A Mechanism Implementing in Nash Equilibria the Optimal Centralized Solution of a Supply Chain Problem

Shruti Sharma

Ph.D. candidate, Electrical Engineering and Computer Science

Volodymyr Babich, Mark Van Oyen

Industrial and Operations Engineering

Demos Teneketzis

Electrical Engineering and Computer Science

University of Michigan, Ann Arbor

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Outline

Overview

Set-up

- A supply chain coordination problem
- Arbitrary number of suppliers and manufacturers
- Decentralized and asymmetric information
- Competitive/selfish/strategic decision makers with no prior beliefs

Our work

- Design of a decentralized negotiation mechanism that,
  - preserves private information of the agents
  - makes the agents willingly participate in the mechanism
  - obtains optimal centralized transactions at Nash equilibrium

Literature survey

- Principal-Agent Models
  - Contracting in Supply Chain Management: Review by Cachon (2003)
  - **Economics:** Myerson (1981, 1982), Grossman and Hart (1983), Guesnerie Laffont (1984), McAfee and McMillan (1986), Maskin and Tirole (1990, 1992)
  - **Operations Management:** Corbett and Tang (1999), Corbett and de Groote (2000), Cachon and Lariviere (2001), Iyer, Schwarz, and Zenios (2005), Yang, Aydin, Babich, and Beil (2008a, b)
  - **Coordination might not be attainable:** Ha (2001), Cakanyildirim, Gan, and Sethi (2006)
- Coordination of cross-functional decisions within a firm: Porteus and Whang (1991), Kouvelis and Lariviere (2000)

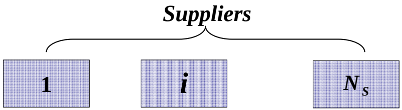
Contribution

- **Generality of model**
  - Arbitrary number of suppliers and manufacturers
  - Complete decentralization of information
  - Competitive/selfish decision makers with no prior beliefs about other agents
- **Developed decentralized negotiation mechanism** that,
  - preserves private information of the agents
  - makes the agents willingly participate in the mechanism
  - obtains optimal centralized transactions at *all* Nash equilibria
  - balances the flow of products and money between the suppliers and manufacturers at equilibria
- **Presented a method to characterize all Nash equilibria**
  - for a given system wide objective, and
  - a given decentralized negotiation mechanism

The supply chain model

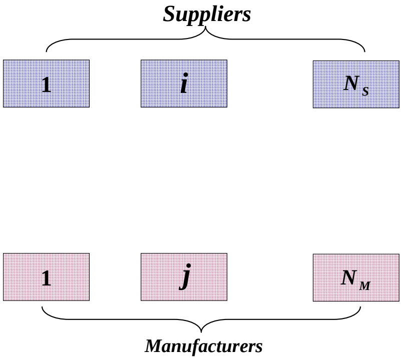
The supply chain model

- N_S suppliers



The supply chain model

- N_S suppliers
- N_M manufacturers



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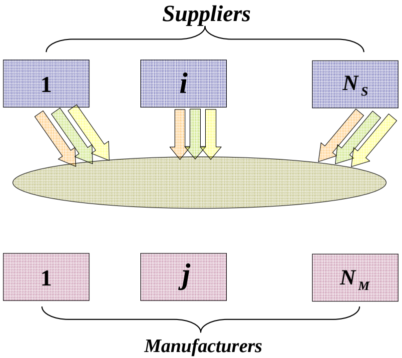

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- Supply vector:
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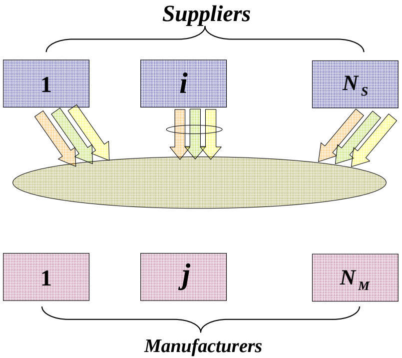

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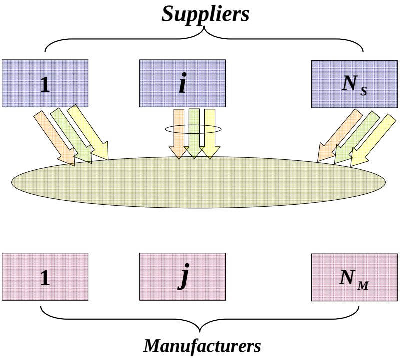
  - Cost of production: $c_i(\boldsymbol{x_i})$



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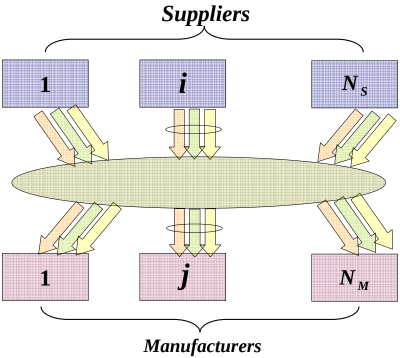

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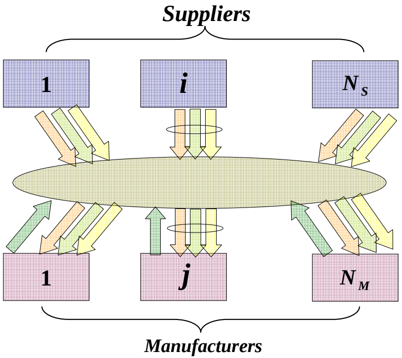
- Payment by manufacturers: $g_j > 0$



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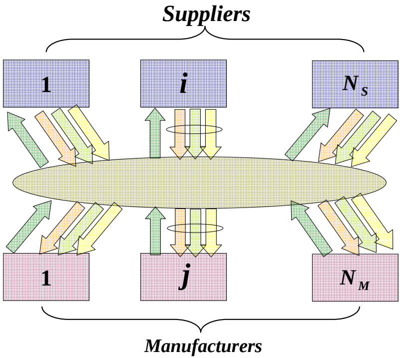
- Payment by manufacturers: $g_j > 0$
- Payment to suppliers: $r_i > 0$



Information available to the agents

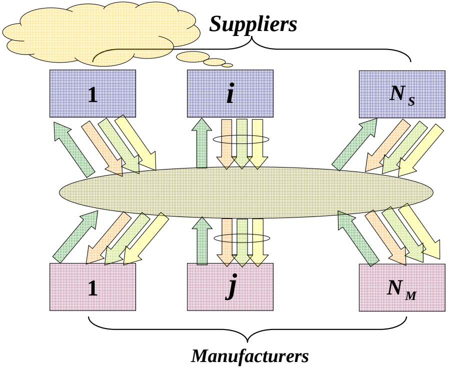
Suppliers:

- Utility of supplier i: $u_i^S(r_i, \boldsymbol{x_i})$

$$= r_i - c_i(\boldsymbol{x_i}) - \left[\frac{1 - l_{\mathcal{D}_i^S}((r_i, \boldsymbol{x_i}))}{l_{\mathcal{D}_i^S}((r_i, \boldsymbol{x_i}))} \right] \quad (1)$$

payment received−cost of production

- c_i is a convex function of $\boldsymbol{x_i}$ with $c_i(\mathbf{0}) = 0$.
- Each supplier's production capability, production capacity, and the cost of production are its private information.
- Suppliers are self utility maximizers / behave strategically.



Information available to the agents

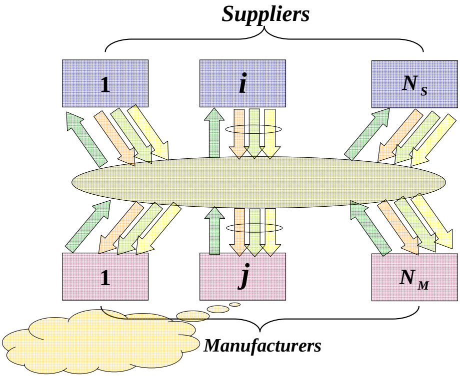
Manufacturers:

- Utility of manufacturer j: $u_j^M(g_j, \boldsymbol{y_j})$

$$= -g_j + v_j(\boldsymbol{y_j}) - \left[\frac{1 - l_{\mathcal{D}_j^M}(\boldsymbol{y_j})}{l_{\mathcal{D}_j^M}(\boldsymbol{y_j})} \right] \quad (2)$$

$-$payment made $+$ value of purchase

- v_j is concave in $\boldsymbol{y_j}$ with $v_j(\boldsymbol{0}) = 0$.
  v_j increases in each element of $\boldsymbol{y_j}$.
- Each manufacturer's purchase value is its private information.
- Manufacturers are self utility maximizers / behave strategically.



The centralized supply chain problem

Problem (P_{SC})

$$\max_{(\boldsymbol{g},\boldsymbol{r},\boldsymbol{x},\boldsymbol{y})} \sum_{i=1}^{N_S} u_i^S(r_i, \boldsymbol{x_i}) + \sum_{j=1}^{N_M} u_j^M(g_j, \boldsymbol{y_j}) \tag{3}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_S} \boldsymbol{x}_i = \sum_{j=1}^{N_M} \boldsymbol{y}_j \tag{4}$$

$$\text{and} \quad \sum_{j=1}^{N_M} g_j = \sum_{i=1}^{N_S} r_i \tag{5}$$

(P_{SC}) obtains a transaction that is balanced in product and money transfers and maximizes the sum of utilities of suppliers and manufacturers.

Solution of Problem (P_{SC}) = Ideal transaction

How to obtain centralized solution

Characteristics of the supply chain model

- **Decentralized information:** Nobody has complete system information.
- **Strategic agents:** The suppliers and manufacturers are selfish.

Solution approach: ***Implementation theory***

Provides guidelines for:

- how the agents should "*communicate*" with one another, and
- how "*the information communicated by the agents should be used to determine the transactions*" so as to induce the selfish agents to communicate information that results in an optimal centralized transaction.

Reference: Implementation theory – Maskin (1985), Jackson (2001), Palfrey (2002), Stoenescu and Teneketzis (2005)

Supply chain problem in implementation theory framework

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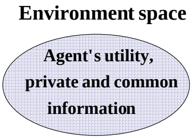

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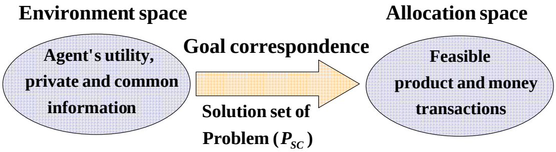

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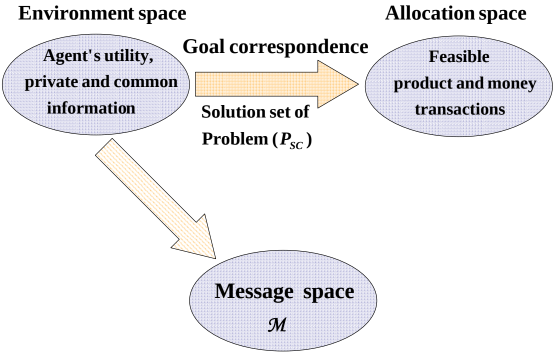

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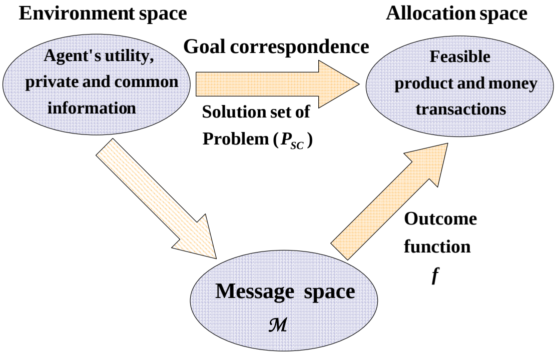
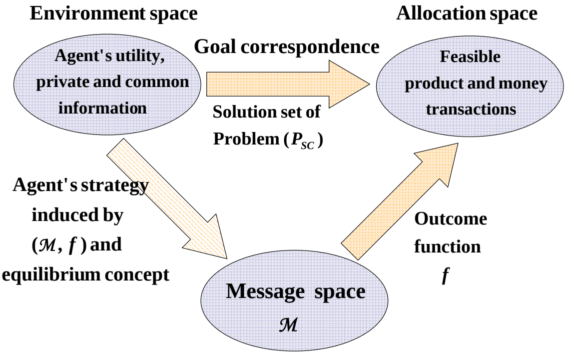


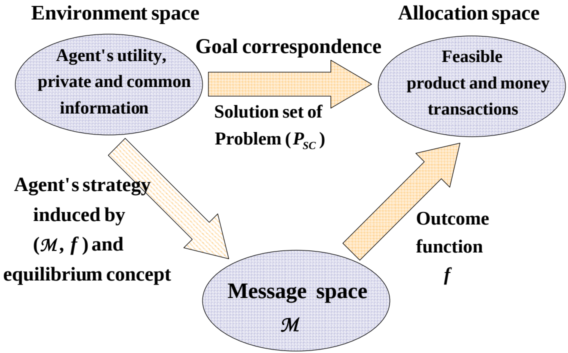
- Decentralized mechanism – Game form: $(\mathcal{M}, f)$

Supply chain problem in implementation theory framework



- Decentralized mechanism – Game form: $(\mathcal{M}, f)$
- Induced game: $(\mathcal{M}, f, \{u_i^S\}_{i=1}^{N_S}, \{u_j^M\}_{j=1}^{N_M})$

Supply chain problem in implementation theory framework



- **Nash equilibrium**: A message profile $\boldsymbol{m}^*$ is a NE if,

$$u_i^S(f(\boldsymbol{m}^*)) \geq u_i^S(f((\boldsymbol{m_i^S}, \boldsymbol{m}^*/i))), \quad \forall\, \boldsymbol{m_i^S} \in \mathcal{M}_i^S, \ \forall\, i \in \{1, 2, \dots, N_S\}$$
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Interpretation of Nash equilibria

Traditional definition of Nash equilibria

– for games of complete information

Difference in supply chain model

- Supply chain model does not result in game of complete information – Agents' utilities are private information
- Suppliers and manufacturers are involved in a message exchange process

Interpretation

The stationary points of the message exchange process should have properties of Nash equilibria.

Desirable properties of a decentralized mechanism

Implementation in Nash equilibria:

A game form $(\mathcal{M}, f)$ “fully implements the goal correspondence π in Nash equilibria” if, for all problem environments,

Set of transactions at all Nash equilibria $=$ Set of optimal centralized transactions

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Individual rationality:

A game form $(\mathcal{M}, f)$ is individually rational if, for all suppliers and manufacturers,

Utility at all Nash equilibria $\geq$ Utility before/without participating in the negotiation process specified by the game form

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A game form $(\mathcal{M}, f)$ is individually rational if, for all suppliers and manufacturers,

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Goal:

To design an individually rational game form $(\mathcal{M}, f)$ for the Supply Chain problem that implements in Nash Equilibria the goal correspondence π corresponding to (P_{SC}).

A game form for the supply chain problem

Message space:

$$\textbf{Suppliers:} \quad \boldsymbol{m}_i^S \quad := \quad (\boldsymbol{x}_i, \boldsymbol{p}_i^S); \quad \boldsymbol{x}_i \in \mathbb{R}^L, \ \boldsymbol{p}_i^S \in \mathbb{R}_+^L \tag{6}$$

(Supply vector, Price vector) proposal for L products

$$\textbf{Manufacturers:} \quad \boldsymbol{m}_j^M \quad := \quad (\boldsymbol{y}_j, \boldsymbol{p}_j^M); \quad \boldsymbol{y}_j \in \mathbb{R}^L, \ \boldsymbol{p}_j^M \in \mathbb{R}_+^L \tag{7}$$

(Purchase vector, Price vector) proposal for L products

(8)

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Inspiring paper:

"Outcome functions yielding Walrasian and Lindahl allocations at Nash equilibrium points", Hurwicz, 1979

A game form for the supply chain problem (cont')

Outcome function:

$$\hat{\boldsymbol{x}}_i(\boldsymbol{m}) = \boldsymbol{x}_i - \frac{1}{(N_S - 1)} \sum_{\substack{k \in \mathcal{N}_S \\ k \neq i}} \boldsymbol{x}_k + \frac{1}{N_S} \sum_{j \in \mathcal{N}_M} \boldsymbol{y}_j \quad (9)$$

Deviation from average supply + Average demand

$$\hat{\boldsymbol{y}}_j(\boldsymbol{m}) = \boldsymbol{y}_j - \frac{1}{(N_M - 1)} \sum_{\substack{k \in \mathcal{N}_M \\ k \neq j}} \boldsymbol{y}_k + \frac{1}{N_M} \sum_{i \in \mathcal{N}_S} \hat{\boldsymbol{x}}_i(\boldsymbol{m}) \quad (10)$$

Deviation from average demand + Average supply

$$\hat{r}_i(\boldsymbol{m}) = \boldsymbol{p}^{\boldsymbol{S}}_{-\boldsymbol{i}}(\boldsymbol{m})^T \hat{\boldsymbol{x}}_i - \left(\boldsymbol{p}^{\boldsymbol{S}}_{\boldsymbol{i}} - \boldsymbol{p}^{\boldsymbol{S}}_{-\boldsymbol{i}}(\boldsymbol{m})\right)^T \left(\boldsymbol{p}^{\boldsymbol{S}}_{\boldsymbol{i}} - \boldsymbol{p}^{\boldsymbol{S}}_{-\boldsymbol{i}}(\boldsymbol{m})\right) \quad (11)$$

$$\text{where } \boldsymbol{p}^{\boldsymbol{S}}_{-\boldsymbol{i}}(\boldsymbol{m}) := \frac{1}{(N_S - 1 + N_M)} \left(\sum_{\substack{k \in \mathcal{N}_S \\ k \neq i}} \boldsymbol{p}^{\boldsymbol{S}}_{\boldsymbol{k}} + \sum_{j \in \mathcal{N}_M} \boldsymbol{p}^{\boldsymbol{M}}_{\boldsymbol{j}} \right) \quad (12)$$

$$\hat{g}_j(\boldsymbol{m}) = \boldsymbol{p}^{\boldsymbol{M}}_{-\boldsymbol{j}}(\boldsymbol{m})^T \hat{\boldsymbol{y}}_j + \left(\boldsymbol{p}^{\boldsymbol{M}}_{\boldsymbol{j}} - \boldsymbol{p}^{\boldsymbol{M}}_{-\boldsymbol{j}}(\boldsymbol{m})\right)^T \left(\boldsymbol{p}^{\boldsymbol{M}}_{\boldsymbol{j}} - \boldsymbol{p}^{\boldsymbol{M}}_{-\boldsymbol{j}}(\boldsymbol{m})\right) \quad (13)$$

$$\text{where } \boldsymbol{p}^{\boldsymbol{M}}_{-\boldsymbol{j}}(\boldsymbol{m}) := \frac{1}{(N_M - 1 + N_S)} \left(\sum_{\substack{k \in \mathcal{N}_M \\ k \neq j}} \boldsymbol{p}^{\boldsymbol{M}}_{\boldsymbol{k}} + \sum_{i \in \mathcal{N}_S} \boldsymbol{p}^{\boldsymbol{S}}_{\boldsymbol{i}} \right) \quad (14)$$

Equilibrium price does not depend on agent's own message (15)

Quadratic penalty term forces the agents to agree on one price (16)

Results

Theorem 1:

Let $\boldsymbol{m}^*$ be a Nash equilibria of the game specified by the game form and the agents' utility functions. Let $(\hat{\boldsymbol{g}}(\boldsymbol{m}^*), \hat{\boldsymbol{r}}(\boldsymbol{m}^*), \hat{\boldsymbol{x}}(\boldsymbol{m}^*), \hat{\boldsymbol{y}}(\boldsymbol{m}^*)) =: (\hat{\boldsymbol{g}}^*, \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$ be the transaction at $\boldsymbol{m}^*$ determined by the game form. Then,

(a) $(\hat{\boldsymbol{g}}^*, \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$ is individually rational, and

(b) $(\hat{\boldsymbol{g}}^*, \hat{\boldsymbol{r}}^*, \hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$ is an optimal solution of Problem (P_{SC}).

Theorem 2:

Given the optimum supply and purchase vector $(\hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$ of Problem (P_{SC}), there exists at least one Nash equilibria $\boldsymbol{m}^*$ of the game corresponding to the proposed game form and the agents' utility functions such that, $(\hat{\boldsymbol{x}}(\boldsymbol{m}^*), \hat{\boldsymbol{y}}(\boldsymbol{m}^*)) = (\hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$.

Furthermore, given $(\hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$, the set of all Nash equilibriathat result in $(\hat{\boldsymbol{x}}^*, \hat{\boldsymbol{y}}^*)$ can be characterized.

Conclusion

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- Studied a decentralized supply chain coordination problem with arbitrary number of suppliers and manufacturers under competitive set up.
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Future scope

- We have a constructive proof for the existence of Nash equilibria.
- We do not have an algorithm to show how to converge to the Nash equilibria.
- Orthogonal/greedy search is not guaranteed to converge because the resulting game is not supermodular.

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Developing algorithms or supermodular games that lead to the optimum centralized transactions.

Thank You!

Questions?

Contact:

- Shrutivandana Sharma
  - **email: svandana@umich.edu**
  - **web: http://www-personal.umich.edu/∼svandana**
- Volodymyr Babich
  - **email: babich@umich.edu**
  - **web: http://www-personal.umich.edu/∼babich**
- Demosthenis Teneketzis
  - **email: teneket@eecs.umich.edu**
  - **web: http://www.eecs.umich.edu/∼teneket**
- Mark P. Van Oyen
  - **email: vanoyen@umich.edu**
  - **web: http://www-personal.umich.edu/∼vanoyen**