

Economics 431
Summer 2007
Midterm Exam
Answer Key

Part I True or false (40 points total - Credit given for explanation)

1) (6 points) If a profit-maximizing competitive firm produces such a quantity q that $MC(q) = AC(q)$, it must be earning zero economic profit.

True. Profit equals

$$\pi = pq - C(q) = q(p - AC(q))$$

The profit is maximized at a quantity where

$$p = MC(q)$$

Since $MC(q) = AC(q)$,

$$p - AC(q) = p - MC(q) = 0.$$

therefore,

$$\pi = 0.$$

2) (4 points) In a competitive industry with identical firms that have increasing marginal costs, the higher is the fixed cost, the higher is the long-run equilibrium price.

True. In the long-run equilibrium, the price is equal to minimum average cost. The higher is the fixed cost, the higher is the minimum average cost.

3) (5 points) Consider a monopolist with a *constant* marginal cost. The higher is the elasticity of demand at the market price chosen by the monopolist, the higher is the monopolist's profit to sales ratio.

False.

$$MR = p \left(1 - \frac{1}{\eta} \right)$$

At monopoly price

$$MR = p \left(1 - \frac{1}{\eta} \right) = c = MC.$$

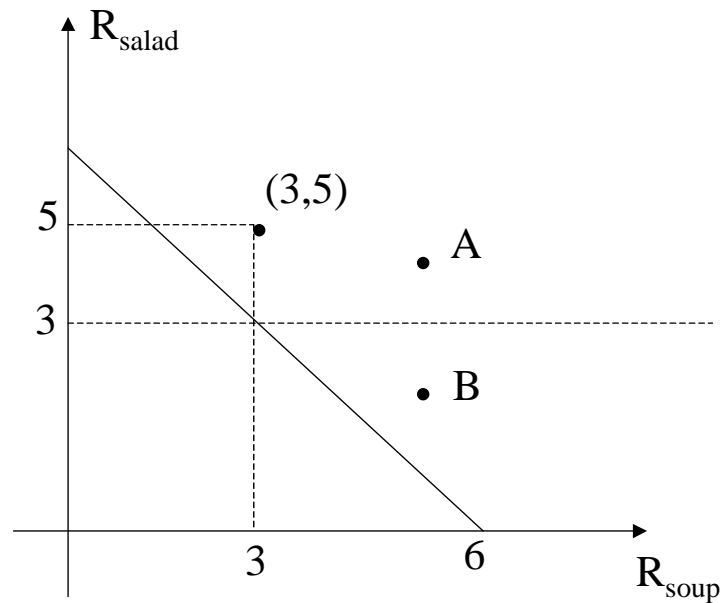
The profit to sales ratio

$$\frac{(p - c)Q}{pQ} = \frac{p - c}{p} = \frac{1}{\eta}$$

equal the *inverse* elasticity.

4) (10 points) A restaurant initially offers only soup and salad combinations for \$ 6 per combo. This restaurant will increase its sales if it starts offering soup alone for \$ 3, salad alone for \$ 5 and the combo for \$ 6. (For full credit, depict your answer on a diagram with reservation price for soup on the horizontal axis and reservation price for salad on the vertical axis).

False. It may be the case that some customers who previously bought the whole lunch would now go just for soup alone or salad alone. On the figure, both customer $A(5,4)$ and customer $B(5,2)$ used to buy the whole lunch, because $5 + 4 > 6$ and $5 + 2 > 6$. Now A continues to buy the whole lunch, but B now buys only soup.



5) (8 points) The industry is composed of n firms with the same constant marginal costs who are Cournot competitors. Due to the new technology, every firm's marginal cost goes down by the same amount. Then the Herfindahl index for this industry must go up because it is proportional to markup over marginal cost.

False After the technological change, everybody's marginal costs are again the same across firms, so everybody's market shares are still the same. Herfindahl index

$$H = \sum_{i=1}^n s_i^2$$

does not change. As for the formula

$$H = \eta \frac{p - c}{p}$$

you have to take into account not only the fall in the marginal cost, but the fall in equilibrium price as well. It exactly offsets the effect of lower c in the formula for Herfindahl index.

6) (7 points) Consider an industry of $N \geq 2$ firms with different marginal costs, which play Bertrand price competition game. That is, the firm with the lowest price gets all the market. If firms charge the same price and it happens to be the lowest price, they share the market equally. Then, in any Nash equilibrium of this game, at least one firm makes zero profit. (Hint: can there be a Nash equilibrium where all firms make positive profits?)

True. Suppose that all firms make positive profits. This implies that *every* firm prices above marginal cost *and* that *every* firm sells something. This can only be the case if all firms charge the same price, because otherwise the firm with the lowest price would take all the market and all others would make zero profits. But if all firms charge the same price that is above marginal cost, any firm has an incentive to cut the price by ε and capture all the market.

Formally, the only situation where everyone makes positive profits is when

$$p_i = p > c_i \text{ and } q_i = \frac{Q(p)}{N} \text{ for firm } i.$$

But then, playing p is not the best response to everybody else playing p . There cannot be a Nash equilibrium where everyone makes positive profits.

Part II. (75 points total) Long questions

7) (30 points) Suppose that you are the managing director of a pharmaceutical company that sells a unique patented drug to hospitals and drug stores. You are free to charge different per-unit prices at these two markets. Let p be the price per unit of drug and q be the quantity demanded. The hospitals demand curve is described by

$$p = 12 - q$$

and the drug stores demand curve is given by

$$p = 8 - q.$$

The marginal cost of producing the drug is constant and equal to $c = 2$ per unit.

a) **(8 points)** What is the price that you will charge to hospitals? To drug stores?

Hospitals:

$$\begin{aligned} MR &= 12 - 2q_H = 2 = MC \\ q_H &= 5; p_H = 7 \end{aligned}$$

Drug stores:

$$\begin{aligned}MR &= 8 - 2q_D = 2 = MC \\ q_D &= 3; p_D = 5\end{aligned}$$

c) **(10 points)** If you were to charge a uniform price to *all* the buyers, what would it be? (No credit for guessing)

At price p the total quantity demanded is

$$Q = 12 - p + 8 - p = 20 - 2p$$

The inverse demand is

$$\begin{aligned}p &= 10 - \frac{Q}{2} \\ MR &= 10 - Q = 2 = MC \\ Q &= 8; p = 6\end{aligned}$$

d) **(12 points)** If the Antitrust Division cares about the sum of your company's profits plus the total consumer surplus of all the buyers, do you think it should ban price discrimination? (Computation required for full credit. Hint: compute the consumer surplus *separately* for drug stores and hospitals, even if they buy at the same price, and then add them up.)

Welfare under 3-d degree price discrimination:

$$\begin{aligned}W = \Pi + CS &= (p_H - c)q_H + (p_D - c)q_D + CS_H + CS_D = \\ &= 5 \cdot 5 + 3 \cdot 3 + \frac{1}{2}q_H(12 - p_H) + \frac{1}{2}q_D(8 - p_D) = \\ &= 5 \cdot 5 + 3 \cdot 3 + \frac{1}{2}25 + \frac{1}{2}9 = 51\end{aligned}$$

Welfare under uniform price:

$$W = \Pi + CS = (p - c)Q + \frac{1}{2}q_H(12 - p) + \frac{1}{2}q_D(8 - p) = 32 + 18 + 2 = 52$$

Welfare is higher under the uniform price. The Antitrust Division would recommend charging the uniform price.

8) (25 points) Suppose that you are the CEO of Cellco, a wireless communications company located in Valleytown. Since Valleytown is surrounded by mountains from all sides, only your company can provide cellular phone service for the local consumers.

There are two types of customers: business customers (B) who value each minute of communication at $v_B = \$0.15$, and personal use customers (P) who value each minute of communication at $v_P = \$0.10$. That is, if customer of type i (i is either B or P) talks q minutes per month and pays a monthly fee p , her utility is

$$U_i = v_i q - p.$$

You do not know which customer is which type and would like to offer two different plans: a business plan with $q_B = 1200$ included minutes for the flat monthly fee p_B and a personal use plan with $q_P = 800$ included minutes for the flat monthly fee p_P . You want each type of customer to take the plan designed for this type. For simplicity, assume that your customers never want extra minutes.

a) **(7 points)** Your marketing manager proposes to charge business customers $p_B = \$180$ and personal use customers $p_P = \$80$. Will it work? Why or why not?

It won't work. Business customers would rather take the plan designed for the personal use customers:

$$U_B(q_B, p_B) = v_B q_B - p_B = 0.15 \cdot 1200 - 180 = 0$$

$$U_B(q_P, p_P) = v_B q_P - p_P = 0.15 \cdot 800 - 80 = 40.$$

b) **(10 points)** Can you find the profit-maximizing way to price the two plans, so that each type of customer takes the plan designed for this type? (Show all work)

The business customer must prefer his plan:

$$U_B(q_B, p_B) \geq U_B(q_P, p_P)$$

$$v_B q_B - p_B \geq v_B q_P - p_P$$

$$p_B \leq p_P + v_B (q_B - q_P)$$

$$p_B \leq p_P + 0.15 \cdot 400 = p_P + 60$$

In order to maximize profit, you can charge all the surplus away from the personal use customers and make the business customers just indifferent between their plan and personal use plan:

$$U_P(q_P, p_P) = v_P q_P - p_P = 0$$

$$p_P = 0.1 \cdot 800 = 80.$$

$$p_B = p_P + 60 = 140.$$

c) **(8 points)** Flipping through the Valleytown Yellow Pages, you realize that you have $n_B = 100$ business customers. What is the minimum number of personal use customers that you need to have in order to prefer to offer two different plans as opposed to just a business plan?

If you just offer a business plan, can charge \$180 to each business customer. If offer two plans, charge \$140 for the business plan and \$80 for the personal use plan. Prefer two plans if

$$140n_B + 80n_P \geq 180n_B$$

$$n_P \geq 50.$$

9) (20 points) Suppose that you are the President of Little Maggie Cookie Company. The customers who buy Little Maggie cookies have different tastes x for fat content, with x ranging from 0 to 1. For each x , there is an equal number of customers with taste x . The total number of customers is $N = 100$. If x is the customer's most preferred fat content, and the cookies have fat content z , then customer x is willing to pay

$$3 - |z - x|$$

for the box of cookies. The marginal cost of producing one box of cookies of any fat content is equal to 1. Throughout the problem, assume that you would always like to serve all the market.

a) **(7 points)** Right now Little Maggie offers just one type of cookies - regular, with $z = \frac{1}{2}$. Assume that you would like to serve all the market. What price will you set for the box of cookies?

In order to serve all the market and maximize profits, you must set the highest price such that the consumers with extreme tastes, $x = 0$ and $x = 1$ still buy the cookies.

$$3 - \frac{1}{2} - p = 0$$

$$p = 2.5$$

b) **(13 points)** Suppose that *in addition* to regular cookies $z = \frac{1}{2}$ you are also considering offering the low fat variety, that is $z_{LF} = \frac{1}{4}$. What price will you set for your new cookies? What are the sales of low fat cookies? If running the low fat production line entails a flow fixed cost $F = 6$, will you stick with just the regular cookies or choose to introduce the low fat variety as well?

Since still need to serve all the market, will not change the price for regular cookies. Then, set the price of low fat cookies to 2.75. $\frac{1}{4}$ of all customers who now buy low fat cookies. Sales of low fat cookies are 25. Low fat buyers pay an additional 0.25 per box. Marginal cost is the same, so have to compare extra revenue to fixed cost. Extra revenue $0.25 \cdot 25 = 6.25$. This exceeds the fixed cost of 6, so it will be profitable to introduce the low fat variety.

Reference Guide

Marginal revenue:

$$MR(q) = \frac{d}{dq} [p(q)q] = p(q) + p'(q)q = p(q) \left[1 + \frac{1}{\varepsilon_p(q)} \right] = p(q) \left[1 - \frac{1}{\eta_p(q)} \right],$$

where $\eta_p(q)$ is the positive of price elasticity of demand when the quantity sold is q .

Monopoly problem with linear demand

Profit maximization problem for the monopolist facing linear demand and constant marginal cost:

$$p = A - BQ$$

$$\pi = pQ - cQ = (p - c)Q = (A - c - BQ)Q$$

Condition for profit maximization

$$MR = A - 2BQ_M = c = MC$$

Monopoly quantity and monopoly price

$$Q_M = \frac{A - c}{2B}; p_M = \frac{A + c}{2}.$$

Profit and consumer surplus

$$\pi_M = \frac{(A - c)^2}{4B}; CS = \frac{1}{2}(A - p_M)Q_M = \frac{(A - c)^2}{8B}.$$

Cournot equilibrium, $n \geq 1$ firms with linear demand $p = A - BQ$ and constant MC c_i

Best response function of firm i :

$$p - c_i = Bq_i$$

s_i - equilibrium market share of firm i

c_i - marginal cost of firm i

$$H = \sum_{i=1}^n s_i^2 - \text{Herfindahl index for the industry}$$

Generalized inverse elasticity rule

$$\frac{H}{\eta} = \frac{p - \sum_{i=1}^n c_i s_i}{p}$$