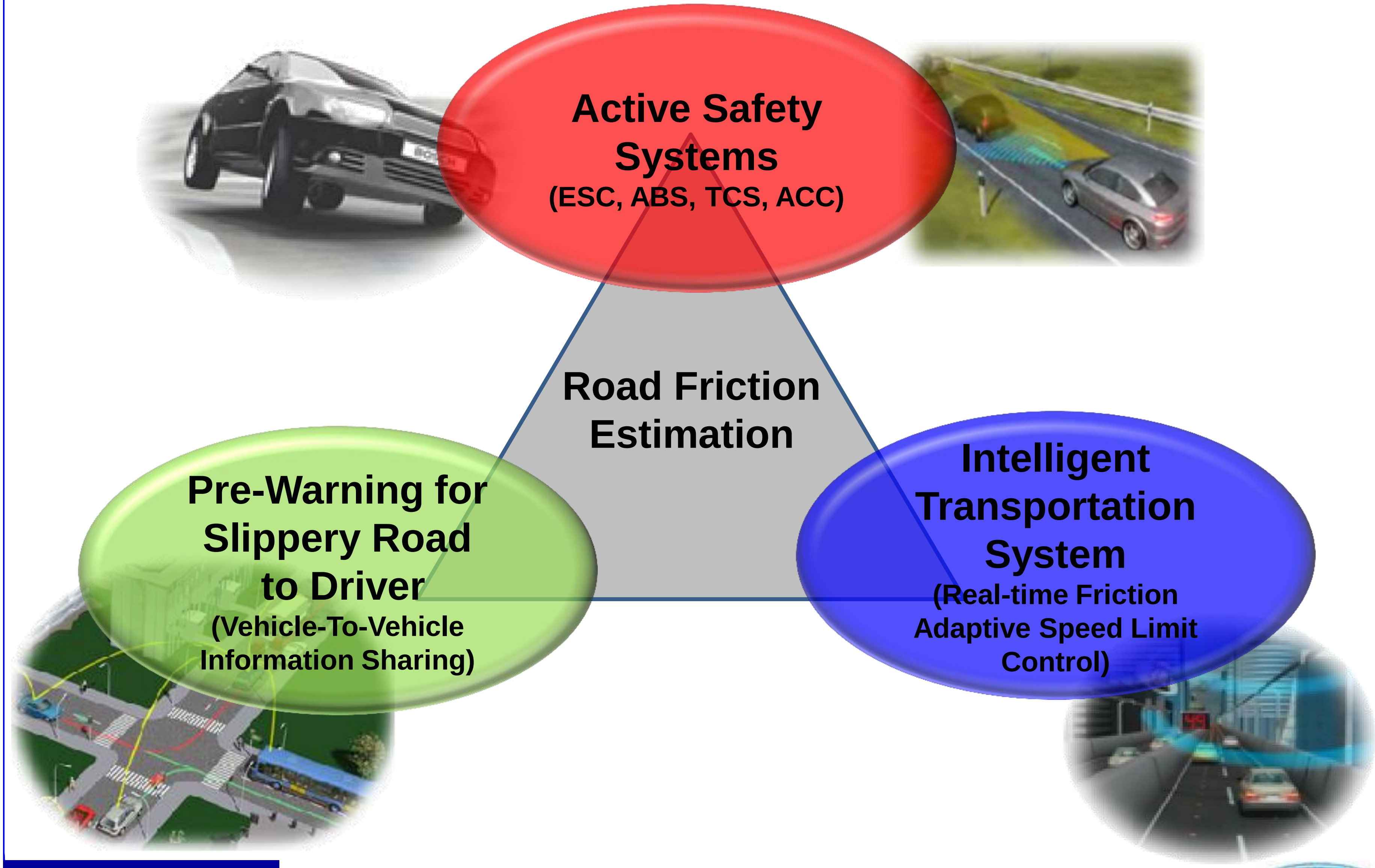
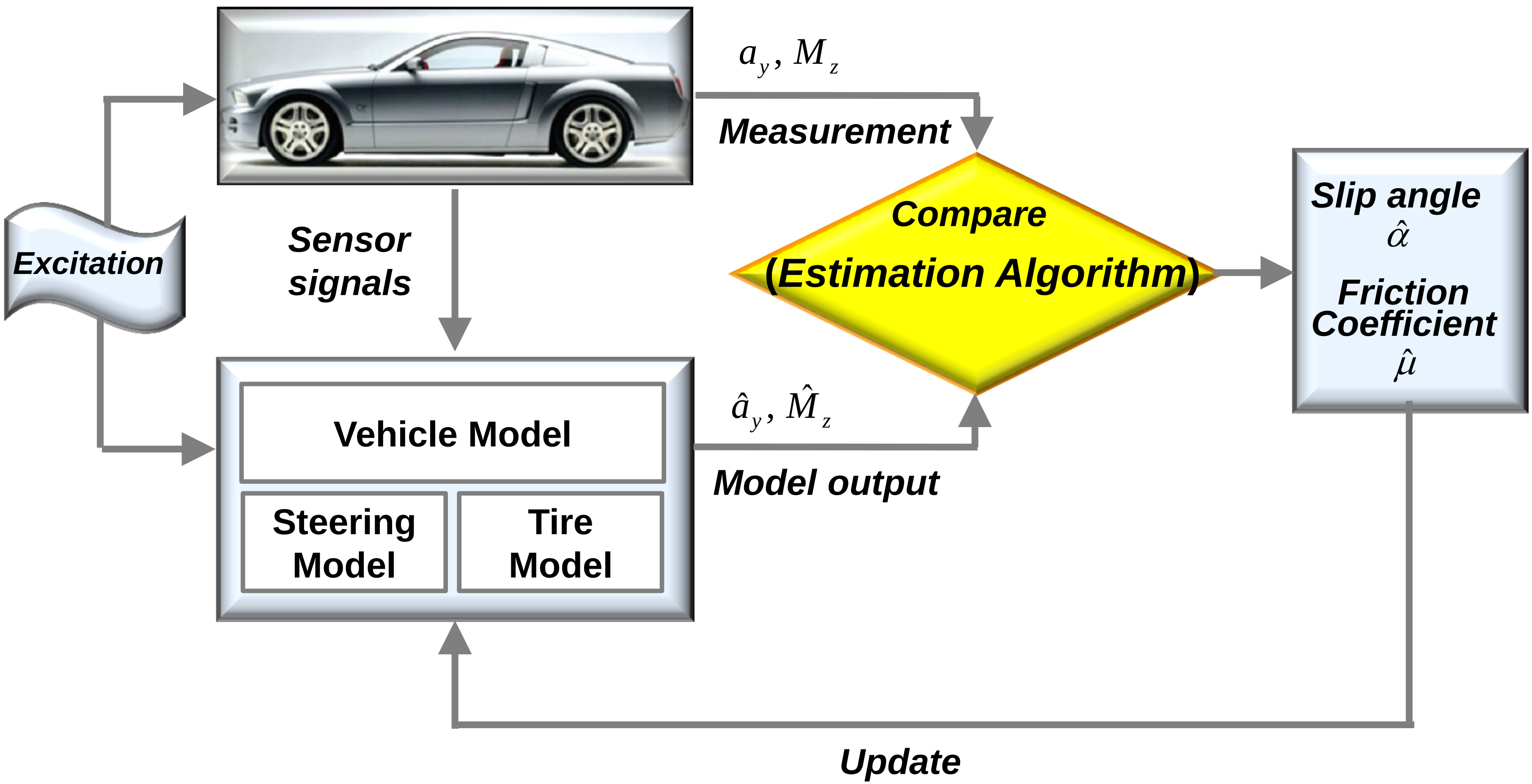
Benefits of Knowledge of Road Friction



Lateral Excitation Based Algorithms



Parameter and State Estimation for Nonlinear Systems

System

$$\dot{x} = f_0(x,u,\theta) + \Delta f(x,u,\theta),$$

$$y = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} h_{1,0} + \Delta h_1 \\ h_{2,0} + \Delta h_2 \end{bmatrix}.$$

Objective

Estimate state x and parameter θ

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} f_0(\hat{x},u,\hat{\theta}) \\ 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \left(y - \hat{y}_0 \right)$$

How can we determine gain L_1 and L_2?

Stability

$$\hat{x} \to x \;\; and \;\; \hat{\theta} \to \theta \;\; as \; t \to \infty$$

Robustness

Robust stability to uncertainties

Stability at An Operation State

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} f_0(\hat{x},u,\hat{\theta}) \\ 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - \hat{y}_0)$$

$$\dot{\hat{z}} = F_0(\hat{z},u) + L(\hat{z},u)\left(H(z,u) - H_0(\hat{z},u)\right)$$

Lyapunov Stability

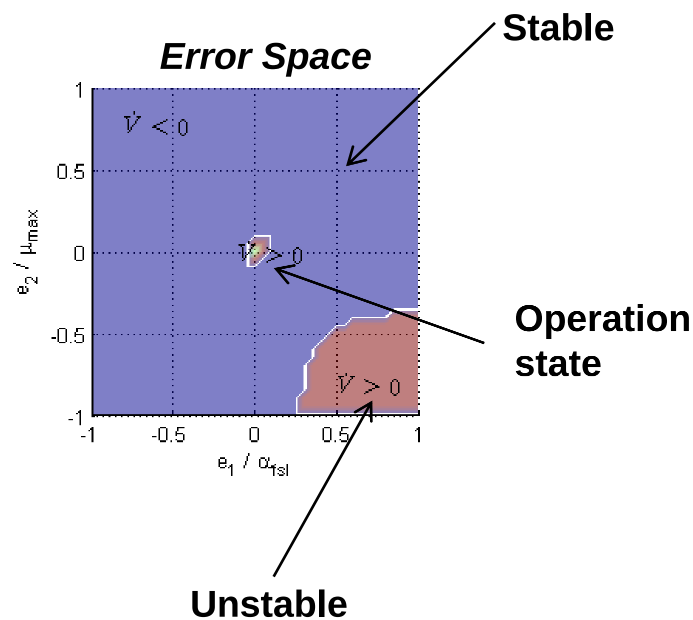
$$V(e,z,u) = e^T P e > 0$$

$$\dot{V}(e,z,u) = 2e^T P \dot{e} = 2e^T P\left\{F - \hat{F}_0 - L\left(H - \hat{H}_0\right)\right\} < 0$$

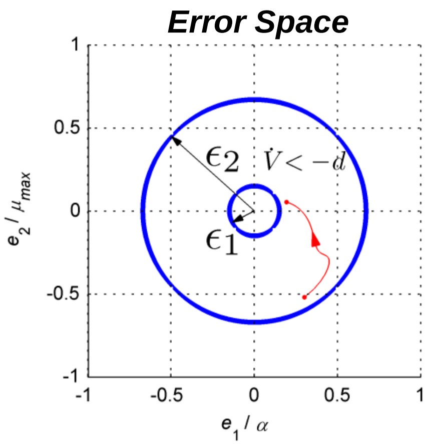
for all uncertainties(ΔF and ΔH)

in error space

with respect to a given operation state.



Observer Requirement



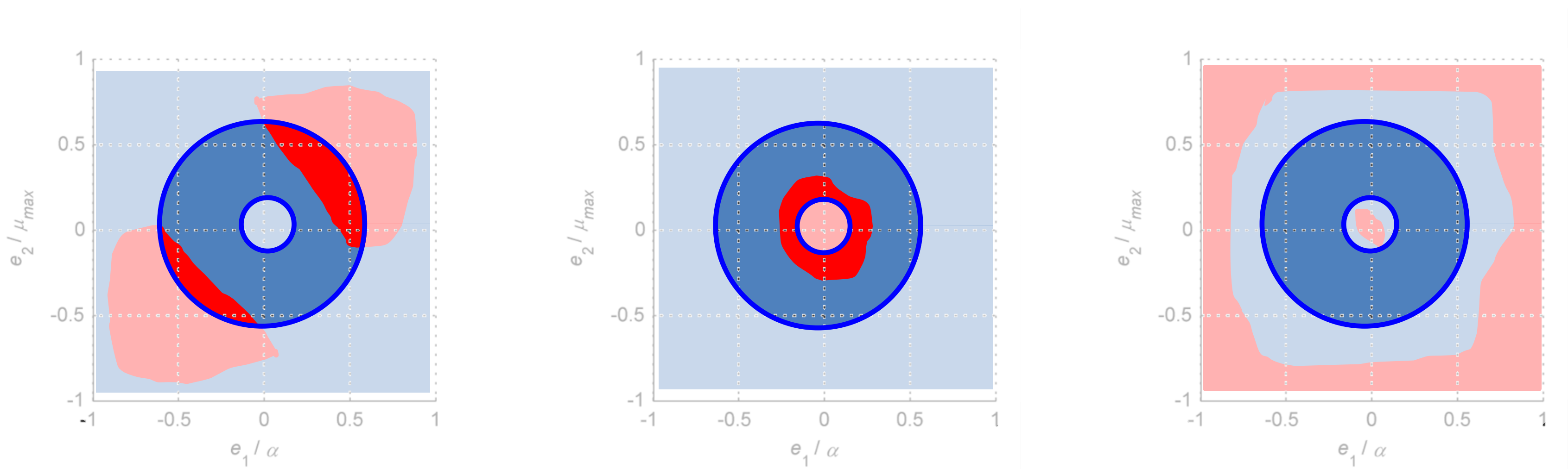
d: convergence parameter

ε_1: steady state error

ε_2: stable error bound

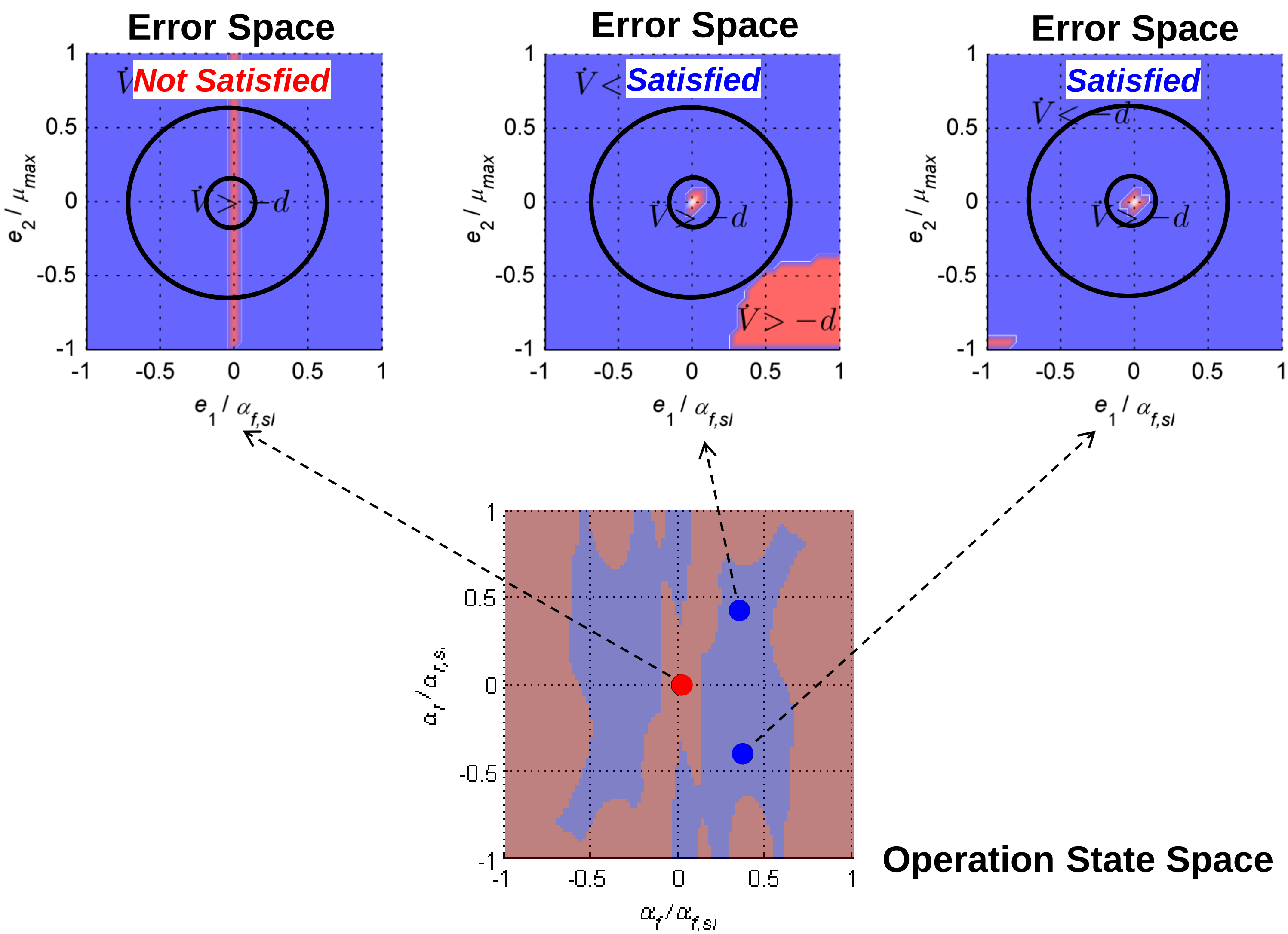
Observer requirement (design parameters)

Inside of donut: all error will converge to the inner circle



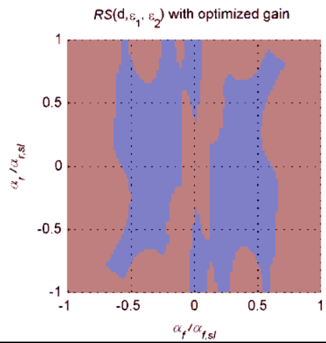
If inside of the donut is all blue,
then the observer is robustly stable at the given operation state.

Stability in an Operation State Space



Observer Gain for Robust Stability

Find L that maximizes the *blue area* under the given requirements.



$$p_1^*, k_1^* \sim k_4^* = \arg\max_{k_1 \sim k_4} \{ Blue\ \ Area \},$$

$$P = \begin{bmatrix} p_1 & 0 \\ 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} l_1 & l_2 \\ l_3 & l_4 \end{bmatrix}.$$

$$l_1 = \left(k_1 \frac{\partial h_1}{\partial x} + \frac{\partial f}{\partial x} \Big/ \frac{\partial h_1}{\partial x} \right)_{x=\hat{x}, \theta=\hat{\theta}}, \quad l_2 = k_2 \left(\frac{\partial h_2}{\partial x} \right)_{x=\hat{x}, \theta=\hat{\theta}},$$

$$l_3 = k_3 \left(\frac{\partial h_1}{\partial \theta} \right)_{x=\hat{x}, \theta=\hat{\theta}}, \quad l_4 = k_4 \left(\frac{\partial h_2}{\partial \theta} \right)_{x=\hat{x}, \theta=\hat{\theta}},$$

Robust Observer Design Synthesis

① System

$$\dot{x} = f_0 + \Delta f, \quad y = \begin{bmatrix} h_1 + \Delta h_1 \\ h_2 + \Delta h_2 \end{bmatrix}.$$

② Observer

$$\dot{\hat{z}} = F_0(\hat{z},u) + L(\hat{z},u)\big(H(z,u) - H_0(\hat{z},u)\big)$$

③

- Derive Gain Matrix
- Set d, ε_1 and ε_2 considering observer requirements. (Donut shape)
- Set Plant Uncertainties

$$\mathcal{F} = \bigcup \Delta F, \quad \mathcal{H} = \bigcup \Delta H.$$

④ Gain Optimization

Optimization to determine k_1, k_2, k_3, and k_4.

Friction/Slip Angle Estimation

$$\dot{\hat{\alpha}}_f = \left(\frac{1}{mV_x} + \frac{a^2}{I_z V_x}\right)\hat{F}_{yf} + \left(\frac{1}{mV_x} - \frac{ab}{I_z V_x}\right)\hat{F}_{yr} - r - \dot{\delta} + l_1\left(ma_y - \left(\hat{F}_{yf} + \hat{F}_{yr}\right)\right) + l_2\left(\tau_a - \hat{\tau}_a\right),$$

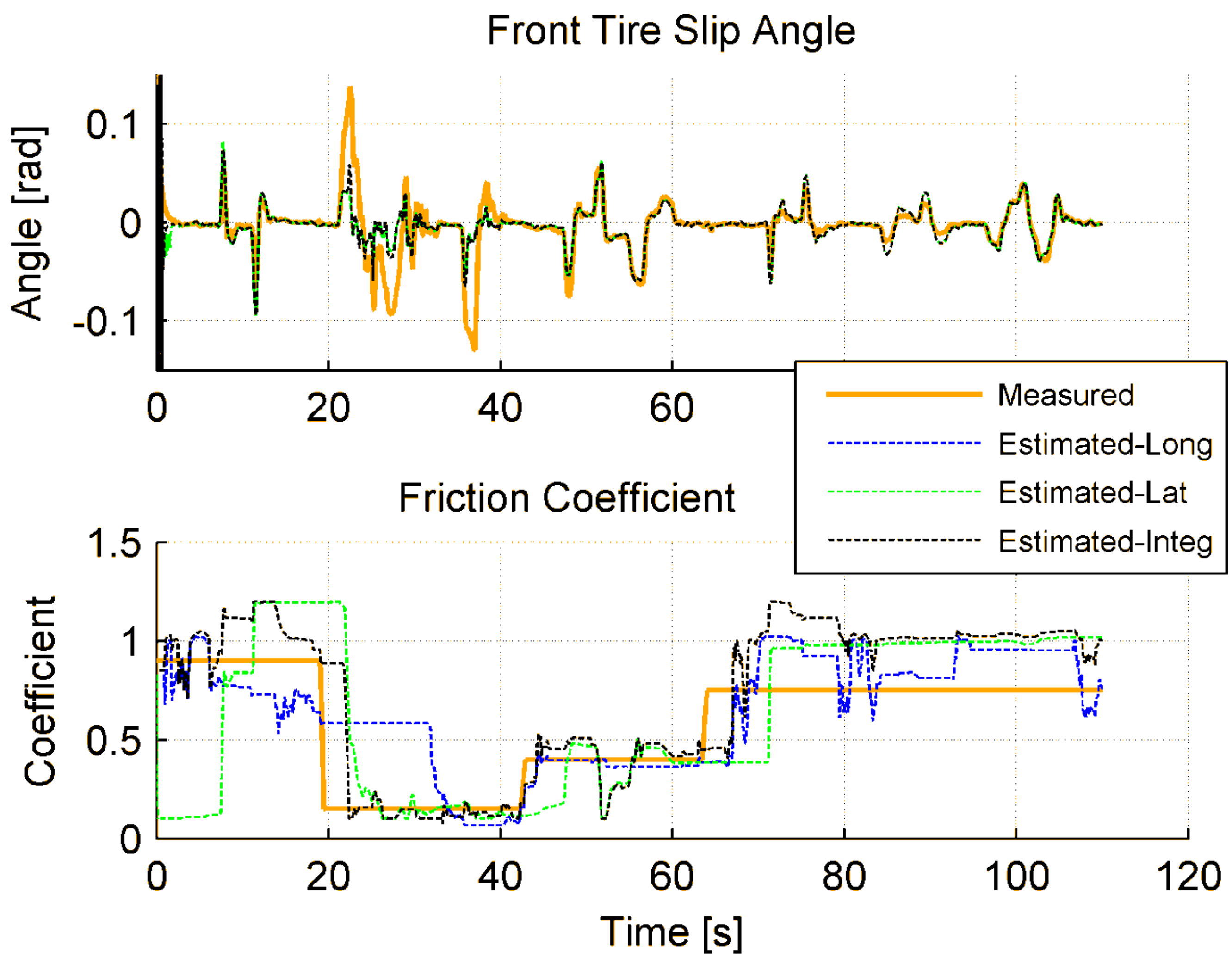
$$\dot{\hat{\mu}} = l_3\left(ma_y - \left(\hat{F}_{yf} + \hat{F}_{yr}\right)\right) + l_4\left(\tau_a - \hat{\tau}_a\right),$$

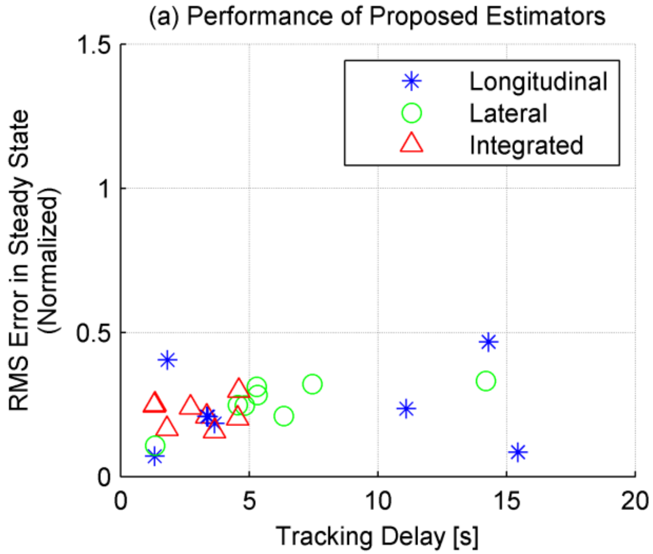
$$l_1 = \left(k_1 \frac{\partial h_1}{\partial x} + \frac{\partial f}{\partial x} \Big/ \frac{\partial h_1}{\partial x}\right)_{x=\hat{x},\theta=\hat{\theta}}, \quad l_2 = k_2\left(\frac{\partial h_2}{\partial x}\right)_{x=\hat{x},\theta=\hat{\theta}},$$

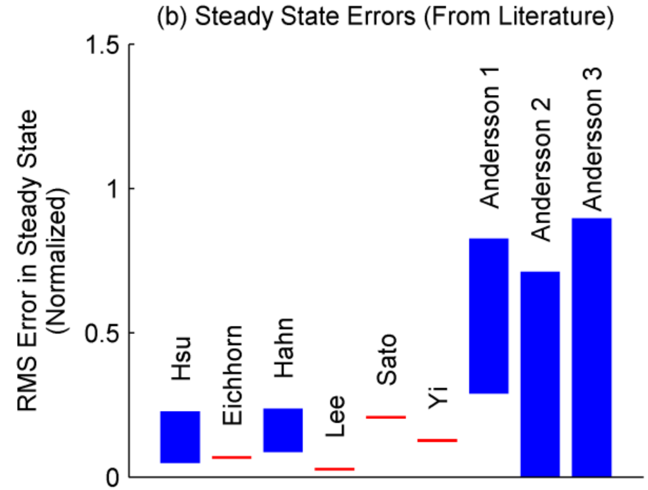
$$l_3 = k_3\left(\frac{\partial h_1}{\partial \theta}\right)_{x=\hat{x},\theta=\hat{\theta}}, \quad l_4 = k_4\left(\frac{\partial h_2}{\partial \theta}\right)_{x=\hat{x},\theta=\hat{\theta}},$$

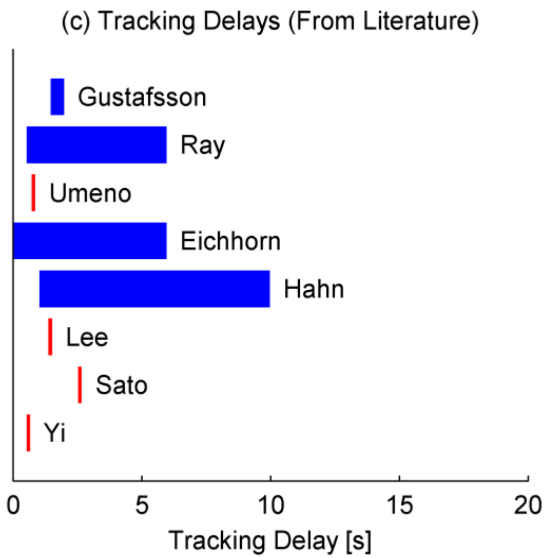
$$k_1 = 2.5\times 10^{-9},\ k_2 = 2.8\times 10^{-6},$$
$$k_3 = 1.8\times 10^{-8},\ k_4 = 1.9\times 10^{-4}.$$

Estimation Result (Test 8)









Gustafsson: Longitudinal
Ray: Longitudinal
Umeno: Wheel Vibration
Hsu: Self Aligning Moment
Eichhorn: Tread Deformation
Hahn: Lateral+GPS
Lee: Longitudinal
Sato: Camera+Temperature
Yi: Longitudinal
Andersson 1: Lateral
Andersson 2: Longitudinal
Andersson 3: Camera

Blue Bar: Ranges of Values
Red Bar: Single Value