1.1. hexagonal close packing

1.1.1. Prove that the lattice constants for hcp satisfy $c \approx 1.633 \, a$

Hint: If we put the origin (0,0,0) at the center of one equilateral triangle in plane A, the three corner points of this triangle shall have coordinates

\[ \left(0, \frac{\sqrt{3} \, a}{2}, 0\right), \left(\frac{a}{2}, -\frac{\sqrt{3} \, a}{6}, 0\right), \text{ and } \left(-\frac{a}{2}, -\frac{\sqrt{3} \, a}{6}, 0\right). \]

Above the center of this triangle, we put another sphere at $\left(0, 0, \frac{c}{2}\right)$, which is one of the points in plane B. The distance between any two of these four points is $a$. One can determine the value of $c$ using this condition.

1.1.2. Packing fraction

The volume of a primitive cell for a hexagonal lattice is $\sqrt{3} \, a^2 \, c / 2$ (as shown in the lecture note) and each primitive cell contains 2 spheres (one in layer A and the other in layer B). For close packing, $a = 2 \, R$. Prove that the packing fraction is $\frac{2 \times \text{volume of one sphere}}{\text{volume of the primitive cell}} \approx 0.740$

1.2. Chapter 2, page 43, problem 1, Interplanar separation

1.3. Chapter 2, page 44, problem 2, Hexagonal space lattice

1.4. Chapter 2, page 44, problem 4, Width of diffraction maximum

1.5. Chapter 2, page 44, problem 5, Structure factor of diamond