Economics 431
Fall 1999
Vertical mergers

A vertical merger is a merger between a firm that sells the final good to consumers (the downstream firm, or D) and its supplier (the upstream firm, or U). The nature of their relationship is the following. Assume both firms are monopolies.. Suppose that upstream firm supplies Q_U units of intermediate good to the downstream firm, and the downstream firm makes Q_D units of final good from it and sells it to the final consumers. For simplicity assume that each unit of downstream good is produced from one unit of upstream good, so

$$Q_D = Q_U = Q.$$

That is, if the downstream firm wants to produce Q units of final goods, it must purchase Q units of intermediate good from the upstream firm. The marginal cost of producing the intermediate good is c, and the price that U charges is r. The price that D charges the final consumer is p and r plays the role of the marginal cost for the downstream firm (r is the cost of producing one unit of final good, because it requires a purchase of intermediate good that costs r). The demand on the final goods market is linear:

$$p = A - BQ$$

The downstream firm maximizes its profit from producing the final good given r:

$$\pi_D = \max_Q (A - BQ - r)Q$$

$$p = \frac{A + r}{2} \tag{1}$$

$$A - BQ = \frac{A + r}{2}$$

$$r = A - 2BQ \tag{2}$$

The last equation describes the relationship between the price charged for the intermediate good and the quantity that firm D demands. It describes the demand for intermediate good (firm U's product). Now we know firm U's demand, and we will determine what price it will charge:

$$\pi_U = \max_Q (r - c)Q = \max_Q (A - 2BQ - c)Q$$

$$r = \frac{A + c}{2} = p_M$$

$$A - 2BQ = \frac{A + c}{2}$$

$$Q = \frac{A-c}{4B} = \frac{Q_M}{2}$$

note that r equals monopoly price (i.e. the price that the upstream firm would charge if it also controlled the production of the final good could sell to consumers). But now there is another monopoly downstream that marks up the price even higher. Because of that, the quantity sold is less than monopoly quantity (namely, it is half of monopoly quantity). How well does the chain of monopolies do in terms of profits?

$$\pi_U = (r-c)\frac{Q_M}{2} = \frac{A-c}{2}\frac{A-c}{4B} = \frac{(A-c)^2}{8B} = \frac{\pi_M}{2} \tag{3}$$

$$\pi_D = (p-r)\,\frac{Q_M}{2} = \frac{A-r}{2}\frac{A-c}{4B} = \frac{A-c}{4}\frac{A-c}{4B} = \frac{(A-c)^2}{16B} = \frac{\pi_M}{4}$$

The sum of their profits is less than monopoly profit. The chain of monopolies does *worse* than a single monopoly.. This problem is known as *double marginalization.*

If U and D are integrated, their joint profit goes up to π_M. There is always a profit incentive to merge vertically, if both upstream and downstream firms are monopolies.

Merger for price discrimination

Consider an upstream monopoly U that sells to two downstream firms D_1 and D_2. Suppose that downstream firms sell to different markets: D_1 sells to the high-demand (low elasticity) market

$$p_1 = A_1 - BQ$$

and D_2 sells to the low-demand (high elasticity) market

$$p_2 = A_2 - BQ$$

where

$$A_1 > A_2$$

Firm U wants to charge a high price at market 1 and a low price at market 2 (draw a diagram to convince yourself). Sometimes the third degree price discrimination is impossible because of resale: if a monopoly U charges different prices to different customers D_1 and D_2, the ones that have a lower price can buy more goods and sell to the ones that face a higher price.

One way to avoid it is for firm U to merge with one of the downstream firms. But which one: D_1 or D_2?

Initially, U charges the uniform price r_u to maximize its total profit from selling to D_1 and D_2

$$\pi_u = \max_r (r_u - c)\,(Q_1 + Q_2)$$

From (2)

$$\begin{aligned} r_u &= A_1 - 2BQ_1 \\ r_u &= A_2 - 2BQ_2 \end{aligned} \tag{4}$$

$$\pi_u = \max_r (r_u - c)(Q_1 + Q_2) = \frac{1}{B}(r_u - c)\left(\frac{A_1 + A_2}{2} - r_u\right)$$

Let

$$\frac{A_1 + A_2}{2} = \overline{A} \text{ - the mid-point between demand intercepts}$$

The profit-maximizing uniform price is

$$r_u = \frac{\overline{A} + c}{2} \tag{5}$$

Consider two alternative mergers: U with D_1 and U with D_2. The pre-merger and post-merger profits are summarized in the following table. Post-merger, the integrated firm $U + D_i$ ($i = 1, 2$) will make monopoly profit at market i (π_M^i) plus firm U will make a half of monopoly profit at the other market, because it is an upstream supplier for the remaining downstream firm

	$U + D_1$	$U + D_2$
Pre-merger sum of profits	$\pi_u + \pi_D^1$	$\pi_u + \pi_D^2$
Psost-merger sum of profits	$\pi_M^1 + \frac{1}{2}\pi_M^2$	$\pi_M^2 + \frac{1}{2}\pi_M^1$

Define the profit gain from merger $\Delta\pi$ as an increase in post-merger profit over the pre-merger profit of the parts:

$$\begin{aligned} \Delta\pi_{U1} &= \pi_M^1 + \frac{1}{2}\pi_M^2 - \left(\pi_u + \pi_D^1\right) \\ \Delta\pi_{U2} &= \pi_M^2 + \frac{1}{2}\pi_M^1 - \left(\pi_u + \pi_D^2\right) \end{aligned}$$

Firm U will go for the merger with a larger profit gain. In particular, if

$$\Delta\pi_{U2} > \Delta\pi_{U1}$$

or, equivalently

$$\Delta\pi_{U2} - \Delta\pi_{U1} > 0$$

firm U would rather merge with firm D_2.

$$\Delta\pi_{U2} - \Delta\pi_{U1} = \pi_M^2 + \frac{1}{2}\pi_M^1 - \left(\pi_u + \pi_D^2\right) - \left(\pi_M^1 + \frac{1}{2}\pi_M^2\right) + \left(\pi_u + \pi_D^1\right) =$$

$$= \pi_M^2 + \frac{1}{2}\pi_M^1 - \pi_u - \pi_D^2 - \pi_M^1 - \frac{1}{2}\pi_M^2 + \pi_u + \pi_D^1 =$$

$$\underbrace{\left(\frac{1}{2}\pi_M^2 - \pi_D^2\right)}_{\text{Net profit gain from merger with } D2} - \underbrace{\left(\frac{1}{2}\pi_M^1 - \pi_D^1\right)}_{\text{Net profit gain from merger with } D1} . \tag{6}$$

The interpretation of this expression is the following. Suppose that firm U buys out either D_1 or D_2. The price it pays for firm D_i equals this firm's profit π_D^i. If firm

U buys D_i, in will enjoy monopoly position on market i. Compared to the situation when it is just an upstream firm for market i, firm U's profit will go up by $\frac{1}{2}\pi_M^i$ (from $\frac{1}{2}\pi_M^i$ to π_M^i). This profit gain is weighted against the price firm U pays for it: π_D^i. The more profitable merger is the one with a bigger net profit gain.

It turns out (after doing some tedious algebra) that it is always better to merge with D_2 - the downstream firm with more elastic demand.

The effects of this merger are the following:

- Quantity sold at the elastic market (market 2) increases (final goods price falls). This happens because by merging into market 2, firm U surmounts the double marginalization problem.
- Quantity sold at the inelastic market (market 1) decreases. This is because firm U now charges firm D_1 a price higher than r_u.
- As with many price discrimination cases, the effect of merger on the social welfare is ambiguous, because quantity increases in one market, but decreases in the other one.