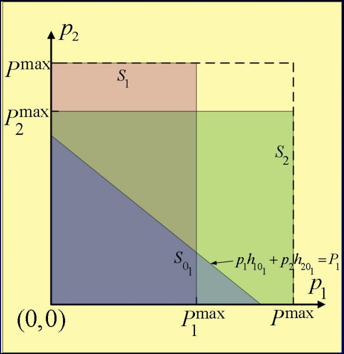
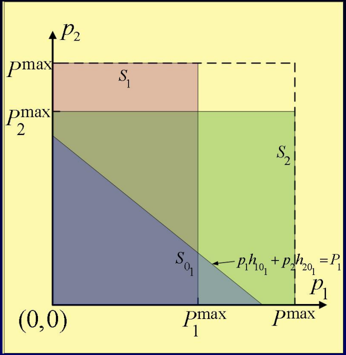
Formulation as an externality problem
The ingredients (cont')



- Technically possible power profiles $S_i := \{p \mid p_i \in \mathcal{P}_i,\ \ p_{-i} \in \mathcal{P}^{M-1}\},\ i \in \mathcal{M}$
- k – semi-feasible power profiles

$$S_{0_k} := \{p \mid \sum_{i=1}^{M} p_i h_{i0_k} \leq P_k,\ \ p_i \in \mathcal{P}\ \forall\, i\},\ k \in \mathcal{K}$$



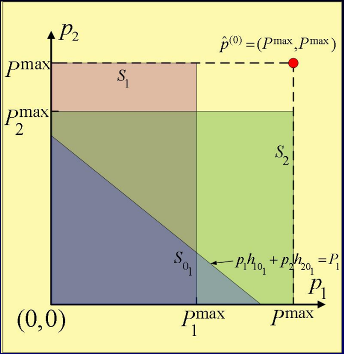
- Technically possible power profiles $S_i := \{p \mid p_i \in \mathcal{P}_i,\ \ p_{-i} \in \mathcal{P}^{M-1}\},\ i \in \mathcal{M}$
- k – semi-feasible power profiles

$$S_{0_k} := \{p \mid \textstyle\sum_{i=1}^{M} p_i h_{i0_k} \leq P_k,\ \ p_i \in \mathcal{P}\ \forall\, i\},\ k \in \mathcal{K}$$

- Feasible power profiles

$$S = \textstyle\bigcap_{i \in \mathcal{M} \cup 0_{\mathcal{K}}} S_i = \{p \mid \sum_{i=1}^{M} p_i h_{i0_k} \leq P_k,\ \ k \in \mathcal{K},\ \ p_i \in \mathcal{P}_i\ \ \forall\, i \in \mathcal{M}\}$$

The *externality algorithm* for decentralized power allocation
(A modification of Lions-Temam (1971) algorithm)



0) **Initialization:**

- Users (including users $0_{\mathcal{K}}$) agree upon a common power profile

$$p^{(0)} \in \{p \mid p_i \in \mathcal{P} \;\forall\; i\} \tag{13}$$

The externality algorithm (cont')

0) **Initialization (cont'):**

- A sequence of modification parameters $\{\tau^{(n)}\}_{n=1}^{\infty}$ is chosen that satisfies,

$$0 < \tau^{(n+1)} \leq \tau^{(n)}, \qquad \forall\ n \geq 1 \tag{14}$$

$$\lim_{n \to \infty} \tau^{(n)} = 0 \tag{15}$$

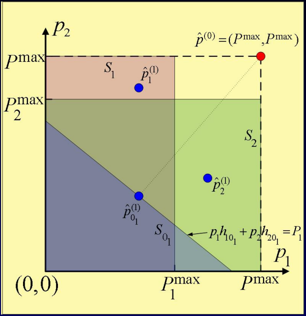
$$\lim_{N \to \infty} \sigma^{(N)} = \lim_{N \to \infty} \sum_{n=1}^{N} \tau^{(n)} = \infty \tag{16}$$

- The counter n is set to 0.

Example

$\tau^{(n)} = \frac{1}{n}$ or $\tau^{(n)} = \frac{1}{\sqrt{n}}$ for real n satisfies (14) – (16).

The externality algorithm (cont')



1) ***n* th iteration:** (Individual optimization)

- User i, $i = 1, 2, \ldots, M$, solves

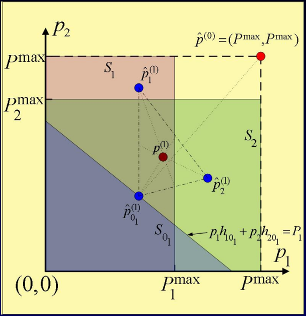
$$\hat{p}_i^{(n+1)} = \operatorname{argmax}_{p \in S_i} \quad U_i(p) - \frac{1}{\tau^{(n+1)}} \|p - p^{(n)}\|^2 \tag{17}$$

- MC_i (user 0_i), $i = \{0_1, 0_2, \ldots, 0_K\}$, solves

$$\hat{p}_i^{(n+1)} = \operatorname{argmax}_{p \in S_{0_k}} \quad - \frac{1}{\tau^{(n+1)}} \|p - p^{(n)}\|^2 \tag{18}$$

- Individual optimals $\hat{p}_i^{(n+1)}$ $\forall$ i are broadcast to all the users.

The externality algorithm (cont')

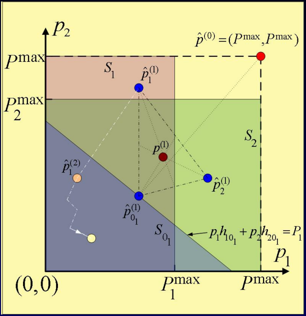


2) Calculation of user and time averages

- Upon receiving $\hat{p}_i^{(n+1)}$ $\forall$ i, users compute for $(n+1)th$ iteration

$$p^{(n+1)} = \frac{1}{M+K} \sum_{i \in \mathcal{M} \cup 0_{\mathcal{K}}} \hat{p}_i^{(n+1)} \qquad (19)$$

The externality algorithm (cont')



- $p^{(n+1)}$ is used as a reference point in the $(n+1)th$ iteration.

The externality algorithm (cont')

2) (cont')

- User i, $i \in \mathcal{M} \cup 0_{\mathcal{K}}$, also computes the weighted averages

$$\hat{w}_i^{(N+1)} = \frac{1}{\sigma^{(N+1)}} \sum_{n=1}^{N+1} \tau^{(n)} \hat{p}_i^{(n)}, \qquad i \in \mathcal{M} \cup 0_{\mathcal{K}}$$

$$= \frac{1}{\sigma^{(N+1)}} \left(\sigma^{(N)} \hat{w}_i^{(N)} + \tau^{(N+1)} \hat{p}_i^{(N+1)} \right), \qquad (20)$$

where $$\sigma^{(N+1)} = \sum_{n=1}^{N+1} \tau^{(n)} = \sigma^{(N)} + \tau^{(N+1)} \qquad (21)$$

- The average calculated in (20) is stored in users' memories.
- The counter n is increased to $n+1$ and the process repeats from Step 1).
- For $(n+1)$th iteration, $\tau^{(n+2)} \leq \tau^{(n+1)}$ is chosen from the predefined sequence in Step 0).

Convergence to optimal solution

Theorem

The externality algorithm results in a power allocation which is the unique global optimum of the centralized problem ($\mathbf{A_C}$).

The optimal allocation is obtained as the limit of the sequences $\{\hat{w}_i^{(N)}\}_{N=1}^{\infty}, \; i \in \mathcal{M} \cup 0_{\mathcal{K}}$, all of which converge to the same limit.

Summary

- Investigated the decentralized power allocation problem for a wireless mesh network with multiple ITCs.
- Formulated the power allocation problem from an externality perspective.
- Proposed a decentralized algorithm for solving it.
- The proposed algorithm obtains a globally optimal power allocation.
- It provides a guaranteed convergence to the optimum solution.
- It satisfies the informational constraints of the problem.

Further Scope

- Modifying the present algorithm to make it applicable for more general class of utility functions.
- Extending the analysis for time-varying channels.
- Designing mechanisms that implement optimal centralized power allocations in Nash Equilibria.

THANK YOU!

Questions and Comments?

Reference

- S. Sharma and D. Teneketzis, “An externality based decentralized optimal power allocation scheme for wireless mesh networks”, *Control Group Report CGR-07-02, Department of EECS, University of Michigan, Ann Arbor.*

  (Submitted for IEEE journal publication.)

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Proof of Theorem

- For simplicity, we prove the Theorem only for a single ITC.
- We call the single MC in this case as user 0 and associate with it, a semi-feasible set S_0

$$S_0 := \{p \mid \sum_{i=1}^{M} p_i h_{i0} \leq P, \quad p_i \in \mathcal{P} \ \forall\, i\} \tag{22}$$

- Since $\hat{p}_i^{(n+1)}$ is the optimal solution of,

$$\hat{p}_i^{(n+1)} \;=\; \text{argmax}_{p \in S_i} \quad U_i(p) - \frac{1}{\tau^{(n+1)}} \|p - p^{(n)}\|^2, \quad i \in \mathcal{M} \cup \{0\} \tag{23}$$

it follows that, $\forall\, p \in S_i, \;\; i \in \mathcal{M} \cup \{0\}$,

$$\tau^{(n+1)} U_i(\hat{p}_i^{(n+1)}) - \|\hat{p}_i^{(n+1)} - p\|^2 + \|p^{(n)} - p\|^2 - \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \\ \geq \tau^{(n+1)} U_i(p) \tag{24}$$

- Adding (24) over all i gives, $\forall\, p \in S \;=\; \cap_{i \in \mathcal{M} \cup \{0\}} S_i$,

$$\tau^{(n+1)} \sum_{i=0}^{M} U_i(\hat{p}_i^{(n+1)}) - \sum_{i=0}^{M} \|\hat{p}_i^{(n+1)} - p\|^2 + (M+1)\|p^{(n)} - p\|^2 \\ - \sum_{i=0}^{M} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \;\geq\; \tau^{(n+1)} \sum_{i=0}^{M} U_i(p) \;=\; \tau^{(n+1)} U(p) \tag{25}$$

Proof of Theorem (cont')

- Since $\|\cdot\|^2$ is a convex function,

$$\|p^{(n+1)} - p\|^2 \leq \frac{1}{M+1} \sum_{i=0}^{M} \|\hat{p}_i^{(n+1)} - p\|^2 \tag{26}$$

- Replacing the second term in (25) using (26), adding over $n = 0, 1, \ldots, N-1$, and dividing by $M+1$ we obtain, $\forall\, p \in S$,

$$\frac{1}{M+1} \sum_{n=0}^{N-1} \tau^{(n+1)} \sum_{i=0}^{M} U_i(\hat{p}_i^{(n+1)}) - \|p^{(N)} - p\|^2$$
$$- \frac{1}{M+1} \sum_{i=0}^{M} \sum_{n=0}^{N-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \;\geq\; \frac{\sigma^{(N)}}{M+1} U(p) - \|p^{(0)} - p\|^2 \tag{27}$$

- By concavity of $U_i(p)$ in p,

$$\frac{1}{M+1} \sum_{i=0}^{M} \sum_{n=0}^{N-1} \tau^{(n+1)} U_i(\hat{p}_i^{(n+1)}) \;\leq\; \frac{\sigma^{(N)}}{M+1} \sum_{i=0}^{M} U_i(\hat{w}_i^{(N)}), \quad i \in \mathcal{M} \cup \{0\} \tag{28}$$

Since $S_i,\ i \in \mathcal{M} \cup \{0\}$, is a convex set and $\hat{p}_i^{(n+1)} \in S_i\ \forall\, n$, it follows that $\hat{w}_i^{(N)} = \frac{1}{\sigma^{(N)}} \sum_{k=1}^{N} \tau^{(k)} \hat{p}_i^{(k)} \in S_i,\ i \in \mathcal{M} \cup \{0\}$.

Proof of Theorem (cont')

- Substituting (28) in (27) and multiplying by $(M+1)/\sigma^{(N)}$ we get, $\forall\ p \in S$,

$$\sum_{i=0}^{M} U_i(\hat{w}_i^{(N)}) - \frac{M+1}{\sigma^{(N)}} \|p^{(N)} - p\|^2 - \sum_{i=0}^{M} \frac{1}{\sigma^{(N)}} \sum_{n=0}^{N-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \geq U(p) - \frac{M+1}{\sigma^{(N)}} \|p^{(0)} - p\|^2 \qquad (29)$$

- $\because\ S_i,\ i \in \mathcal{M} \cup \{0\}$, and S are compact,
  numerators of the 2nd terms on both LHS and RHS of (29) are bounded.
- The 2nd terms go to zero as $N \to \infty$, $\because\ 1/\sigma^{(N)} \to 0$ as $N \to \infty$.
- Numerator of the 3rd term on the LHS of (29) grows with N.
- $\because\ S_i,\ i \in \mathcal{M} \cup \{0\}$, is compact, $\hat{w}_i^{(N)} \in S_i$, and $U_i(\cdot)$ is a continuous function on S_i, $\exists\ C_0, C_1\ :\ \forall\ N$,

$$\begin{aligned} \|\hat{w}_i^{(N)}\| &\leq C_0, \qquad i \in \mathcal{M} \cup \{0\} \\ U_i(\hat{w}_i^{(N)}) &\leq C_1, \qquad i \in \mathcal{M} \cup \{0\} \end{aligned} \qquad (30)$$

- $\therefore$ the 1st terms on both the LHS and the RHS of (29) are bounded, and so the third term on the LHS.

Proof of Theorem (cont')

- $\exists\ C_2 : \forall\ N$
$$\frac{1}{\sigma(N)} \sum_{n=0}^{N-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \leq C_2 \tag{31}$$
- $\because$ S_i is compact, (30) $\Rightarrow \exists$ a subsequence $\{\hat{w}_i^{(N')}\}_{N'=1}^{\infty}$ of $\hat{w}_i^{(N)}$, such that $\hat{w}_i^{(N')} \longrightarrow \hat{w}_i^* \in S_i,\ i \in \mathcal{M} \cup \{0\}$.

Next we show that,

1. The subsequence $\{w^{(N'-1)}\}_{N'=1}^{\infty}$ of $w^{(N)}$ converges to a limit w^*.
2. $\hat{w}_i^* = \hat{w}_j^* = w^* \quad \forall\ i, j \in \mathcal{M} \cup \{0\}$.

Lemma

$$\lim_{N' \to \infty} \|\hat{w}_i^{(N')} - w^{(N'-1)}\|^2 = \|\hat{w}_i^* - w^*\|^2 = 0, \quad \forall\ i$$

- We must show that, $\forall\ i$,

$$\forall\ \epsilon > 0,\ \exists\, N_0' \ :\ \forall\ N' \geq N_0',\ \ \|\hat{w}_i^{(N')} - w^{(N'-1)}\|^2 \leq \epsilon, \quad (32)$$

- $\because\ \tau^{(n)} \rightarrow 0$, we can find an n_0 such that,

$$\tau^{(n_0)} \leq \frac{\epsilon}{2C_2} \quad (33)$$

- Knowing n_0 we can compute the finite quantity,

$$A_{0_i} = \sum_{n=0}^{n_0-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \quad (34)$$

- $\because\ \sigma^{(N)} \rightarrow \infty$ as $N \rightarrow \infty$, $\exists\ N_{0_i}'$ large enough such that,

$$\sigma^{(N_{0_i}')} \geq \frac{2\tau^{(1)} A_{0_i}}{\epsilon} \quad (35)$$

- $\because\ \sigma^{(N)}$ is increasing in N,

$$\forall\ N' \geq N_0' = \max_i N_{0_i}', \qquad \sigma^{(N')} \geq \sigma^{(N_0')} \geq \sigma^{(N_{0_i}')} \quad \forall i \quad (36)$$

- $\because\ \tau^{(n+1)} \leq \tau^{(n)} \quad \forall\ n,$

$$\frac{1}{\sigma^{(N')}} \sum_{n=0}^{N'-1} \tau^{(n+1)} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2$$
$$\leq \frac{\tau^{(1)}}{\sigma^{(N')}} \sum_{n=0}^{n_0-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 + \frac{\tau^{(n_0)}}{\sigma^{(N')}} \sum_{n=n_0}^{N'-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \qquad (37)$$

- Substituting (31) and (34) in (37) and using (36) we get,

$$\|\hat{w}_i^{(N')} - w^{(N'-1)}\|^2 \leq \frac{1}{\sigma^{(N_0')}} \tau^{(1)} A_{0_i} + \tau^{(n_0)} C_2$$
$$\leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon \qquad (38)$$

The second inequality in (38) follows from (33) and (35).

This proves the Lemma.

Proof of Theorem (cont')

- Subsequences $\hat{w}_i^{(N')}, i \in \mathcal{M} \cup \{0\}$, converge to the same limit $\hat{w}_i^* = w^*$.
- $\because \hat{w}_i^* \in S_i, \; i \in \mathcal{M} \cup \{0\}, \quad \Rightarrow w^* \in S = \bigcap_{i \in \mathcal{M} \cup \{0_1, 0_2, \ldots, 0_K\}} S_i$.
- $\Rightarrow w^*$ is a **feasible solution** for Problem (**A**).

Next we show that,

- w^* is an **optimal solution** of Problem (**A**).

- Since $\| \cdot \|^2$ is a convex function, for $i \in \mathcal{M} \cup \{0\}$,

$$\begin{aligned} \|\hat{w}_i^{(N')} - w^{(N'-1)}\|^2 &\le \frac{1}{\sigma(N')} \sum_{n=0}^{N'-1} \tau^{(n+1)} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \\ &\le \frac{1}{\sigma(N')} \sum_{n=0}^{N'-1} \|\hat{p}_i^{(n+1)} - p^{(n)}\|^2 \;\text{ for }\; \tau^{(n+1)} \le 1 \qquad (39) \end{aligned}$$

- Substituting (39) in (29) and taking the limit as $N' \to \infty$,

Proof of Theorem (cont')

$$\lim_{N' \to \infty} \{-\sum_{i=0}^{M} \|\hat{w}_i^{(N')} - w^{(N'-1)}\|^2 + \sum_{i=0}^{M} U_i(\hat{w}_i^{(N')})\} \quad \geq \quad U(p)$$

or

$$\sum_{i=0}^{M} U_i(w^*) \quad \geq \quad U(p) \qquad (40)$$

- The inequality in (40) follows from (38).

This proves the optimality of w^* and the optimality of the externality algorithm for decentralized power allocation.