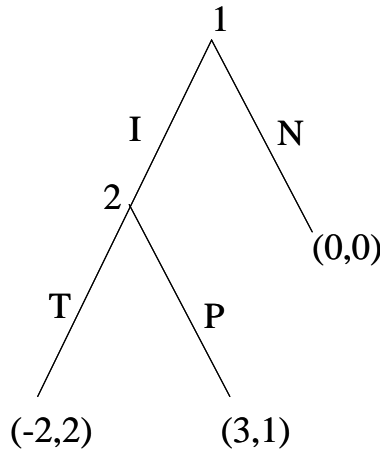


Economics 431
Winter 2002
Final exam
Answer Key

1) (30 points) Consider the following game between an investor (Player 1) and an entrepreneur (Player 2). Player 1 moves first and decides whether to invest (I) 2 million dollars in the entrepreneur's venture or stay out (N). If Player 1 does invest, Player 2 (entrepreneur) then decides whether to take the money and run away (T) or produce (P) and generate profits for himself and the investor. The tree of the game and the payoffs (investor's payoff is the first number, entrepreneur's payoff is the second number) are depicted below.



a) (8 points) Write down the payoff matrix of this game and find all of its Nash equilibria

	T	P
I	$-2, 2$	$\underline{3}, 1$
N	$\underline{0}, \underline{0}$	$0, 0$

There is a unique Nash equilibrium (N, T).

b) (7 points) Suppose that the game in part a) is repeated three times. Does this repeated game have a subgame perfect equilibrium that involves playing (I, P) in at least one period? Explain.

The *only* subgame perfect equilibrium of the repeated game is to play (N, T) every time.

c) (15 points) Now suppose that the game in part a) is repeated indefinitely and the probability adjusted discount factor is equal to δ . Write down a complete description of a strategy (for both players), such that the action profile (I, P) is played every period in a subgame perfect equilibrium. For what values of δ the strategy you described is a subgame perfect equilibrium? For full credit, you should check for all relevant deviations.

Player 1

Play I in the first period

For the history that had only (I, P) in all of the previous periods, play I this period.

For all other histories, play N this period

Player 2

Play P in the first period

For the history that had only (I, P) in all of the previous periods, play P this period.

For all other histories, play T this period

A strategy profile is a subgame perfect equilibrium if neither player has a profitable deviation after any history.

Player 1

After (IP, IP, \dots, IP) :

$$\underbrace{\frac{3}{1-\delta}}_{\text{Play } I} > \underbrace{0}_{\text{Play } N}$$

There are no profitable deviations for any δ

After all other histories

$$\underbrace{0}_{\text{Play } N} > \underbrace{-2 + 0 \frac{\delta}{1-\delta}}_{\text{Play } I} = -2$$

There are no profitable deviations for any δ

Player 2

After (IP, IP, \dots, IP) :

$$\underbrace{\frac{1}{1-\delta}}_{\text{Play } P} > \underbrace{2 + 0}_{\text{Play } T}$$

which is the case when

$$\delta > \frac{1}{2}$$

After all other histories, Player 2 gets zero forever no matter what he does, so no other action raises his payoff.

2) (20 points) The market has linear demand given by

$$p = 20 - Q$$

and three firm with constant marginal costs that differ across firms. Firms 1 and 2 have both marginal cost equal to $c = 8$, but firm 3 has a higher marginal cost equal to $d = 10$. Assume that pre-merger and post-merger firms play a Cournot game.

a) (10 points) Do firms 1 and 2 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly.

Pre-merger:

$$p = \frac{1}{4}(A + 2c + d) = 11.5$$

$$p - c = \frac{1}{4}(A - 2c + d) = 3.5$$

$$p - d = \frac{1}{4}(A + 2c - 3d) = 1.5$$

$$\pi_1 = \pi_2 = \frac{1}{16B}(A - 2c + d)^2 = 12.25$$

Post-merger

$$p = \frac{1}{3}(A + c + d) = 12.6667$$

$$p - c = \frac{1}{3}(A - 2c + d) = 4.6667$$

$$\pi_{1+2} = \frac{1}{9B}(A - 2c + d)^2 = 21.7778 < 24.5 = \pi_1 + \pi_2$$

b) (10 points) Do firms 1 and 3 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly. (Hint: will the newly merged firm find it optimal to produce any output at marginal cost d ?)

Pre-merger

$$\pi_1 = \frac{1}{16B}(A - 2c + d)^2 = 12.25$$

$$\pi_3 = \frac{1}{16B}(A + 2c - 3d)^2 = 2.25$$

Post-merger: no output is produced at the plant with marginal cost d . Firm 1 + 3 will have marginal cost c , and firm 2 will have the same marginal cost.

$$\pi_{1+3} = \frac{(A - c)^2}{9B} = 16 > 14.5 = \pi_1 + \pi_3$$

3) (15 points) Initially, the upstream firm U supplies two downstream firms D_1 and D_2 and charges uniform price r_u . Under uniform pricing the downstream firms D_1 and D_2 make $\pi_D^1 = 3$ and $\pi_D^2 = 18$ respectively. Monopoly profit at market 1 is $\pi_M^1 = 24$, monopoly profit at market 2 is $\pi_M^2 = 48$. Firm U wants to merge with one of the downstream firms in order to price-discriminate (charge the other firm a price different from r_u).

a) (10 points) Suppose that the upstream firm wants to merge with either D_1 or D_2 . Calculate explicitly which merger is more profitable. What is the post-merger profit of firm $U + D_i$, where i is either 1 or 2? (Hint: let pre-merger profit of firm U be π_U . In the end, your answer should not depend on π_U . Don't forget that post-merger firm $U + D_i$ also gets profits from being a monopolistic upstream supplier for the other downstream firm D_j . In equilibrium, this profit equals to $\frac{1}{2}\pi_M^j$)

If U merges with D_1 . Pre merger:

$$\pi_u + \pi_D^1 = \pi_u + 3$$

Post-merger

$$\pi_M^1 + \frac{\pi_M^2}{2} = 24 + \frac{1}{2} \cdot 48 = 48$$

Difference between pre-merger profit and post merger profit:

$$48 - 3 - \pi_u = 45 - \pi_u$$

If U merges with D_2 . Pre merger:

$$\pi_u + \pi_D^2 = \pi_u + 18$$

Post-merger

$$\pi_M^2 + \frac{\pi_M^1}{2} = 48 + \frac{1}{2} \cdot 24 = 60$$

Difference between pre-merger profit and post merger profit:

$$60 - 18 - \pi_u = 42 - \pi_u$$

It is more profitable to merge with D_1 .

b) (5 points) Suppose that the most profitable merger did take place. How did the quantities of final good sold at market 1 and market 2 change compared to the pre-merger situation? Explain.

The motive for merger is price discrimination. Under price discrimination, the profit-maximizing price on one market must be higher than r_U and the profit-maximizing price on the other market must be lower than r_U . The double marginalization problem is resolved on market 1 via merger. Therefore, this market has a lower price and a higher quantity, compared to pre-merger situation. The other market, market 2, has a price that is higher than r_U .

Alternatively, the most profitable merger is into the market with a higher elasticity of demand. Therefore, it must be the case that market 1 has higher elasticity of demand. This means that market 2 has a lower elasticity of demand than markets 1 and 2 combined. Then it must be the case that the post-merger price on market 2 is higher than r_U and the quantity is lower.

4) (20 points) An upstream manufacturer whose marginal cost is $c = 6$ sells his product to two retailers who are Cournot competitors. First, retailers simultaneously and independently decide whether or not to launch an ad campaign. If *at least one* retailer pays for the ad campaign, market demand is high:

$$p = A_H - Q, \text{ where } A_H = 24$$

If neither one launches the ad campaign, demand is low

$$p = A_L - Q, \text{ where } A_L = 15$$

The ad campaign costs $S = 28$. Next, given the high or low demand, retailers play a Cournot quantity game. Assume that each retailer gets the product from the manufacturer for the price $r = c = 6$ and pays a flat franchise fee equal to T .

a) (10 points) What are the retailers' equilibrium profits if both pay S , if one of them pays S and if neither of them pays S ? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

	Pay S		Don't Pay S
Pay S	$\frac{(A_H-c)^2}{9} - S - T, \frac{(A_H-c)^2}{9} - S - T$	$\frac{(A_H-c)^2}{9} - S - T, \frac{(A_H-c)^2}{9} - T$	$\frac{(A_H-c)^2}{9} - S - T, \frac{(A_H-c)^2}{9} - T$
Don't Pay S	$\frac{(A_H-c)^2}{9} - T, \frac{(A_H-c)^2}{9} - S - T,$	$\frac{(A_L-c)^2}{9} - T, \frac{(A_L-c)^2}{9} - T$	
	Pay S	Don't Pay S	
	Pay S	$8 - T, 8 - T$	$8 - T, \underline{36 - T}$
	Don't Pay S	$\underline{36 - T}, 8 - T$	$\underline{9 - T}, \underline{9 - T}$

In the Nash equilibrium, neither retailer pays S . Total maximum franchise fee

$$2T = \frac{2}{9} (A_L - c)^2 = 18.$$

b) (10 points) Suppose the manufacturer cannot launch the ad campaign himself and cannot force the retailers to pay for it. However, the manufacturer can impose a resale price maintenance agreement on each retailer, saying that they are to sell the good for the price equal to $p^* = 15$. In this case, each retailer will have half of the market. What are the retailers' equilibrium profits if both pay S , if one of them pays S and if neither of them pays S ? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

Total quantity:

$$Q = A - p^*$$

Each retailer sells

$$\frac{Q}{2} = \frac{1}{2}(A - p^*)$$

	Pay S		Don't Pay S
Pay S	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T, \frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T, \frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T, \frac{(p^*-c)(A_H-p^*)}{2} - T$
Don't Pay S	$\frac{(p^*-c)(A_H-p^*)}{2} - T, \frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - T, \frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - T, \frac{(p^*-c)(A_H-p^*)}{2} - T$
	Pay S	Don't Pay S	
Pay S	$12.5 - T, 12.5 - T$	$12.5 - T, 40.5 - T$	
Don't Pay S	$40.5 - T, 12.5 - T$	$0 - T, 0 - T$	

There are two Nash equilibria, but in each of them involves someone paying for the ad campaign.

All equilibrium profits can potentially be extracted as a franchise fee:

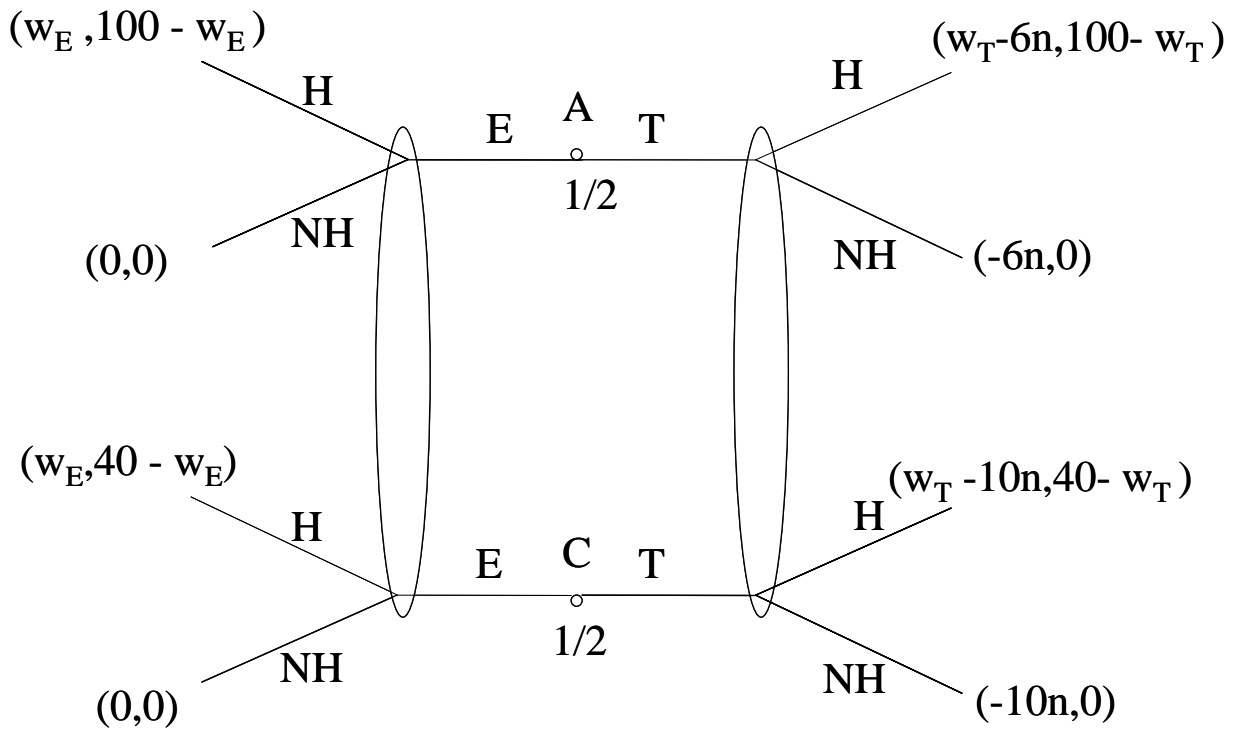
$$40.5 + 12.5 = 53$$

If the manufacturer must charge everyone the same T , he can only extract 25.

5) (30 points) Suppose that college students are of two types when it comes to qualities most desired by employers. One half of all students is of type A (able) and the other half is of type C (challenged). Employers are willing to pay up to 100 to type A student and up to 40 to type C student. Before the student goes on the job market, she chooses whether or not to take a tough major that consists of n difficult quantitative courses. Each student must sacrifice some party time in order to take a tough course. Suppose that this sacrifice costs 6 per course to type A , but it costs more, 10 per course, to type C .

Formally, the game is between one student and one employer, and it proceeds as follows. Nature chooses student type: A or C . The student (Player 1) knows her type, but the employer does not. Each student type chooses whether to take only easy courses (E) or to take n tough courses (T). The student then announces her salary requirement w . The employer observes whether or not the student has taken

tough courses, and decides whether to hire the student and pay her w . The game and the payoffs are depicted on the figure below.



a) (10 points) Construct an equilibrium where only A student plays T and C student plays E . Specify the employer's beliefs when he observes someone who plays T and someone who plays E . What should be the salary requirement of student A ? What should be the salary requirement of student C ?

Employer believes that the student who takes tough courses is type A for sure. With this belief, the student who plays T must request 100 in order to maximize her payoff.

Employer believes that the student who takes easy courses is type C for sure. With this belief, the student who plays E must request 40 in order to maximize her payoff.

b) (10 points) For what values of n does type A have an incentive to play T ? (Hint: if type A plays E , how would employer's beliefs change? Based on that, what should be the salary requirement of type A ?)

If type A also plays E , the employer only sees the students who take easy courses, and he believes that only a half of them are type A . Therefore, he is willing to pay up to

$$\frac{1}{2}100 + \frac{1}{2}40 = 70$$

to such a student. If type A plays E , he should request 70. Then, type A has an incentive to play T if it yields a higher equilibrium payoff

$$\underbrace{100 - 6n}_{\text{if type } A \text{ plays } T} \geq \underbrace{70}_{\text{if type } A \text{ plays } E}$$

$$n \leq 5$$

c) (10 points) For what values of n does type C have an incentive to play E ? (Hint: if type C plays T instead, how would employer's beliefs change? Based on that, what should be the salary requirement of type C who plays T ?). Find all values of n for which the strategy profile (TE, HH) described in part a) is a Bayesian Nash equilibrium.

If type C plays T , the employer only sees the students who take tough courses, and he believes that only a half of them are type C . Therefore, he is willing to pay up to

$$\frac{1}{2}100 + \frac{1}{2}40 = 70$$

to such a student. If type C plays T , he should request 70 and be hired. Then, type C has an incentive to play E if it yields a higher equilibrium payoff

$$\underbrace{40}_{\text{if type } C \text{ plays } E} \geq \underbrace{70 - 10n}_{\text{if type } C \text{ plays } T}$$

$$n \geq 3$$

For the strategy profile (TE, HH) to be an equilibrium, n must be 3, 4, or 5.

Reference Guide

Present value calculations

$$1 + \delta + \delta^2 + \dots = \frac{1}{1 - \delta}$$

$$\delta + \delta^2 + \delta^3 + \dots = \frac{\delta}{1 - \delta}$$

Cournot game between n firms with different marginal costs c_i

Demand:

$$p = A - BQ$$

Profit maximization condition of firm i :

$$p - c_i = Bq_i$$

Equilibrium price:

$$p = \frac{A + \sum_{i=1}^n c_i}{n + 1}$$

Alternative expressions for equilibrium profit of firm i :

$$\pi_i = (p - c_i) q_i = Bq_i^2 = \frac{1}{B} (p - c_i)^2$$