Economics 431
Winter 2002
Second Midterm Exam
Answer Key

1) (15 points) A local market for bottled water is served by 5 firms, whose marginal costs per bottle of water are constant and are given in the following table

Firm	1	2	3	4	5
c, \$	0.80	0.70	0.85	0.65	0.85

The market price for a bottle of water is \$ 1.21. Assume that you can use Cournot model with linear demand to describe how this market operates.

a) **(7 points)** Which firm has the highest market share? Explain.

Profit maximization by a Cournot oligopolist implies

$$p - c_i = Bq_i$$

$$\frac{p - c_i}{Q} = \frac{Bq_i}{Q} = Bs_i$$

The firm with the smallest marginal cost (firm 4) has the largest market share.

b) **(8 points)** Estimate the elasticity of demand for bottled water at market price.

$$p - c_i = Bq_i$$

$$np - \sum c_i = BQ$$

$$\frac{np - \sum c_i}{p} = \frac{BQ}{p} = \frac{1}{\eta}$$

Generalized inverse elasticity rule reads

$$\frac{p - \frac{1}{n}\sum c_i}{p} = \frac{1}{n}\frac{1}{\eta}$$

$$\frac{1.21 - 0.77}{1.21} = \frac{4}{11} = \frac{1}{5}\frac{1}{\eta}$$

$$\eta = \frac{11}{20} = 0.55.$$

2) (15 points) The first digital phone book (national phone directory) was made by a firm called Pro CD and sold for \$ 10,000 per copy in 1986. Since then, *one* *other* firm, American Business Information, started making the same product. Prices dropped lower and lower, and now a national phone directory is available for free.

a) **(8 points)** How can a presence of just one other competitor result in a very low price for a product? Explain. What features of the market can account for this outcome?

This is the market satisfying the assumptions of a Bertrand game. Two firms make identical products, and each firm can serve all the market. The optimal pricing strategy is to undercut the competitor's price. The only Nash equilibrium is to price at marginal cost, zero, in this case. The essential feature of the market that can account for this result is that each firm can serve all the market.

b) **(7 points)** Now consider the market for digital encyclopedias. This market, again, is served by two firms: Microsoft (which makes Encarta encyclopedia) and Britannica. Both Britannica and Encarta are now offered for \$ 89.99 on a CD and for \$ 85 as an on-line subscription. Why aren't digital encyclopedias free? What feature of the market makes a non-zero price sustainable?

This is the case of differentiated products. Encarta and Britannica are not the same product. Some consumers like one product, and some like the other one. Therefore, the *spatial Bertrand* model applies, and the price above marginal cost is the only Nash equilibrium.

3) (20 points) GigaTech is *the only* manufacturer of the cutting-edge new generation phone handsets that sell for \$ 200. SneakyCom is one among *many* providers of prepaid calling cards that sell for \$ 20. The customers who buy GigaTech phones and/or prepaid calling cards have the following reservation prices:

Customer's name	Reservation price for a phone, R_1	Reservation price for a card, R_2
A	205	10
B	210	15
C	180	25
D	190	40

Consumer utility from a product equals their reservation price minus the price of the product. Consumers do not buy unless they get positive utility.

a) **(5 points)** Suppose that the calling card and the phone are offered separately at prices $p_1 = 200$ for the phone and $p_2 = 20$ for the card. Which consumer buys which product(s).

A and B buy phone only, C and D buy card only.

b) **(8 points)** SneakyCom CEO approaches GigaTech CEO with the following plan. Instead of selling phones and cards separately, they can offer the phone and the card *only* as a package for the price of \$220 per package. Then consumers who like calling cards a lot may also buy phones, and GigaTech will be able to sell more phones. Which consumers buy which product(s) if the phone and the card are bundled? (Hint: calling cards are still available separately from other providers) What happens to the sales of phones?

A buys nothing
B buys a phone and card package
C buys card only
D buys card only:

$$190 + 40 - 220 = 10 < 20 = 40 - 20$$

Phone sales go down.

c) **(7 points)** Reconcile your result in b) with the fact that most new cars are offered only together with tires as one package. Will the dealers sell more cars if they offer them without tires and let the customers buy their most preferred tires elsewhere? Explain.

Cars and tires are *complementary* products. Tires are useless without cars, and cars are useless without tires. If tires and cars are not bundled, tire and car manufacturers eat away at each other's demand curve, which results in a higher price for a car plus tires, and necessarily lower sales. If cars are bundled with tires, setting the price for the bundle is under one party's control. This makes the price of car plus tires lower, and the quantity sold greater.

4) (30 points) Consider a senate race game. Player 1 (the incumbent senator) decides whether to launch an ad campaign or not (A, N). Player 2 (challenger) *simultaneously and independently* decides whether to enter the race or stay out (I, O). The payoffs from each action combination are given by (Player 1's payoff is the first entry in each cell).

		Player 2	
		I	O
Player 1	A	$1, 1$	$3, 3$
	N	$2, 4$	$4, 2$

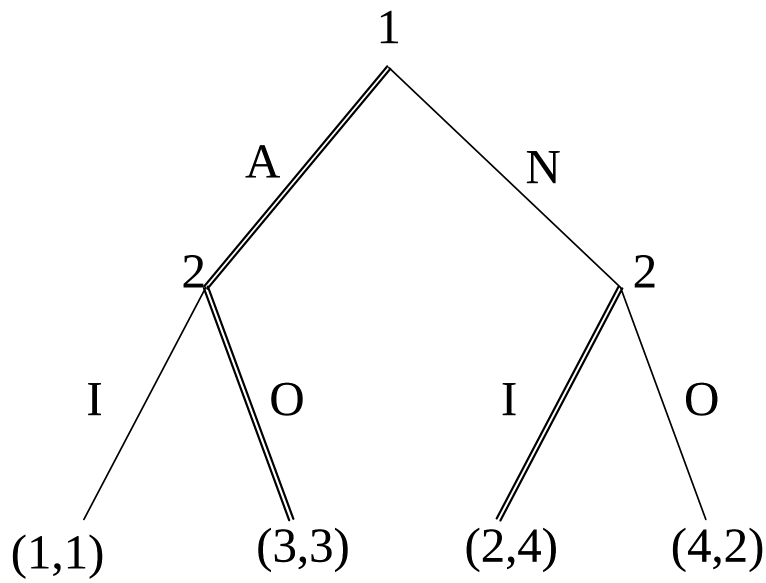
a) **(8 points)** Find all the Nash equilibria of this game. Which player(s), if any, have a dominant strategy?

		Player 2	
		I	O
Player 1	A	$1, 1$	$3,\underline{3}$
	N	$\underline{2},\underline{4}$	$\underline{4}, 2$

The only Nash equilibrium is (N, I). Player 1 has a dominant strategy, player 2 does not.

b) **(10 points)** Now suppose that Player 1 moves first by choosing either A or N. Player 2 *observes* player 1's action and then chooses I or O. For every action combination, the players' payoffs are the same as in the above payoff matrix. Draw a tree of this new game. How many strategies does player 1 have and what are they? How many strategies does player 2 have and what are they? Find all the subgame perfect equilibria of this game.

$S_1 = \{A, N\}$
$S_2 = \{II, IO, OI, OO\}$



Subgame perfect equilibrium is (A, OI).

c) **(12 points)** Now find *all* the Nash equilibria of the game in part b). Are there any Nash equilibria that are not subgame perfect? Which Nash equilibrium is more believable? Explain. Does Player 1 receive a higher payoff in a static game of part a) or in a dynamic game of part b)? Why do you think this difference arises?

	II	IO	OI	OO
A	$1, 1$	$1, 1$	$\underline{3}, \underline{3}$	$3, \underline{3}$
N	$\underline{2}, \underline{4}$	$\underline{4}, 2$	$2, \underline{4}$	$\underline{4}, 2$

We can see that there are two Nash equilibria: (A, OI) and (N, II). In the first one, player 2's strategy prescribes him to always do what's best for him, no matter which node of the game he is in. The second strategy involves bluff, or non-credible threat. Player 2 says, "I will always enter, even if it hurts me". Knowing this, it is in player 1's best interest not to advertise. Player 2 expects player 1 not to advertise, and so he knows the game will never reach the point where his bluff is called, so II is a best response to N.

Which Nash equilibrium is more believable? In the second equilibrium, player 2 makes a threat that he has no incentive to carry out. So why should player 1 believe him? "I don't care what you say you'll do, I will just advertise, and we'll see what happens". Every time a player plays a strategy he himself has no incentive to stick to, we have this credibility problem.

Player 1 gets a higher payoff of 3 in the subgame perfect equilibrium of the dynamic game. By moving first, he is able to take advantage of the different responses of player 2 to his different actions, and therefore can better manipulate the outcome of the game.

5) (30 points) Market demand is given by $p = 12 - Q$. There are two firms: the incumbent firm (I) and the entrant firm (E). Incumbent moves first by choosing quantity q_I from the interval $[0, 4]$. The entrant observes q_I and decides whether or not to enter and how much to produce if he enters (q_E). There is *no fixed cost of entry.* If the entrant decides to stay out, his profit is zero and the incumbent enjoys a monopoly position. Suppose that both incumbent and entrant have identical marginal costs equal to $c = 8$.

a) **(10 points)** What is the subgame perfect equilibrium of this game? What are the quantities produced by the incumbent and entrant? What are their profits?

The entrant always enters, because he can earn at least zero profit no matter what quantity the incumbent produces. Then this is a Stackelberg game.

$$q_I = q_M = \frac{A - c}{2B} = 2$$

$$q_E = \frac{A - c}{4B} = 1$$

$$\begin{aligned} \pi_I &= \frac{\pi_M}{2} = \frac{(A-c)^2}{8B} = 2 \\ \pi_E &= \frac{\pi_M}{4} = \frac{(A-c)^2}{16B} = 1 \end{aligned}$$

b) **(6 points)** What is the minimum quantity that must be produced by the incumbent to deter entry (to make entry unprofitable)? Will the incumbent ever try to deter entry by increasing quantity?

The dominant strategy for the entrant is

$$q_E^*(q_I) = \frac{A - c}{2B} - \frac{q_I}{2}$$

Whenever this strategy tells the entrant to produce positive quantity, the entrant is able to earn positive profit and therefore chooses to enter. Then the only quantity that can deter entry is

$$q_I = \frac{A - c}{B} = 4.$$

The incumbent will never choose this quantity because it yields zero profit, he can do better with lower quantities.

Now suppose that before the game begins the incumbent can purchase new equipment: he can either stay with the old equipment that leaves his marginal cost at $c = 8$ or spend an additional amount $K = 5$ on new equipment which cuts his marginal cost to $c_L = 6$. If the incumbent does not purchase new equipment, his marginal cost stays at $c = 8$, and the game proceeds as in part a).

c) **(14 points)** If the incumbent had purchased new equipment and anticipates entry, what quantity does he produce? Will he deter entry? What is the incumbent's payoff? At the beginning of the game, will the incumbent choose to purchase new equipment? (Hint: does the entrant's dominant strategy change in any way when the incumbent's marginal cost changes?)

The entrant's dominant strategy depends on the *entrant's* marginal cost, not the incumbent's. It is still

$$q_E^*(q_I) = \frac{A-c}{2B} - \frac{q_I}{2} = 2 - \frac{q_I}{2}$$

The incumbent chooses q_I so that it maximizes

$$\max_{q_I} \left(A - c_L - Bq_I - Bq_E^*(q_I)\right) \cdot q_I =$$

$$= \max_{q_I} \left(A - c_L - Bq_I - B\left(\frac{A-c}{2B} - \frac{q_I}{2}\right)\right) \cdot q_I =$$

$$= \max_{q_I} \left(12 - 6 - q_I - \left(\frac{12-8}{2} - \frac{q_I}{2}\right)\right) \cdot q_I =$$

$$= \max_{q_I} \left(4 - \frac{q_I}{2}\right) \cdot q_I$$

$$q_I^* = 4,\ q_E^* = 0$$

Entry is deterred. The incumbent has a payoff of

$$\pi_I = (p - c_L) \cdot q_I^* = (12 - q_I^* - c_L) \cdot q_I^* = 2 \cdot 4 = 8.$$

At the beginning of the game, if the incumbent purchases new equipment, it involves no up front spending and leaves him with a payoff of 2. If he buys new equipment, he spends 5 but gets a profit of 8,leaving him with a net payoff of 3. The incumbent chooses to buy new equipment.

Reference Guide

Linear Demand

$$p = A - BQ$$

Elasticity

$$\eta = \left| \frac{dQ}{dp} \frac{p}{Q} \right| = \frac{1}{B} \frac{p}{Q}.$$

Cournot Oligopoly

Firm i chooses q_i to maximize its profit given the outputs of all other firms:

$$\max_{q_i} \left(A - c_i - BQ_{-i} - Bq_i\right) q_i$$

In equilibrium, each firm's output must satisfy the condition for profit maximization:

$$A - c_i - BQ_{-i} - 2Bq_i = 0$$

or

$$p - c_i = Bq_i.$$

Stackelberg Game

Demand is linear, $p = A - BQ$, firm 1 is the leader, firm 2 is the follower, their marginal costs are constant and both equal c.

Follower's dominant strategy	$q_2^* (q_1) = \frac{A-c}{2B} - \frac{q_1}{2}$
Equilibrium quantity, leader	$q_1^* = \frac{A-c}{2B}$
Equilibrium quantity, follower	$q_2^* = \frac{A-c}{4B}$
Equilibrium profit, leader	$\pi_1^* = \frac{(A-c)^2}{8B}$
Equilibrium profit, follower	$\pi_2^* = \frac{(A-c)^2}{16B}$

Maximization problem

$$\max_q (a - bq)q$$

has the solution

$$q^* = \frac{a}{2b}$$