Economics 431
Winter 2002
Final Exam
April 25, 2002

Print your name here ______________________________

Your UM ID number[1] ___________________________

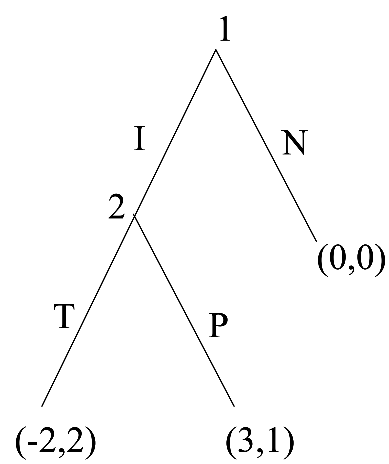
Instructions:

- Do not open the exam until you are told to do so.
- *Once the exam begins*, check that you have all the pages. There should be 13 pages including this one.
- *Once the exam begins*, print your name in capital letters on top of each page to receive credit for it.
- This is a closed book, closed notes exam.
- You have 100 minutes (1 hour and 40 minutes) to take the exam.
- Answer the questions in the space provided. To get credit on word questions, you should provide a brief explanation of your answer. Please write concisely and to the point. Feel free to use diagrams, but label them properly. If your answer involves doing math, show all work (this way you will get partial credit in case your ideas are correct but your math is not).
- If you run out of space on a particular question, you may use the back side of the same page. Clearly indicate on the front of the page that your answer is on the back; and on the back, give the number of question you are answering.

Good Luck!

[1] The underlined 8 digits on the face of your M-Card

1) (30 points) Consider the following game between an investor (Player 1) and an entrepreneur (Player 2). Player 1 moves first and decides whether to invest (I) 2 million dollars in the entrepreneur's venture or stay out (N). If Player 1 does invest, Player 2 (entrepreneur) then decides whether to take the money and run away (T) or produce (P) and generate profits for himself and the investor. The tree of the game and the payoffs (investor's payoff is the first number, entrepreneur's payoff is the second number) are depicted below.



a) (8 points) Write down the payoff matrix of this game and find all of its Nash equilibria

b) (7 points) Suppose that the game in part a) is repeated three times. Does this repeated game have a subgame perfect equilibrium that involves playing (I, P) in at least one period? Explain.

c) (15 points) Now suppose that the game in part a) is repeated indefinitely and the probability adjusted discount factor is equal to δ. Write down a complete description of a strategy (for both players), such that the action profile (I, P) is played every period in a subgame perfect equilibrium. For what values of δ the strategy you described is a subgame perfect equilibrium? For full credit, you should check for all relevant deviations.

(contunue here)

2) (20 points) The market has linear demand given by

$$p = 20 - Q$$

and three firm with constant marginal costs that differ across firms. Firms 1 and 2 have both marginal cost equal to $c = 8$, but firm 3 has a higher marginal cost equal to $d = 10$. Assume that pre-merger and post-merger firms play a Cournot game.

a) (10 points) Do firms 1 and 2 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly.

b) (10 points) Do firms 1 and 3 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly. (Hint: will the newly merged firm find it optimal to produce any output at marginal cost d?)

3) (15 points) Initially, the upstream firm U supplies two downstream firms D_1 and D_2 and charges uniform price r_u. Under uniform pricing the downstream firms D_1 and D_2 make $\pi_D^1 = 3$ and $\pi_D^2 = 18$ respectively. Monopoly profit at market 1 is $\pi_M^1 = 24$, monopoly profit at market 2 is $\pi_M^2 = 48$. Firm U wants to merge with one of the downstream firms in order to price-discriminate (charge the other firm a price different from r_u).

a) (10 points) Suppose that the upstream firm wants to merge with either D_1 or D_2. Calculate explicitly which merger is more profitable. What is the post-merger profit of firm $U + D_i$, where i is either 1 or 2? (Hint: let pre-merger profit of firm U be π_U. In the end, your answer should not depend on π_U. Don't forget that post-merger firm $U + D_i$ also gets profits from being a monopolistic upstream supplier for the other downstream firm D_j. In equilibrium, this profit equals to $\frac{1}{2}\pi_M^j$)

b) (5 points) Suppose that the most profitable merger did take place. How did the quantities of final good sold at market 1 and market 2 change compared to the pre-merger situation? Explain.

4) (20 points) An upstream manufacturer whose marginal cost is $c = 6$ sells his product to two retailers who are Cournot competitors. First, retailers simultaneously and independently decide whether or not to launch an ad campaign. If *at least one* retailer pays for the ad campaign, market demand is high:

$$p = A_H - Q, \text{ where } A_H = 24$$

If neither one launches the ad campaign, demand is low

$$p = A_L - Q, \text{ where } A_L = 15$$

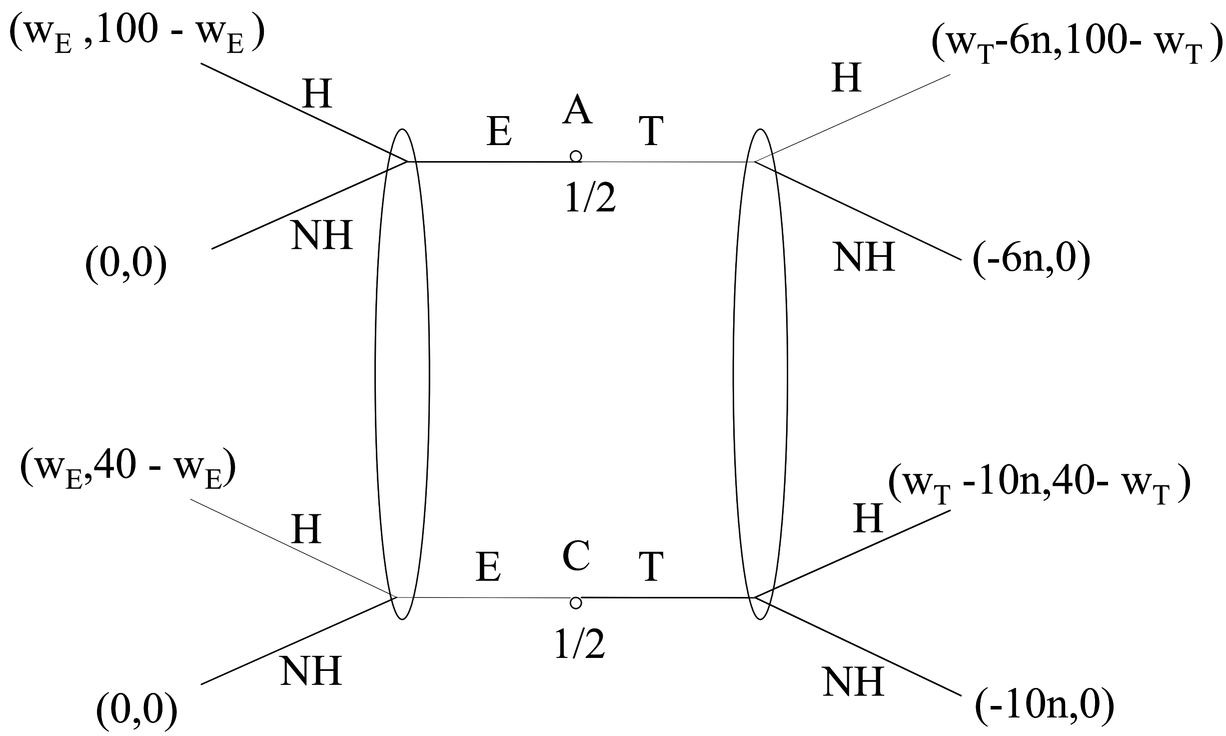
The ad campaign costs $S = 28$. Next, given the high or low demand, retailers play a Cournot quantity game. Assume that each retailer gets the product from the manufacturer for the price $r = c = 6$ and pays a flat franchise fee equal to T.

a) (10 points) What are the retailers' equilibrium profits if both pay S, if one of them pays S and if neither of them pays S? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

b) (10 points) Suppose the manufacturer cannot launch the ad campaign himself and cannot force the retailers to pay for it. However, the manufacturer can impose a resale price maintenance agreement on each retailer, saying that they are to sell the good for the price equal to $p^* = 15$. In this case, each retailer will have half of the market. What are the retailers' equilibrium profits if both pay S, if one of them pays S and if neither of them pays S? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

5) (30 points) Suppose that college students are of two types when it comes to qualities most desired by employers. One half of all students is of type A (able) and the other half is of type C (challenged). Employers are willing to pay up to 100 to type A student and up to 40 to type C student. Before the student goes on the job market, she chooses whether or not to take a tough major that consists of n difficult quantitative courses. Each student must sacrifice some party time in order to take a tough course. Suppose that this sacrifice costs 6 per course to type A, but it costs more, 10 per course, to type C.

Formally, the game is between one student and one employer, and it proceeds as follows. Nature chooses student type: A or C. The student (Player 1) knows her type, but the employer does not. Each student type chooses whether to take only easy courses (E) or to take n tough courses (T). The student then announces her salary requirement w. The employer observes whether or not the student has taken tough courses, and decides whether to hire the student and pay her w. The game and the payoffs are depicted on the figure below.



a) (10 points) Construct an equilibrium where only A student plays T and C student plays E. Specify the employer's beliefs when he observes someone who plays T and someone who plays E. What should be the salary requirement of student A? What should be the salary requirement of student C?

b) (10 points) For what values of n does type A have an incentive to play T? (Hint: if type A plays E, how would employer's beliefs change? Based on that, what should be the salary requirement of type A?)

c) (10 points) For what values of n does type C have an incentive to play E? (Hint: if type C plays T instead, how would employer's beliefs change? Based on that, what should be the salary requirement of type C who plays T?). Find all values of n for which the strategy profile (TE, HH) described in part a) is a Bayesian Nash equilibrium.

Reference Guide

Present value calculations

$$1+\delta+\delta^2+...=\frac{1}{1-\delta}$$

$$\delta+\delta^2+\delta^3+...=\frac{\delta}{1-\delta}$$

Cournot game between n firms with different marginal costs c_i

Demand:

$$p = A - BQ$$

Profit maximization condition of firm i:

$$p - c_i = Bq_i$$

Equilibrium price:

$$p = \frac{A + \sum_{i=1}^{n} c_i}{n+1}$$

Alternative expressions for equilibrium profit of firm i:

$$\pi_i = (p - c_i)\, q_i = Bq_i^2 = \frac{1}{B}\,(p - c_i)^2$$