So far we almost always worked in full-information environment. Clearly, assuming that every actor on the market knows prices charged by every firm, firms know each other’s marginal costs, consumers know the quality of the product, etc.

How much did we miss when we made this assumption? What important phenomena did we not capture?

We have extensively studied market inefficiency associated with imperfect competition: firms who are not price-takers produce a less than optimal level of output in an attempt to create artificial scarcity.

In this environment, not all possible beneficial exchanges between producers and consumers are realized: there are some consumers who are willing to pay above the good’s marginal cost, but never get the good, because the price is too high.

There is another important source of market inefficiency that firms may want to take advantage of: imperfect information.

Now we will study how limited information can disrupt a market. This study is important because its results contradict the strongest conclusions of standard economic models based on perfect information.

In particular, we will start with showing that competitive markets are inefficient when information is asymmetric.

A classic example is the market for used cars. Used cars are of high or low quality. Sellers know the quality of the car. Buyers do not know the quality of a particular car, but do know what fraction of the cars have high quality.

Let
\[ \delta = \frac{1}{2} \] - fraction of high quality cars
\[ v_H = 200 \] - willingness to pay for high quality car
\[ v_L = 100 < v_H \] - willingness to pay for high quality car
\[ c_H = 180 < v_H \] - how much the high quality car is worth to its current owner
\[ c_L = 80 < v_L \] - how much the low quality car is worth to its current owner

Under full information both types of cars are traded at the price that is between buyer’s and seller’s valuations.
Under asymmetric information, both cars must sell for the same price $p$. Since high quality cars are indistinguishable from low quality cars, each car has the same price. Suppose that both types of cars are put on the market.

\[ p = \delta v_H + (1 - \delta) v_L = 150 \]

Observe that

\[ p = \delta v_H + (1 - \delta) v_L = v_L + \delta (v_H - v_L) > v_L > c_L \]

At price $p$, all low quality cars are traded.

\[ p > c_L \]

\[ p = 150 < 180 = c_H \]

That is, there is no equilibrium where consumers buy good cars.

Formally, consider the following game between buyer and seller.

1. Nature randomly selects a car and gives it to the seller. Seller has a good car with probability $\delta$ and a bad car with probability $1 - \delta$.

2. Seller’s strategy: What to do if car is good, what to do if car is bad, what price to charge. Let $S$ be the action ”offer car for sale” and $N$ be the action ”don’t offer car for sale”.

...
3. Seller always charges the highest price that makes the buyer just indifferent between buying and not buying.

If both types of cars are sold: \((SS, p = 150)\) this is not an equilibrium, because seller does not play his best response: he gets a negative payoff from putting good cars on the market.

If only good cars are sold \((SN, p = 200)\), the seller with a bad car has an incentive to put it on the market.

Finally, if only bad cars are sold, this is an equilibrium:
It turns out that only low quality cars are traded. Bad cars drive out good cars. This is because usually the lower is the price, the more is the demand. Here low price conveys bad news about quality: consumers know that only bad cars can possibly be sold for a low price.

**How the warranty solves the lemons problem**
Any car with warranty can be sold for

\[ p_w = v_H \]

because consumer makes no distinction between a good car and a bad car with a warranty. Suppose that consumer believes that a car without a warranty is low quality (because warranty costs nothing for a high quality car, no high quality car will ever be offered without a warranty. Then, the one offered without the warranty must necessarily be low quality). Then a car without a warranty can be sold for

\[ p_0 = v_L \]
If warranty is costly

\[ w > v_H - v_L \]

only good cars carry it. Warranty perfectly discloses quality. (Separating equilibrium). Cars with no warranty are bad. Seller prefers not to offer bad cars with warranty, because he would rather sell a bad car for \( v_L \) than sell it for \( v_H \) and pay \( w \) for the warranty.

\[
(p_0, v_H - p_0) \\
(c_H, 0) \\
(p_0, v_L - p_0) \\
(c_L, 0) \\
\]

Pooling equilibrium when \( w < v_H - v_L \)

If warranty is cheap

\[ w < v_H - v_L \]

all cars come with warranty (Pooling equilibrium). It pays to sell a bad car for \( v_H \) with a warranty:

\[ v_H - w > v_L \]
Facts about advertising

1. Some ads contain no info about the product, and don’t even show the product
2. Profitability is positively correlated with ad intensity
3. If we take the same goods, ad to sales ratio varies widely
4. There is a negative correlation between ad intensity and elasticity of demand

Views on advertising

1. Advertising is wasteful (a Nash equilibrium that is not Pareto optimal) It is a best response to advertise when the rival does, but if no one advertises, both firms gain. In fact, why advertise Coca-Cola or Budweiser?

Suppose that market share is proportional to ad share

\[
\pi_1(s_1, s_2) = \frac{s_1}{s_1 + s_2} - cs_1
\]

\[
\frac{\partial \pi_1}{\partial s_1} = \frac{1}{s_1 + s_2} - \frac{s_1}{(s_1 + s_2)^2} - c = 0
\]

\[
\frac{s_2}{(s_1 + s_2)^2} = c
\]

Now use the symmetry condition

\[
\frac{1}{4s} = c; \quad s = \frac{1}{4c}
\]

\[
\pi = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}
\]

2. Advertising serves to coordinate consumers into purchasing the same network good. Some goods give consumers more utility if other people buy the same goods. These are called network goods. An example of a network good is a telephone - the more people own telephones, the more money consumers are willing to pay to have one. VCRs with different tape formats are network goods as well. The more people own a certain VCR format, the easier it is to find videos to rent, for example. This makes one’s VCR more valuable. The VHS tape format introduced by JVC has become the standard in the US only in the 80s. The competing Betamax format from Sony was squeezed out of the market.
Advertising may coordinate consumers into buying *the same* network good thus creating a network standard. This role of advertising is especially important for newly introduced network goods, such as connected Palm Pilots.

Formally, quantity demanded $Q$ depends not only on price $p$ but also on the amount of advertising $S$. Advertising creates a network effect: for a fixed price, quantity demanded rises with the amount of advertising.

$$Q(p, S) = (A - p) g(S)$$

The extent of this effect is characterized by *advertising elasticity of demand* which is equal to percentage change in quantity demanded in response to 1 percent increase in advertising:

$$\eta_S = \frac{dQ}{dS} \frac{S}{Q} = (A - p) \frac{dg}{dS} \frac{S}{(A - p) g(S)} = \frac{S g'(S)}{g(S)}$$

Suppose that $S$ stands for the number of mailings that the producer sends out or for the total length of commercials that he puts on TV in a given period. The marginal cost of one unit of advertising is $\tau$. Let the good be produced by a monopolist who chooses the price and the amount of advertising to maximize profit

$$\max_{p,S} \pi(p, S) = (p - c)Q(p, S) - \tau S$$

$$\frac{\partial \pi}{\partial p} = (p - c) \frac{\partial Q}{\partial p} + Q = (p - c) \frac{Q}{p} \left( \frac{p}{Q} \frac{\partial Q}{\partial p} + \frac{p}{p - c} \right) = 0$$

$$\frac{\partial \pi}{\partial S} = (p - c) \frac{\partial Q}{\partial S} - \tau = (p - c) \frac{Q}{S} \left( \frac{S}{Q} \frac{\partial Q}{\partial S} - \frac{\tau}{p - c} \frac{S}{Q} \right) = 0$$

The first condition is the usual one for the monopolist reflecting the inverse elasticity rule: relative markup is inversely proportional to the elasticity of demand:

$$\frac{p - c}{p} = \frac{1}{\eta_p}.$$ 

The second condition reads that the marginal benefit from advertising, $(p - c) \frac{\partial Q}{\partial S}$, equals the marginal cost, $\tau$. It can be rewritten as

$$\frac{\tau}{p - c} \frac{S}{Q} = \eta_S$$
Multiplying the first equation by the second one, we get
\[
\frac{p - c}{p} \cdot \frac{\tau}{p - c} \cdot \frac{S}{Q} = \frac{\tau S}{pQ} = \frac{\eta S}{\eta_p}.
\]
This reads that the ratio of ad expenditures \(\tau S\) to sales revenue \(pQ\) equals the ratio of advertising elasticity of demand to price elasticity of demand.

**Explaining uninformative advertising**

If advertising does not tell the consumer anything about the product and does not alter consumer tastes, why will there be image advertising? One explanation is that product quality is only revealed by consumption experience. A manufacturer who has low quality uses advertising to convince consumers that he has a high quality product. The message sent by advertising is the following: "I have a good product which will sell for a high price in the future, and because of that now I can afford to blow money on advertising" If only high quality products are advertised, and low quality are not, consumers rationally believe that advertised products are of high quality and are willing to pay more for them.

Formally, let the game proceeds as follows. Nature chooses the quality of the product and gives it to the manufacturer (cost of manufacturing is the same for high and low quality products and equals zero). After that, there are two periods. In the first period, product quality is unknown. If the product is purchased in the first period, consumer knows its quality in the second period. Consumer’s strategy is to decide whether to buy the product in a particular period by looking at the product’s price \(p\) and advertising expenditure \(a\).

Manufacturer’s strategy is to choose whether or not to advertise a high quality product, whether or not to advertise a low quality product and what price to set for the product. If the product is advertised, the advertising expenditure is \(A\), and otherwise it is zero. Advertising is "wasteful": consumers already know that the product exists and already know where to buy it. Because advertising is supposed to convey information about quality in the first period, the price of the product will depend on advertising level (but not on quality)

Let
- \(p_1(A)\) be the price of the advertised product
- \(p_1(0)\) be the price of the unadvertised product
- \(a_i(H)\) advertising expenditure on high quality product in period \(i = 1, 2\)
- \(a_i(L)\) advertising expenditure on low quality product in period \(i = 1, 2\)

**Solving the game**

Start with period 2 when product quality is known. Manufacturer will not advertise the product whose quality is known, because it no longer alters willingness to pay: \(a_2(H) = 0, a_2(L) = 0\). In period 2, the price will depend on quality: a high quality product will sell for \(p_2(H) = v_H\), and a low quality product will sell for
\( p_2(L) = v_L. \) Consumers, therefore, *always* get zero utility in period 2.

The period 1 game is shown on the figure. Suppose that we are looking for a separating equilibrium. That is, we want only high quality products to be advertised. Then, if the price in period 1, \( p_1(A) \) is low enough, then the manufacturer with a low quality good will not be able to afford to advertise. That’s how consumers know that they are offered a high quality product: a high quality manufacturer blows money on advertising and offers an introductory price - an action that is profitable *only* for a high quality firm. So, advertising of low quality products must not be profitable in a separating equilibrium (if quality is low manufacturer is better off not advertising):

\[
p_1(A) - A + v_L \leq 2v_L
\]

We must also check that the high quality manufacturer has no incentive to save the ad expenditure \( A \) and simply offer his product, unadvertised, for the price of low quality product \( v_L \)

\[
v_L + v_H \leq p_1(A) - A + v_H.
\]

Both of these conditions hold for

\[
p_1(A) = v_L + A
\]

This is the only price for which the manufacturer plays the best response to consumer’s strategy for both quality levels. Also, the consumer must have an incentive to buy the advertised product for price \( p_1(A) \). This is the case when he gets non-negative utility from buying:

\[
v_H - p_1(A) = v_H - v_L - A \geq 0
\]
The interpretation of this condition is the following: for the high quality product to be advertised, the advertising expenditure $A$ (which is the cost of signalling the product quality) must not exceed the maximum benefit from such signalling (i.e. the price premium $v_H - v_L$ that a high quality product commands). If this condition does not hold, advertising is too expensive a signalling device, and there is no separating equilibrium.

**Manufacturer’s dilemma (reputation)**

So far we assumed that the manufacturer gets a low quality product as a result of bad luck - the cost of producing high and low quality were the same. What if there is an incentive to skimp on quality to save on cost? If the quality of the product is unknown, and there is only one period, the manufacturer will try to cheat the consumers by selling low quality for high price.

If low quality products are indeed less costly to manufacture, why would anyone ever try to make a high quality product? It is clear that producers of high quality products count on consumers believing that the quality will be high in the future and coming to buy again. How can repeat purchases discipline a producer to stick to high quality?

Let there be two potential quality levels: high quality $q_H = 1$, and low quality $q_L = 0$. It costs $c_0$ to make a product of quality 0 and $c_1 > c_0$ to make a product of quality 1.

Each period, the producer chooses quality $q = \{0, 1\}$ and price $p$.

Consumers get utility $u = vq - p$ when they purchase a good of quality $q$ at price $p$.

If there were only one period, the only equilibrium is to cheat the consumer: produce $q = 0$ and charge $p = v$. If there is a finite and predetermined number of periods, same thing: manufacturer will cheat in the last period. Knowing this, consumers won’t buy in the last period. Then, the only period when someone could buy is the last-but-one period. Manufacturer has nothing to lose, so he will cheat in the last-but-one period as well, etc.

The situation is different when the game is repeated indefinitely. Suppose that the manufacturer has a discount rate $\delta$. The manufacturer’s reputation for high quality matters for repeat purchases: Consumers assume that quality is high in the first period, and assume that quality this period must be the same as last period.

$$E q_t = q_{t-1}$$

Formally, consumer’s strategy consists of expectation (belief) about quality $E q_t$ and purchasing decision given this belief (a rule that tells the consumer whether
to purchase or not depending on $E{q}_t$). Producer’s strategy is a rule that tells him what quality to produce and what price to charge after every history. The proposed subgame perfect equilibrium is for the manufacturer to produce high quality every period and to charge the price $p$. If he ever deviates to low quality, he is to produce the low quality from then on and charge $p_0 = 0$. Formally, for any history that involves producing only high quality in the past, the producer is to produce high quality and charge $p$. For any other history, he is to produce low quality and charge $p_0 = 0$.

We must check that the producer does not want to cheat. Suppose that he decides to produce low quality. If he does this once, the strategy tells him to produce only low quality thereafter. Therefore, the consumers will expect quality to be low forever after and not buy. Producer will get 0. For the proposed strategy to be subgame perfect, the producer must have no incentive to deviate

$$(p - c_1) \left(1 + \frac{1}{1 + r} + \frac{1}{(1 + r)^2} + \ldots \right) = \frac{p - c_1}{1 - \frac{1}{1 + r}} > p - c_0 + 0$$

This can be rewritten as

$$\frac{p - c_1}{r} > (c_1 - c_0)$$

The producer must have no deviations after every history, so he must have no incentive to rebuild his reputation once he has lost it by providing a high quality good for the price equal to zero:

$$0 - c_1 + \delta \frac{p - c_1}{1 - \delta} = -c_1 + \frac{p - c_1}{r} < 0$$

This implies that for the proposed strategies to be subgame perfect, the price must be in the following range

$$c_0 < \frac{p - c_1}{r} < c_1$$

The producer does not cut quality only if high quality implies reputation rent that the producer is afraid of losing.

Note that reputation matters only when consumers believe it matters. Reputation rents are created by consumers. If they expect low quality, the producer will never make high quality.

That is, there is another subgame perfect equilibrium. Different beliefs - different equilibrium. If consumers start from the belief that no matter what, quality will be low in the future, this belief and producing low quality every period is also a subgame perfect equilibrium. Whatever the history, the producer always makes low quality, and the consumers expect him to do so.