

Economics 431
Homework 6
Due Wed, Aug 8

Part I. Horizontal mergers

1) Mergers and economies of scale

When marginal cost is constant and there are no fixed costs, no two firms have a profit incentive to merge unless it is a merger from duopoly to monopoly. The situation is different when marginal costs are increasing: when two firms merge, the new firm can achieve better cost minimization by allocating its output across multiple plants. Then a multi-plant firm has a cost advantage over its one-plant competitors, and this may provide a profit incentive to merge.

The example illustrates this motive for merger: a merged firm will operate multiple plants and will be able produce output at lower average cost than a one-plant firm.

An industry consists of $N = 3$ firms with identical costs . Market demand is

$$p = A - BQ = 150 - Q$$

a) Show that if the total cost function is linear (marginal cost is constant), then it will never pay for two firms to merge if the resulting two firms are again Cournot competitors.

b) Now let the cost function be

$$C(q) = cq + q^2 = 18q + q^2.$$

Suppose that two firms (2 and 3) merge, assume the name of firm 2 and play Cournot against the remaining firm 1. Will there be a profit incentive to merge? Will the merger benefit consumers? (Hint: carefully consider if the merged firm would produce using both original firms' plants or just those of one firm.)

Winter 2002 final, question 2

2) Mergers, cost synergies and welfare

Let the industry initially consist of 4 firms that operate on a market with linear demand given by

$$p = 25 - Q.$$

In order to operate, each firm has to pay a fixed cost $F = 15$ Marginal cost is constant and equals $c = 5$ for every firm Assume that the firms engage in Cournot competition.

a) Show that any two firms in this industry will have a profit incentive to merge.

b) Compute welfare (the sum of all profits net of fixed costs plus consumer surplus) pre-merger (when $N = 4$) and post-merger (when $N = 3$). Does this merger make consumers better off? Does this merger make firms and consumers jointly better off? Why?

3) Capacity motive for merger and market price

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There are no fixed costs. Marginal cost is constant and equals $c = 5$ for every firm. Assume that initially the firms engage in Cournot competition.

a) Suppose that if two firms merge, they become a Stackelberg leader, and the other two firms are followers. Show that this merger leads to a price drop. Also show that the two firms have a profit incentive to merge.

b) Demonstrate that further mergers in this industry are not in the public interest.

Part II Vertical mergers

Winter 2002 final, question 3

Part III Vertical relations

Winter 2002 final, question 4, Fall 2003 final, question 3

Economics 431
Homework 6
Answer Key

Part I. Horizontal mergers

1) Mergers and economies of scale

When marginal cost is constant and there are no fixed costs, no two firms have a profit incentive to merge unless it is a merger from duopoly to monopoly. The situation is different when marginal costs are increasing: when two firms merge, the new firm can achieve better cost minimization by allocating its output across multiple plants. Then a multi-plant firm has a cost advantage over its one-plant competitors, and this may provide a profit incentive to merge.

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a) Suppose that we have a three industry firm with linear demand

$$\begin{aligned} p &= A - BQ \\ MC(q) &= c \end{aligned}$$

and identical linear cost functions across firms:

$$C(q) = cq$$

If three firms play a Cournot equilibrium

$$\pi = \frac{(A - c)^2}{B(N + 1)^2} = \frac{1}{16} \frac{(A - c)^2}{B}$$

If two of the firms merge, they will get the profit of $\frac{1}{9}$ between themselves. This does not exceed the sum of their pre-merger profits

$$\frac{1}{9} < \frac{1}{16} + \frac{1}{16}$$

The two firms will not choose to merge.

Now suppose the cost function is

$$C(q) = cq + q^2$$

The best response function of a Cournot competitor

$$\begin{aligned} p &= A - BQ_{-i} - Bq_i \\ \max_q (A - BQ_{-i} - Bq)q - cq - q^2 \\ A - BQ_{-i} - 2Bq - c - 2q &= 0 \\ A - c - BQ &= q(B + 2) \\ A - c - BQ &= \frac{Q}{N}(B + 2) \\ Q &= \frac{N(A - c)}{BN + B + 2}, \quad q = \frac{(A - c)}{BN + B + 2} \\ p = A - BQ &= A - \frac{N(A - c)}{N + 1 + 2/B} = \frac{A(N + 1 + 2/B) - N(A - c)}{N + 1 + 2/B} = \frac{A + cN + \frac{2A}{B}}{N + 1 + 2/B} \\ \pi &= q^2(B + 2) - q^2 = q^2(B + 1) \\ \pi &= \frac{(A - c)^2(B + 1)}{(BN + B + 2)^2} \end{aligned}$$

Now $A = 150$, $B = 1$, $c = 18$. If $N = 3$

$$\begin{aligned} q &= \frac{(A - c)}{BN + B + 2} = \frac{132}{6} = 22 \\ \pi &= q^2(B + 1) = 968 \end{aligned}$$

Suppose now that two firms merge. The new firm has two plants, A and B , each of which has the cost function

$$C(q) = cq + q^2$$

We must now figure out whether production will happen at one or two plants. With two-plant production, output

$$q = q_A + q_B$$

will be allocated to equate marginal costs at each plant:

$$\begin{aligned} MC_A(q_A) &= MC_B(q_B) \\ c + 2q_A &= c + 2q_B \\ q_A &= q_B = \frac{q}{2} \end{aligned}$$

The two-plant cost function is

$$C_2(q) = C_A\left(\frac{q}{2}\right) + C_B\left(\frac{q}{2}\right) = c\frac{q}{2} + \left(\frac{q}{2}\right)^2 + c\frac{q}{2} + \left(\frac{q}{2}\right)^2 = cq + \frac{q^2}{2}$$

For any quantity to be produced, q ,

$$C_2(q) = cq + \frac{q^2}{2} < C(q) = cq + q^2$$

It is *always* cheaper to produce at two plants. The merged firm has a cost advantage over the remaining firm.

Let us now solve for the resulting Cournot equilibrium. Let us call the merged firm 2, and the remaining firm 1. Best response of firm 2 to the quantity produced by firm 1 is the quantity that maximizes firm 2's profit given q_1

$$\max_{q_2} (A - Bq_1 - Bq_2)q_2 - cq_2 - \frac{q_2^2}{2}$$

(note that firm 2 uses the two plant cost function $C_2(q)$)

$$\begin{aligned} A - Bq_1 - 2Bq_2 &= c + q_2 \\ q_2 &= BR_2(q_1) = \frac{A - c - Bq_1}{2B + 1} \end{aligned}$$

We have found the best response of firm 2 to the quantity produced by firm 1.

Now let us find firm 1's best response function.

$$\max_{q_1} (A - Bq_2 - Bq_1)q_1 - cq_1 - q_1^2$$

(note that firm 1 uses the one-plant cost function $C(q)$)

$$\begin{aligned} A - Bq_2 - 2Bq_1 &= c + 2q_1 \\ q_1 &= BR_1(q_2) = \frac{A - c - Bq_2}{2B + 2} \end{aligned}$$

Now remembering that $A = 150$, $B = 1$, $c = 18$, let us find the Nash equilibrium pair of quantities (q_1^*, q_2^*) (they are where the two best response functions intersect).

$$\begin{aligned} q_2^* &= \frac{132 - q_1^*}{3} = 44 - \frac{q_1^*}{3} \\ q_1^* &= \frac{132 - q_2^*}{4} = 33 - \frac{q_2^*}{4} \end{aligned}$$

$$3q_2^* = 132 - q_1^*$$

$$4q_1^* = 132 - q_2^*$$

$$4(132 - 3q_2^*) = 132 - q_2^*$$

$$396 = 11q_2^*$$

$$q_2^* = 36$$

$$q_1^* = 132 - 3q_2^* = 24$$

Market price

$$p = A - Bq_1^* - Bq_2^* = 150 - 24 - 36 = 90$$

$$\pi_1 = pq_1^* - C(q_1^*) = 90 \cdot 24 - [18 \cdot 24 + 24^2] = 1152$$

$$\pi_2 = pq_2^* - C_2(q_2^*) = 90 \cdot 36 - \left[18 \cdot 36 + \frac{36^2}{2}\right] = 1944$$

Will the two firms want to merge into one? The post-merger profit of the whole firm is greater than the pre-merger sum of profits of the parts:

$$968 + 968 = 1936 < 1944$$

Note that the remaining firm also gains from merger, because price goes up. Consumers, therefore, lose.

2) Mergers, cost synergies and welfare

Let the industry initially consist of 4 firms that operate on a market with linear demand given by

$$p = 25 - Q.$$

In order to operate, each firm has to pay a fixed cost $F = 15$ Marginal cost is constant and equals $c = 5$ for every firm Assume that the firms engage in Cournot competition.

a) Show that any two firms in this industry will have a profit incentive to merge.

For Cournot oligopoly,

$$\pi(N) = \frac{(A - c)^2}{B(N + 1)^2}$$

For two firms to have an incentive to merge, the post-merger profit of the merged firm must exceed the sum of pre-merger profits of the parts.

$$2\pi(N) - 2F < \pi(N - 1) - F$$

$$15 = F > 2\pi(N) - \pi(N - 1) = \frac{(A - c)^2}{B} \left(\frac{2}{(N + 1)^2} - \frac{1}{N^2} \right) =$$

$$= 400 \left(\frac{2}{25} - \frac{1}{16} \right) = 32 - 25 = 7$$

b) Compute welfare (the sum of all profits net of fixed costs plus consumer surplus) pre-merger (when $N = 4$) and post-merger (when $N = 3$). Does this merger make consumers better off? Does this merger make firms and consumers jointly better off? Why?

The merger does not make consumers better off because fewer firms on the market means higher price.

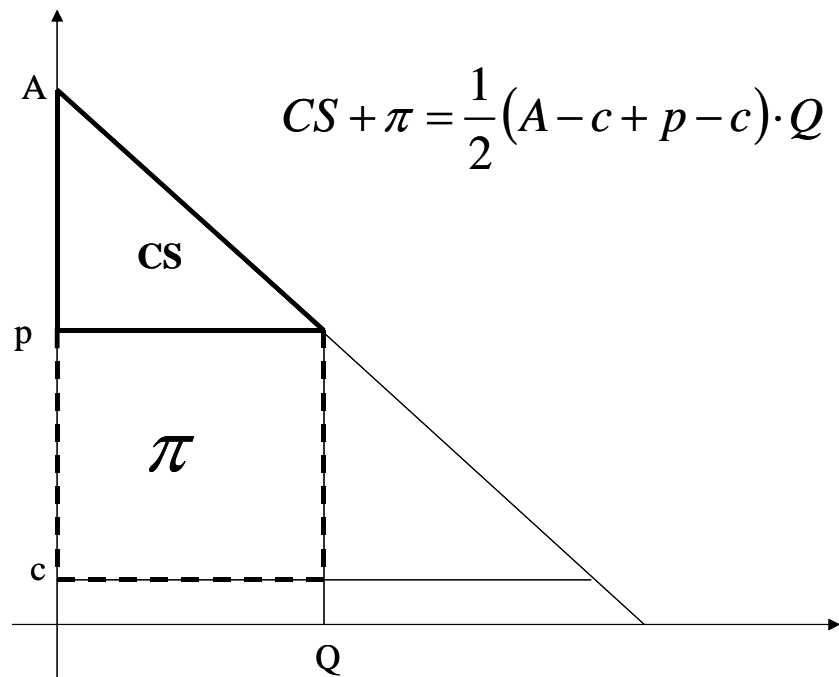
Total quantity produced with N firms on the market

$$Q(N) = \frac{N}{N+1} \frac{(A-c)}{B}$$

Price with N firms on the market

$$p(N) = c + \frac{A-c}{N+1}$$

Welfare with N firms on the market



$$\begin{aligned} W(N) &= CS + N \cdot \pi(N) - N \cdot F = \frac{1}{2} (A - c + (p(N) - c)) Q(N) - F \cdot N = \\ &= \frac{1}{2} (A - c) \left(1 + \frac{1}{N+1} \right) \cdot \frac{N}{N+1} \frac{(A-c)}{B} - F \cdot N \end{aligned}$$

$$= \frac{1}{2} \frac{(A-c)^2}{B} \frac{(N+2)N}{(N+1)^2} - F \cdot N$$

Pre-merger welfare:

$$W(4) = 200 \frac{6 \cdot 4}{5^2} - 15 \cdot 4 = 132$$

Post-merger welfare:

$$W(3) = 200 \frac{5 \cdot 3}{4^2} - 15 \cdot 3 = 142.5$$

Welfare goes up as a result of merger, because the loss of consumer surplus resulting from higher price is outweighed by the gain in firms' profits and saved fixed cost.

3) Capacity motive for merger and market price

Let the industry initially consist of 4 firms that operate on a market with linear demand given by

$$p = 25 - Q.$$

There are no fixed costs. Marginal cost is constant and equals $c = 5$ for every firm. Assume that initially the firms engage in Cournot competition.

a) Suppose that if two firms merge, they become a Stackelberg leader, and the other two firms are followers. Show that this merger leads to a price drop. Also show that the two firms have a profit incentive to merge.

$$\pi_l(L, F) = (p - c) q_l = \frac{A - c}{(L + 1)(F + 1)} \cdot \frac{1}{L + 1} \frac{A - c}{B} = 16$$

$$\pi_f(L, F) = (p - c) q_f = \frac{A - c}{(L + 1)(F + 1)} \cdot \frac{1}{(L + 1)(F + 1)} \frac{A - c}{B}$$

Pre-merger every firm gets

$$\pi_l(4, 0) = 16$$

The sum of pre-merger profits is $16 + 16 = 32$.

Post-merger, the leader gets

$$\pi_l(1, 2) = \frac{10}{3} \cdot \frac{1}{2} \cdot 20 = 33\frac{1}{3} > 2\pi_l(4, 0) = 32$$

Pre-merger

4 leaders, 0 followers

$$p - c = \frac{A - c}{(L + 1)(F + 1)} = 4$$

Post merger

1 leader, 2 followers

$$p - c = \frac{A - c}{(L + 1)(F + 1)} = \frac{20}{(1 + 1)(2 + 1)} = \frac{10}{3} < 4$$

b) Demonstrate that further mergers in this industry are not in the public interest.

Pre-merger

1 leader, 2 followers

$$p - c = \frac{A - c}{(L + 1)(F + 1)} = \frac{10}{3}$$

Post merger

2 leaders, 0 followers

$$p - c = \frac{A - c}{(L + 1)(F + 1)} = \frac{20}{3}$$

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2) (20 points) The market has linear demand given by

$$p = 20 - Q$$

and three firm with constant marginal costs that differ across firms. Firms 1 and 2 have both marginal cost equal to $c = 8$, but firm 3 has a higher marginal cost equal to $d = 10$. Assume that pre-merger and post-merger firms play a Cournot game.

a) (10 points) Do firms 1 and 2 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly.

Pre-merger:

$$p = \frac{1}{4}(A + 2c + d) = 11.5$$

$$p - c = \frac{1}{4}(A - 2c + d) = 3.5$$

$$p - d = \frac{1}{4}(A + 2c - 3d) = 1.5$$

$$\pi_1 = \pi_2 = \frac{1}{16B}(A - 2c + d)^2 = 12.25$$

Post-merger

$$p = \frac{1}{3}(A + c + d) = 12.6667$$

$$p - c = \frac{1}{3}(A - 2c + d) = 4.6667$$

$$\pi_{1+2} = \frac{1}{9B}(A - 2c + d)^2 = 21.7778 < 24.5 = \pi_1 + \pi_2$$

b) (10 points) Do firms 1 and 3 have a profit incentive to merge with each other? Calculate pre-merger and post merger profits explicitly. (Hint: will the newly merged firm find it optimal to produce any output at marginal cost d ?)

Pre-merger

$$\pi_1 = \frac{1}{16B}(A - 2c + d)^2 = 12.25$$

$$\pi_3 = \frac{1}{16B}(A + 2c - 3d)^2 = 2.25$$

Post-merger: no output is produced at the plant with marginal cost d . Firm 1 + 3 will have marginal cost c , and firm 2 will have the same marginal cost.

$$\pi_{1+3} = \frac{(A - c)^2}{9B} = 16 > 14.5 = \pi_1 + \pi_3$$

3) (15 points) Initially, the upstream firm U supplies two downstream firms D_1 and D_2 and charges uniform price r_u . Under uniform pricing the downstream firms D_1 and D_2 make $\pi_D^1 = 3$ and $\pi_D^2 = 18$ respectively. Monopoly profit at market 1 is $\pi_M^1 = 24$, monopoly profit at market 2 is $\pi_M^2 = 48$. Firm U wants to merge with one of the downstream firms in order to price-discriminate (charge the other firm a price different from r_u).

a) (10 points) Suppose that the upstream firm wants to merge with either D_1 or D_2 . Calculate explicitly which merger is more profitable. What is the post-merger profit of firm $U + D_i$, where i is either 1 or 2? (Hint: let pre-merger profit of firm U be π_U . In the end, your answer should not depend on π_U . Don't forget that post-merger firm $U + D_i$ also gets profits from being a monopolistic upstream supplier for the other downstream firm D_j . In equilibrium, this profit equals to $\frac{1}{2}\pi_M^j$)

If U merges with D_1 . Pre merger:

$$\pi_u + \pi_D^1 = \pi_u + 3$$

Post-merger

$$\pi_M^1 + \frac{\pi_M^2}{2} = 24 + \frac{1}{2} \cdot 48 = 48$$

Difference between pre-merger profit and post merger profit:

$$48 - 3 - \pi_u = 45 - \pi_u$$

If U merges with D_2 . Pre merger:

$$\pi_u + \pi_D^2 = \pi_u + 18$$

Post-merger

$$\pi_M^2 + \frac{\pi_M^1}{2} = 48 + \frac{1}{2} \cdot 24 = 60$$

Difference between pre-merger profit and post merger profit:

$$60 - 18 - \pi_u = 42 - \pi_u$$

It is more profitable to merge with D_1 .

b) (5 points) Suppose that the most profitable merger did take place. How did the quantities of final good sold at market 1 and market 2 change compared to the pre-merger situation? Explain.

The motive for merger is price discrimination. Under price discrimination, the profit-maximizing price on one market must be higher than r_U and the profit-maximizing price on the other market must be lower than r_U . The double marginalization problem is resolved on market 1 via merger. Therefore, this market has a lower price and

a higher quantity, compared to pre-merger situation. The other market, market 2, has a price that is higher than r_U .

Alternatively, the most profitable merger is into the market with a higher elasticity of demand. Therefore, it must be the case that market 1 has higher elasticity of demand. This means that market 2 has a lower elasticity of demand than markets 1 and 2 combined. Then it must be the case that the post-merger price on market 2 is higher than r_U and the quantity is lower.

4) (20 points) An upstream manufacturer whose marginal cost is $c = 6$ sells his product to two retailers who are Cournot competitors. First, retailers simultaneously and independently decide whether or not to launch an ad campaign. If *at least one* retailer pays for the ad campaign, market demand is high:

$$p = A_H - Q, \text{ where } A_H = 24$$

If neither one launches the ad campaign, demand is low

$$p = A_L - Q, \text{ where } A_L = 15$$

The ad campaign costs $S = 28$. Next, given the high or low demand, retailers play a Cournot quantity game. Assume that each retailer gets the product from the manufacturer for the price $r = c = 6$ and pays a flat franchise fee equal to T .

a) (10 points) What are the retailers' equilibrium profits if both pay S , if one of them pays S and if neither of them pays S ? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

	Pay S	Don't Pay S
Pay S	$\frac{(A_H-c)^2}{9} - S - T, \frac{(A_H-c)^2}{9} - S - T$	$\frac{(A_H-c)^2}{9} - S - T, \frac{(A_H-c)^2}{9} - T$
Don't Pay S	$\frac{(A_H-c)^2}{9} - T, \frac{(A_H-c)^2}{9} - S - T,$	$\frac{(A_L-c)^2}{9} - T, \frac{(A_L-c)^2}{9} - T$
	Pay S	Don't Pay S
	Pay S	$8 - T, 8 - T$
	Don't Pay S	$8 - T, \underline{36 - T}$
	Pay S	$\underline{36 - T}, 8 - T$
	Don't Pay S	$\underline{9 - T}, \underline{9 - T}$

In the Nash equilibrium, neither retailer pays S . Total maximum franchise fee

$$2T = \frac{2}{9} (A_L - c)^2 = 18.$$

b) (10 points) Suppose the manufacturer cannot launch the ad campaign himself and cannot force the retailers to pay for it. However, the manufacturer can impose a resale price maintenance agreement on each retailer, saying that they are to sell the good for the price equal to $p^* = 15$. In this case, each retailer will have half

of the market. What are the retailers' equilibrium profits if both pay S , if one of them pays S and if neither of them pays S ? Is the ad campaign launched in the Nash equilibrium? In equilibrium, what is the maximum total franchise fee that the manufacturer can extract?

Total quantity:

$$Q = A - p^*$$

Each retailer sells

$$\frac{Q}{2} = \frac{1}{2}(A - p^*)$$

	Pay S		Don't Pay S
Pay S	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T$
Don't Pay S	$\frac{(p^*-c)(A_H-p^*)}{2} - T$	$\frac{(p^*-c)(A_H-p^*)}{2} - S - T$	$\frac{(p^*-c)(A_L-p^*)}{2} - T$
	Pay S	Don't Pay S	
Pay S	<u>$12.5 - T$</u> , $12.5 - T$	<u>$12.5 - T$</u> , <u>$40.5 - T$</u>	
Don't Pay S	<u>$40.5 - T$</u> , <u>$12.5 - T$</u>	$0 - T$, $0 - T$	

There are two Nash equilibria, but in each of them involves someone paying for the ad campaign.

All equilibrium profits can potentially be extracted as a franchise fee:

$$40.5 + 12.5 = 53$$

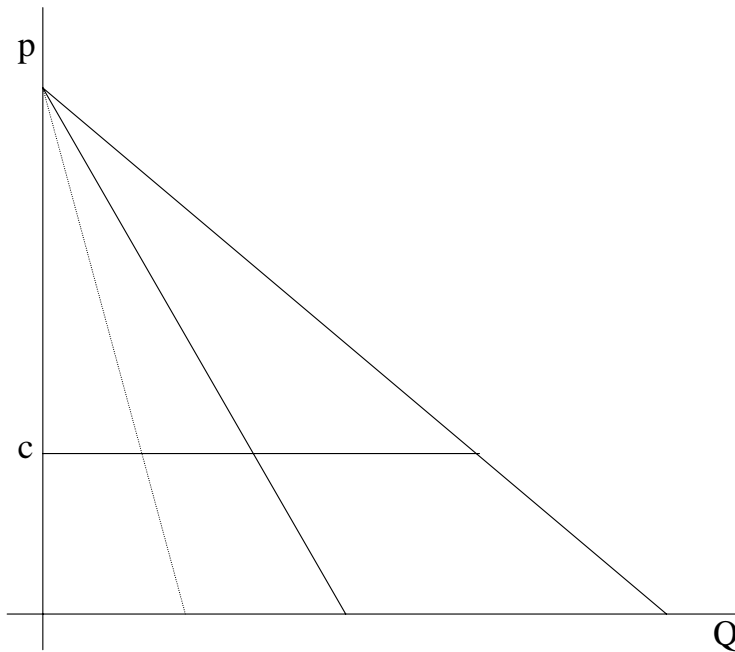
If the manufacturer must charge everyone the same T , he can only extract 25.

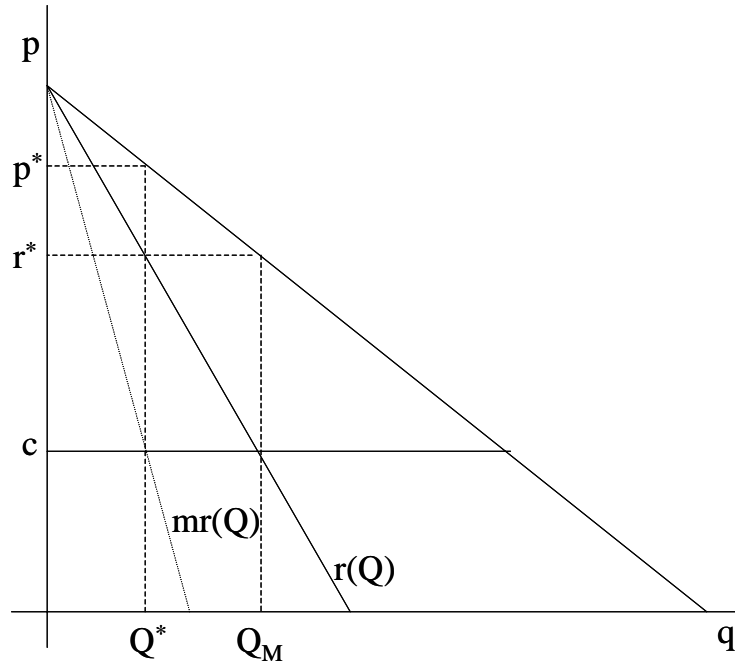
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3) (35 points) There is a monopoly manufacturer and a monopoly retailer. Demand for the retail product is

$$p = 12 - Q.$$

This demand and some other lines are shown on the diagram. Marginal cost of the manufacturer is constant and equal to $c = 4$. The manufacturer charges the retailer a uniform wholesale price r .





a) **(15 points)** Clearly show the following on the diagram and mark it as instructed:

(2 points) Retailer's demand for the manufacturer's product and label it $r(Q)$;

(2 points) Manufacturer's marginal revenue line and label it $mr(Q)$;

(2 points) Profit-maximizing quantity for the manufacturer and label it Q^* ;

(2 points) Wholesale price and label it r^* ;

(2 points) Retail price and label it p^* ;

(5 points) If retailer and manufacturer were maximizing their joint profits, would they still choose to produce Q^* or some other quantity? Calculate this quantity, show it on the diagram and label it Q_M .

They would produce monopoly quantity

$$Q_M = \frac{A - c}{2B} = 4$$

b) **(3 points)** There are various ways to force the retailer to sell Q_M rather than Q^* . One way is a contract that forbids the retailer to charge more than \bar{p} (a price ceiling). Calculate \bar{p} .

$$\bar{p} = 12 - 4 = 8$$

c) **(8 points)** What other contracts between the manufacturer and the retailer can achieve a bigger joint profit for them? Describe at least three other arrangements.

Franchise agreement - the manufacturer charges a flat fee and sells for marginal cost. It is the equivalent of a two-part tariff.

Royalty agreement - the manufacturer sells at marginal cost, but the retailer pays back a fraction of his profits

Vertical merger

Minimum sales quota

d) (**9 points**) In part b), a price ceiling was used to improve the joint profits of manufacturer and retailer. Another very common agreement between manufacturers and retailers is a price *floor* (also known as resale price maintenance agreement). Explain when this restriction is used and why it may improve the manufacturer's profits from a franchise contract. (Hint: you do not have to assume monopolistic retailer).

Suppose that there are several retailers that provide a promotional service for which consumer is willing to pay extra. This service is costly and non-excludable, that is, if one retailer provides the service, the others enjoy its benefits for free. Without a price floor, retailers' profits are low due to competition, and retailers cannot afford to pay for the promotional service. As a result, consumer demand is low, and retailers' profits are low, too. With price floor, retailers' profits go up. This may just be enough to induce retailers to pay for the promotional service. Then consumer demand and retailers' profits are high, and the manufacturer can extract a bigger franchise fee from the retailers.