

**Economics 431**  
**Winter 2002**  
**1st Midterm exam**  
**Answer Key**

**1) (8 points)** Consider a competitive industry with identical firms (in equilibrium). If the fixed costs of all firms go up, the concentration ratio  $CR_4$  has to go up in the new long-run equilibrium.

*True.* The higher is the fixed cost, the larger is the minimum efficient scale. Less firms will stay, so the 4 biggest firms will have to account for a larger fraction of the total market share. In particular, if all firms are identical,

$$CR_4 = \frac{4}{N} \cdot 100\%.$$

**2) (7 points)** As long as consumers are willing to pay a positive price for the good, the more is the quantity produced, the greater is the total surplus from trade.

*False* When quantity exceeds the point where supply crosses demand, consumers value goods at less than the marginal cost of production, and surplus from trade is less than that at the crossing point.

**3) (7 points)** It costs the local telephone company \$2 per month to provide caller ID service to a household. Elasticity of demand for caller ID service equals  $-\frac{4}{3}$  (at any price). Then the telephone company will make more money if it offers caller ID at \$5 a month than if it offers this service at \$8 a month.

*False.*

$$p \left( 1 - \frac{1}{\eta} \right) = c$$
$$p \left( 1 - \frac{1}{4/3} \right) = \frac{p}{4} = 2$$

The profit maximizing price is \$8.

**4) (8 points)** Grocery stores typically have a 10% senior citizen discount, but some tanning salons offer as much as 25%. This must be because the pressure for companies to look charitable is much higher in the services industry than in retail.

*False.* Senior discounts have nothing to do with firms being charitable. Senior discounts are a form of price discrimination, and as such they raise the firms' profits. Elasticity of demand for groceries by seniors is not much higher than anyone else's,

hence a modest discount. But elasticity of demand for tanning among seniors must be *a lot* higher than in the rest of the population - on average, seniors have much less to gain from looking tanned. Hence price discrimination results in a deeper discount.

**5) (20 points)** An industry consists of many identical firms. Each firm's total cost function is

$$C(q) = \frac{1}{8} + \frac{1}{2}q^2$$

and each firm's supply curve is

$$p = q$$

where  $p$  is the market price and  $q$  is quantity supplied.

a) **(10 points)** What is the market price in the *long-run* equilibrium?

Long run equilibrium obtains after all entry and exit decisions have been made. Price equals the minimum average cost

$$p^* = \min_q \left( \frac{1}{8q} + \frac{1}{2}q \right)$$

The minimum is reached where

$$q^2 = \frac{1}{4}, q = \frac{1}{2}$$

$$p^* = \frac{1}{2}.$$

b) **(6 points)** Suppose that  $Q^* = 100$  units of the good are sold in this long-run equilibrium. What is the equilibrium number of firms in the industry?

Each firm supplies  $q = \frac{1}{2}$ . Then there must be a total of  $N = 200$  firms who supply  $Q^* = 100$ .

c) **(4 points)** What is the Herfindahl index for this industry? (If you did not get part *b*, simply assume that the number of firms is equal to  $N$  and leave your answer in letters)

$$H = \sum_{i=1}^N s_i^2 = \sum_{i=1}^N \left( \frac{100}{N} \right)^2 = \frac{10,000}{N} = 5.$$

**6) (20 points)** The same textbook that sells for \$ 70 in the US sells for \$ 5 in India. Suppose you know that if the publisher were to offer this book for the same (uniform) price in the two countries, *no one* in India would buy the book. Assume that the textbook is not offered outside of US and India.

a) **(10 points)** If the publisher had to charge the same price in both countries, what would it be? Are the US consumers harmed by price discrimination in this case? Explain.

The publisher's decision problem on the US market is *identical* under uniform pricing and under price discrimination. If Indian consumers do not buy at the common uniform price, the publisher acts as if she were only dealing with one market - the US. Hence the US price under third degree price discrimination is the same as that under uniform pricing - \$ 70. The US consumers are not harmed by price discrimination, but the Indian consumers and the publisher are made better off.

b) **(10 points)** Assume that the publisher's costs are exactly the same in either country. Suppose that the elasticity of demand for the textbook in the US is  $-\frac{35}{34}$ . What is the elasticity of demand for this textbook in India?

$$p_{US} \left(1 - \frac{1}{\eta_{US}}\right) = p_I \left(1 - \frac{1}{\eta_I}\right)$$

$$70 \cdot \left(1 - \frac{34}{35}\right) = 5 \cdot \left(1 - \frac{1}{\eta_I}\right)$$

$$\frac{2}{5} = 1 - \frac{1}{\eta_I}$$

$$\eta_I = \frac{5}{3}$$

**7) (20 points)** VoiceAce is a manufacturer of unique voice recognition software sold to home and business users. The software is offered in two versions, the Regular with the vocabulary of 20,000 words and the Deluxe with the vocabulary of 50,000 words.

Business customers ( $B$ ) value each word in the software's vocabulary at  $v_B = 1.5$  cents (\$0.015) whereas home use customers ( $H$ ) value each word at 0.8 cents (\$0.008). That is, if customer of type  $i$  ( $i$  is either  $B$  or  $H$ ) has a voice recognition software with a vocabulary of  $q$  words and pays a price  $p$ , her utility is

$$U_i = v_i q - p.$$

a) **(10 points)** Suppose that VoiceAce wants to sell the Regular version to the home use customers and the Deluxe version to the business customers, but cannot tell the customer types apart. What prices should VoiceAce charge for the two versions of software?

Low type ( $H$ ) must be indifferent between buying the Regular and not buying

$$p_R = v_H \cdot 20,000 = 0.008 \cdot 20,000 = \$160$$

High type ( $B$ ) must be indifferent between buying Deluxe and Regular at \$160

$$v_B \cdot 50,000 - p_D = v_B \cdot 20,000 - 160$$

$$p_D = 160 + 0.015 \cdot 30,000 = 610$$

b) **(10 points)** Suppose VoiceAce can make either one version or two versions of software at the *same* cost. Will VoiceAce always find it profitable to offer two versions of the software? Explain. What fraction of VoiceAce total sales (by quantity) must be to home use customers in order for it to offer two versions of the software?

VoiceAce can sell the Deluxe version to business customers only at

$$p_D = 0.015 \cdot 50,000 = \$750$$

For VoiceAce to offer two versions, it must be more profitable than selling to just the business customers for \$750 apiece and selling nothing to home use customers.

$$750n_B \leq 610n_B + 160n_H$$

$$140n_B \leq 160n_H$$

$$n_H \geq \frac{7}{8}n_B$$

$$n_H + \frac{7}{8}n_H \geq \frac{7}{8}(n_B + n_H)$$

$$\frac{n_H}{n_H + n_B} \geq \frac{7}{15} \text{ or } 46.7\%$$

**8) (20 points)** Popsico Inc is a soda manufacturer. The customers who buy Popsico soda have different tastes  $y$  for sugar content, with  $y$  ranging from 0 to 1. For each  $y$ , there is an equal density of customers with taste  $y$ . If  $y$  is the customer's most preferred sugar content, and the drink has sugar content  $x$ , then customer  $y$  is willing to pay

$$1 - 0.4|y - x|$$

for the can of soda. The marginal cost of producing one can of soda of any sugar content is equal to  $c = 0.1$ . Assume that the manufacturer always serves all the market.

a) **(5 points)** When you take over as the CEO of Popsico, it manufactures just one type of soda, the Thirstbuster, with sugar content  $x = \frac{1}{2}$ . Assume that the fixed cost of introducing this product has been already paid in full. What price per can will you set?

The marginal consumer pays the full price equal to 1. The marginal consumer is distance  $\frac{1}{2}$  from the product.

$$p + 0.4 \cdot \frac{1}{2} = 1$$

$$p = 0.8$$

b) **(5 points)** Suppose that your product manager proposes a complete product line overhaul. In particular, his plan is to eliminate the Thirstbuster altogether and to offer two new types of soda instead: Thirstbuster-lite and Thirstbuster-sweet. Each product type will cost  $F = 0.04$  to introduce (this fixed cost is measured in flow terms). What must be the sugar contents of the new products and their prices in order to maximize profit? Is this project going to raise Popsico's profits and, if so, by how much? When doing net profit calculations, assume that producing the old product, the Thirstbuster, entails zero fixed costs.

Old product

$$\pi = \left( V - c - \frac{t}{2} \right) \cdot 1 = 0.7$$

New product line:  $x_L = \frac{1}{4}$ ,  $x_H = \frac{3}{4}$ ,

$$p = V - \frac{t}{4} = 0.9$$

$$\pi = \left( V - c - \frac{t}{4} \right) \cdot \frac{1}{2} + \left( V - c - \frac{t}{4} \right) \cdot \frac{1}{2} - 2F = 0.8 - 2F = 0.72 > 0.7$$

c) **(10 points)** A guy from BigShot Consulting suggests that Popsico can make even more money (compared to part b) if instead of completely overhauling the product line the company keeps selling the Thirstbuster ( $x = \frac{1}{2}$ ) and introduces *just one* other product. This, he reasons, will save some of the fixed costs. If Popsico were to introduce just one other soda, what would the sugar content be? Would the Thirstbuster be offered at the same price as in part a)? What is the market share for the new product going to be? Whose plan results in higher profits for Popsico: the product manager's (in part b) or the consultant's? When doing profit calculations, assume that  $F = 0.04$  and that consumers who are indifferent between two drinks, will purchase the one closest to their taste.

The price for Thirstbuster stays the same, because otherwise Popsico does not serve all the market.

The new product can be located at either  $x = \frac{1}{4}$  or  $x = \frac{3}{4}$  and priced at  $p = 0.9$ .

The market share will equal 25% for the new drink (assuming that consumers who are indifferent will buy the *new* drink).

Profit:

$$\left( V - c - \frac{t}{2} \right) \cdot \frac{3}{4} + \left( V - c - \frac{t}{4} \right) \cdot \frac{1}{4} - F = 0.7 \cdot 0.75 + 0.8 \cdot 0.25 - 0.04 = 0.685 < 0.72$$

The product manager's plan is better. Moreover, not changing the product line at all is also better.

## Reference guide

### Derivatives of some functions

$$\frac{d}{dx}(x^2) = 2x$$
$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

### Marginal revenue for a monopolist

$$MR(q) = p(q) \cdot \left(1 - \frac{1}{\eta(p)}\right),$$

where  $p(q)$  is the market price when quantity sold is  $q$  and

$$\eta(p) = \left| \frac{dq_D}{dp} \cdot \frac{p}{q_D} \right|$$

is the positive of the elasticity of demand.