

Economics 431
Homework 7
Due Mon, August 13

Part I Review of vertical relations

End of chapter problems 18.1, 18.3

Part II Asymmetric information about product quality, signalling games

Winter 2002 Final, question 5

Fall 2003 final, question 5

Advertising

End of chapter problems 20.1, 20.2

Fall 2003 final, question 4

Solutions to End of the Chapter Problems:

Problem 18.1

(a)

Since demand is given by $Q = 30 - p$, inverse demand is given by $p = 30 - Q$.

Profit of the Volvo dealer is given by

$$\pi^D(p, w) = pQ - wQ = (30 - Q)Q - wQ$$

Maximization with respect to Q will give

$$\begin{aligned}\frac{\partial \pi^D}{\partial Q} &= 30 - 2Q - w = 0 \Rightarrow Q = \frac{30 - w}{2} = 15 - \frac{w}{2} \Rightarrow p = 15 + \frac{w}{2} \\ \Rightarrow \pi^D &= \left(15 + \frac{w}{2}\right) \left(15 - \frac{w}{2}\right) - w \left(15 - \frac{w}{2}\right) = \frac{w^2}{4} - 15w + 225\end{aligned}$$

(b)

Since the quantity sold is the quantity purchased, we can simply invert the equation for Q derived in part (a) to get a demand for cars as a function of Q . This will give $w = 30 - 2Q$

Revenue and Marginal Revenue of the Volvo manufacturer are given by

$$R^M = 30Q - 2Q^2 \Rightarrow MR^M = 30 - 4Q$$

Equate marginal revenue with marginal cost to obtain optimal level of Q . Find $Q = 6.25$ and $w = 17.5$.

Based on $Q = 6.25$, find the retail price. $p = 30 - Q = 23.75$

Verify that the profits of the dealer and the manufacturer are:

$$\pi^D = 39.0625, \pi^M = 78.125, \Rightarrow \pi^D + \pi^M = 117.1875$$

(c)

Now compute the integrated firm's profit using the marginal cost of production as the cost of the car.

$$\pi^I(p, 5) = (30 - Q)Q - 5Q \Rightarrow \frac{\partial \pi^I}{\partial Q} = 30 - 2Q - 5 = 0$$

$$\Rightarrow Q = 12.5, p = 17.5, \pi^I = 156.25$$

Observe that double marginalization leads to a higher price and lower quantity.

Problem 18.3

(a)

With an RPM agreement, the monopolist can choose the price as if it were a fully integrated seller. Equate marginal revenue with marginal cost to obtain $Q = 40, p = 50$

(b)

CS = 800

(c)

ABC's profit = 1200

Problem 20.1

The information given provides point estimates for the demand elasticity and the advertising elasticity. Using the Dorfman-Steiner condition (equation 10.10) and the targeted level of sales one can find the optimal level of advertising. The Dorfman-Steiner equation is

$$\text{Advertising Expenditure} / \text{Sales Revenue} = \frac{\eta_S}{\eta_P} = \frac{0.5}{2.0} = \frac{1}{4}. \text{ Therefore, advertising expenditure} =$$
$$(\text{sales revenue}) / 4 = 5,000,000$$

So the firm should commit 5 million dollars to advertising.

Problem 20.2

(a)

The demand elasticity is given by

$$\eta_P = -\frac{\partial Q}{\partial P} \frac{P}{Q} = \frac{1}{2} P^{-\frac{3}{2}} S^{\frac{1}{4}} \frac{P}{Q} = \frac{1}{2}$$

The advertising elasticity is given by

$$\eta_S = -\frac{\partial Q}{\partial S} \frac{S}{Q} = \frac{1}{2} P^{-\frac{1}{2}} S^{\frac{3}{4}} \frac{S}{Q} = \frac{1}{4}$$

The advertising-to-sales ratio is given by

$$\frac{\eta_S}{\eta_P} = \frac{0.25}{0.5} = \frac{1}{2}$$

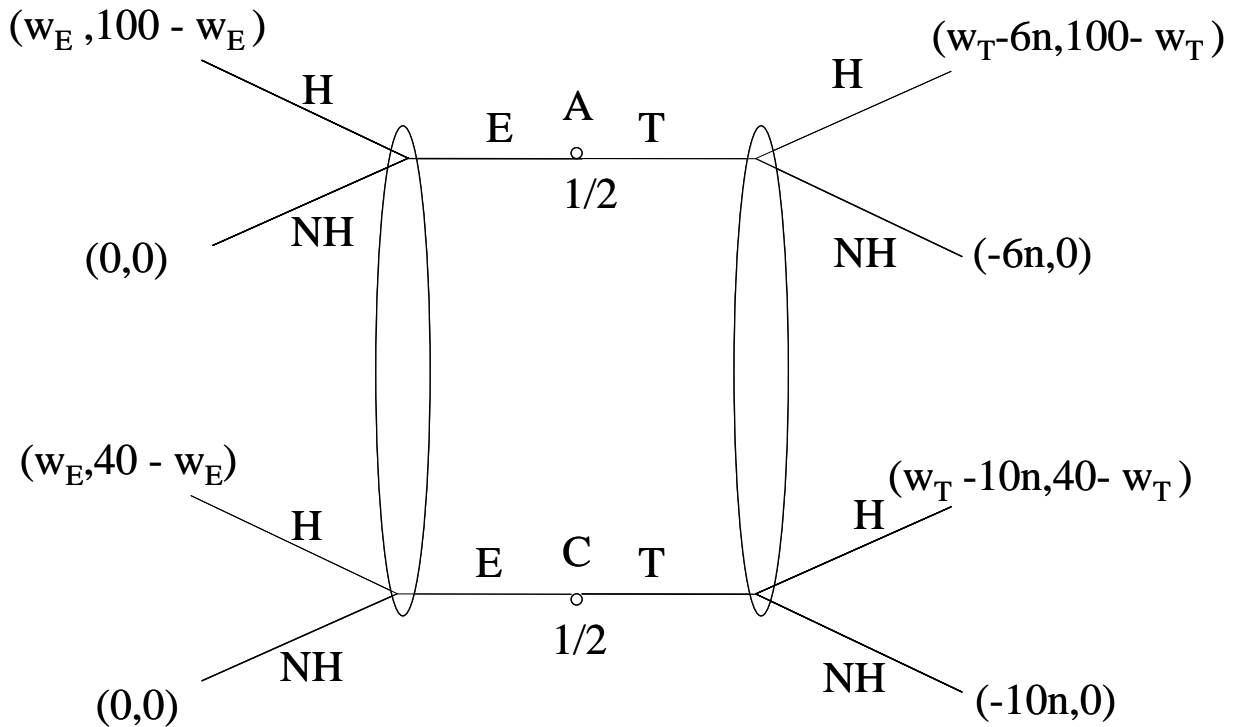
(b)

The answer is no since the data is in terms of expenditure. As the cost of advertising goes up, the expenditure will go up but the data is in terms of expenditure and so the cost as a percent of sales revenue doesn't matter.

Winter 2002 Final Exam

5) (30 points) Suppose that college students are of two types when it comes to qualities most desired by employers. One half of all students is of type *A* (able) and the other half is of type *C* (challenged). Employers are willing to pay up to 100 to type *A* student and up to 40 to type *C* student. Before the student goes on the job market, she chooses whether or not to take a tough major that consists of n difficult quantitative courses. Each student must sacrifice some party time in order to take a tough course. Suppose that this sacrifice costs 6 per course to type *A*, but it costs more, 10 per course, to type *C*.

Formally, the game is between one student and one employer, and it proceeds as follows. Nature chooses student type: *A* or *C*. The student (Player 1) knows her type, but the employer does not. Each student type chooses whether to take only easy courses (*E*) or to take n tough courses (*T*). The student then announces her salary requirement w . The employer observes whether or not the student has taken tough courses, and decides whether to hire the student and pay her w . The game and the payoffs are depicted on the figure below.



a) (10 points) Construct an equilibrium where only *A* student plays *T* and *C* student plays *E*. Specify the employer's beliefs when he observes someone who plays

T and someone who plays E . What should be the salary requirement of student A ? What should be the salary requirement of student C ?

Employer believes that the student who takes tough courses is type A for sure. With this belief, the student who plays T must request 100 in order to maximize her payoff.

Employer believes that the student who takes easy courses is type C for sure. With this belief, the student who plays E must request 40 in order to maximize her payoff.

b) (10 points) For what values of n does type A have an incentive to play T ? (Hint: if type A plays E , how would employer's beliefs change? Based on that, what should be the salary requirement of type A ?)

If type A also plays E , the employer only sees the students who take easy courses, and he believes that only a half of them are type A . Therefore, he is willing to pay up to

$$\frac{1}{2}100 + \frac{1}{2}40 = 70$$

to such a student. If type A plays E , he should request 70. Then, type A has an incentive to play T if it yields a higher equilibrium payoff

$$\underbrace{100 - 6n}_{\text{if type } A \text{ plays } T} \geq \underbrace{70}_{\text{if type } A \text{ plays } E}$$

$$n \leq 5$$

c) (10 points) For what values of n does type C have an incentive to play E ? (Hint: if type C plays T instead, how would employer's beliefs change? Based on that, what should be the salary requirement of type C who plays T ?). Find all values of n for which the strategy profile (TE, HH) described in part a) is a Bayesian Nash equilibrium.

If type C plays T , the employer only sees the students who take tough courses, and he believes that only a half of them are type C . Therefore, he is willing to pay up to

$$\frac{1}{2}100 + \frac{1}{2}40 = 70$$

to such a student. If type C plays T , he should request 70 and be hired. Then, type C has an incentive to play E if it yields a higher equilibrium payoff

$$\underbrace{40}_{\text{if type } C \text{ plays } E} \geq \underbrace{70 - 10n}_{\text{if type } C \text{ plays } T}$$

$$n \geq 3$$

For the strategy profile (TE, HH) to be an equilibrium, n must be 3, 4, or 5.

Fall 2003 Final Exam

4) (15 points)

a) (6 points) Suppose that the monopolist maximizes profit by choosing the price for the good and the amount of "persuasive" advertising. Then, at the profit maximum, which of the following will have a larger impact on quantity demanded: a 1% cut in price or a 1% increase in advertising? Explain. (Hint: how does the price elasticity of demand η_P compare to the advertising elasticity of demand η_S ?)

We know that any profit maximum satisfies

$$\frac{\tau S}{pQ} = \frac{\eta_S}{\eta_P}$$

Also, because the monopolist makes at least zero profit, $\tau S < pQ$. Then it must be that $\eta_S < \eta_P$.

A price cut must always have a stronger effect on quantity.

b) (9 points) Assume that the monopolist has total sales (i.e. revenue) of \$ 60 million, spends \$ 5 million on advertising and makes a profit (net of advertising expenditures) of \$ 15 million. Calculate the price elasticity of demand, η_P , and advertising elasticity of demand, η_S .

$$\pi = (p - c)Q - \tau S$$

$$\frac{(p - c)Q}{pQ} = \frac{1}{\eta_P}$$

$$\frac{\tau S}{pQ} = \frac{\eta_S}{\eta_P}$$

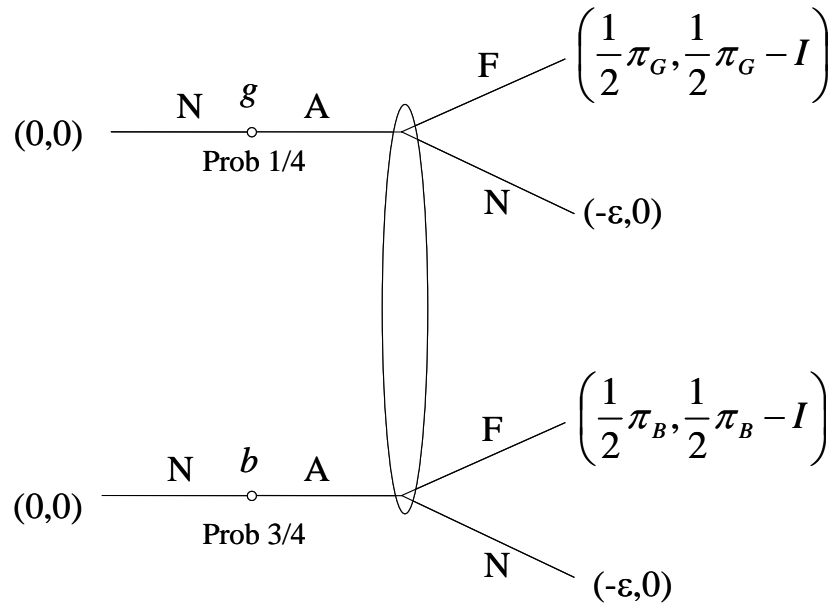
$$(p - c)Q = \pi + \tau S = 15 + 5 = 20$$

$$\frac{1}{\eta_P} = \frac{(p - c)Q}{pQ} = \frac{20}{60} = \frac{1}{3}, \eta_P = 3$$

$$\eta_S = \eta_P \frac{\tau S}{pQ} = 3 \frac{5}{60} = 0.25.$$

5) (35 points) Consider a game between the entrepreneur (Player 1) and the investor (Player 2). The entrepreneur needs an investment of $I = 90$ to implement a project and promises to repay the investor with one half of whatever profits the project generates (this is called equity financing). The project can be either good or bad. It is good with probability $\frac{1}{4}$ and bad with probability $\frac{3}{4}$. A good project generates a payoff $\pi_G = 200$, and a bad project generates a payoff $\pi_B = 80$. The

entrepreneur knows whether the project is good or bad, but the investor does not. The investor has to decide whether to finance the project or not (F or N), based on his beliefs about the quality of the project. The entrepreneur decides whether to apply for financing (A or N), depending on the quality of his project. There is a small application cost $\varepsilon > 0$ that the entrepreneur pays if he applies but is not funded.



a) **(3 points)** In terms of the sum of payoffs, which project(s) is it efficient to fund? Explain.

It is efficient to fund only the good project, funding the bad project reduces the sum of payoffs.

b) **(6 points)** Is there an equilibrium where only the good project is funded? Explain

This can only be the case if the bad type does not apply. But if the project is funded, the bad entrepreneur has an incentive to apply as well, because doing so increases his payoff. There cannot be an equilibrium where only the good type applies.

c) **(6 points)** Is there an equilibrium where both projects are funded? (Hint: if both entrepreneur types apply, what is the investor's expected payoff from funding the project?)

Both entrepreneurs must apply if both projects are to be funded. The investor's payoff is negative.

$$\frac{3}{4} \left(\frac{1}{2} \pi_B \right) + \frac{1}{4} \left(\frac{1}{2} \pi_G \right) - I = -35 < 0$$

If both types apply, the investor is better off not funding the project.

d) **(8 points)** What is the equilibrium of this game?

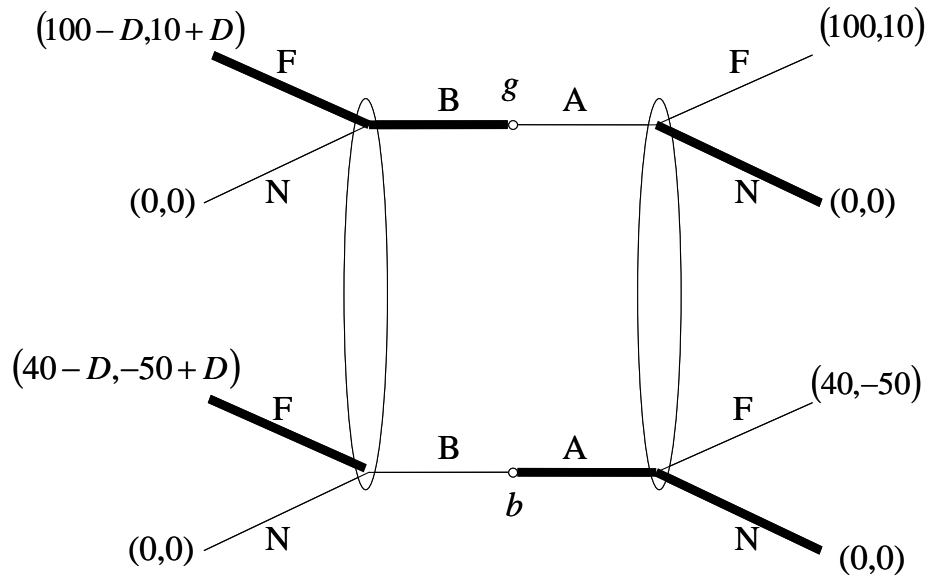
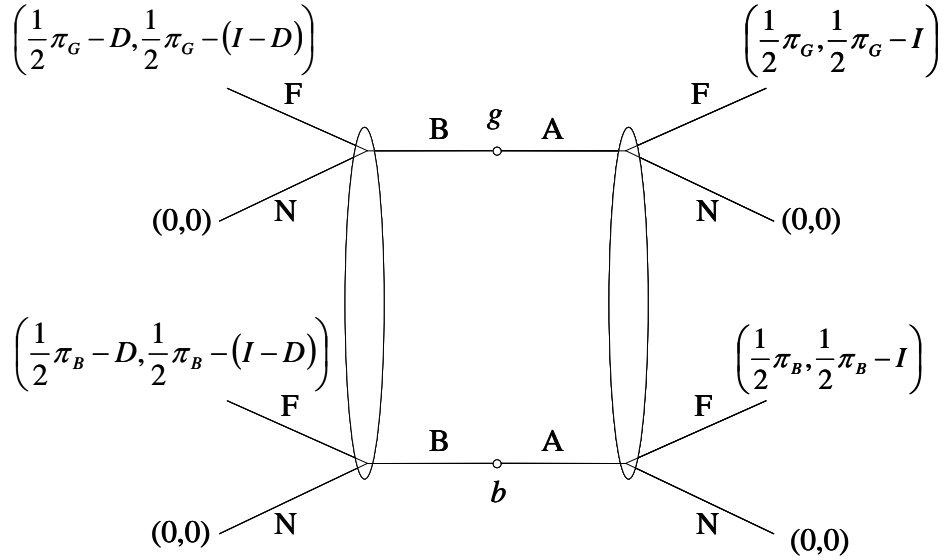
The investor does not fund the project, and neither type applies for funding. For the investor to prefer not to fund the project, he has to believe that the payoff from funding is negative. That is, the investor has to believe that the project is bad with sufficiently high probability. For example, if the investor believes that the project is bad with probability $\frac{3}{4}$, not funding the project will be a best response.

Any belief ρ that gives the investor a negative payoff from funding the project is consistent with equilibrium:

$$\rho \left(\frac{1}{2} \pi_B \right) + (1 - \rho) \left(\frac{1}{2} \pi_G \right) - I = 40\rho + 100(1 - \rho) - 90 = 10 - 60\rho < 0, \text{ i.e. } \rho > \frac{1}{6}$$

Finally, both entrepreneur types play the best response by not applying, because if either applies, he is denied funding and pays the application cost.

e) **(12 points)** Now suppose that the entrepreneur has two sources of funding. As before, he can ask the investor for $I = 90$ and promise $\frac{1}{2}\pi$ in return (i.e. choose action A). Alternatively, the entrepreneur can take action B - borrow an amount $D < 90$ from the bank (a debt which he has to repay regardless of the profitability of his project), and then ask the investor to put up the rest of the funds, $I - D$, in exchange for a promise to pay $\frac{1}{2}\pi$. The investor does not observe the quality of the project, but he does observe what action is taken by the entrepreneur. Formally, the actions and payoffs are depicted on the figure below. Assume that the application cost ε is zero. Find an equilibrium where the good type borrows (takes action B) and the bad type does not (takes action A). How much must the good type borrow in this equilibrium (that is, find all the values of D consistent with this separating equilibrium)? Explain why funding part of the project with debt is a good signal for the investor.



If only the good type takes action B , investor has to believe that a type who took action B is good for sure. Therefore, the best response to B is F .

If only the bad type takes action A , investor has to believe that a type who took action A is bad for sure. Therefore, the best response to B is N .

Is B the best response for the good type?. If he chooses A his project is not funded and he gets 0, but if he chooses B , his payoff is $100 - D > 0$. B is the best response for the good type.

Is A the best response for the bad type?. If he instead chooses B his project is funded and he gets $40 - D$, but if he chooses A , his payoff is 0. A is the best response for the bad type when $0 > 40 - D$, that is when

$$D > 40.$$

Funding part of the project by debt is a good signal for the investor, because the entrepreneur puts his money where his mouth is - a bad type cannot afford this much debt.