Economics 431
Homework 5
Due Wed,

Part I. The tipping game (this example due to Stephen Salant)

The customer comes to a restaurant for dinner every day and is served by the same waiter. The waiter can give either good or bad service. The customer *observes* the quality of the service and decides whether or not to tip.

Waiter's strategy is quality of service: good G or bad B.

Customer's strategy must tell him what to do when the service is good and what to do when the service is bad. The customer, therefore, has 4 strategies: Always tip no matter what the service (TT), tip only if good service (TN), tip only if bad service (NT) and never tip (NN). Good service is worth \$ 10 to the customer, bad service is worth 0. The size of the tip is fixed at \$ 5. Giving good service costs the waiter \$ 2, whereas giving bad service costs the waiter 0. The payoff matrix for this game is the following:

	TT	TN	NT	NN
G	3,5	3,5	-2,10	-2,10
B	5,-5	0,0	5,-5	0,0

a) What is the Nash equilibrium of this game if played only once?

Suppose the customer comes to the restaurant every day and repeatedly plays the same game with the waiter.

b) If the customer knows he is permanently moving out of town in three weeks and tells this to the waiter, what is the subgame perfect equilibrium of the repeated game?

c) Suppose that on any given day both customer and waiter know that the customer will return for dinner tomorrow with probability p and will never return with probability $1 - p$. Also assume that the interest rate is zero (what is the discount factor then?)

Consider the following agreement between the customer and the waiter

The waiter: Starts with giving good service. Continues to give good service in the current period if and only if has been giving good service in the past and has been always tipped in the past. Otherwise, gives bad service forever.

The customer: Starts with tipping only for good service (TN). Continues to tip only for good service in the current period provided that he had always tipped in the past and had been given only good service in the past. Otherwise, never tips (NN) in the current period.

For which values of p the strategies just described is a subgame perfect equilibrium of the repeated game?

d) Let the game be exactly the same as in part c). Let waiter's strategy be the same as in part c). However, now the customer's strategy is different:

The customer starts with always tipping, no matter what the service (TT). Continues to always tip in the current period provided that he had always tipped in the past and had been given only good service in the past. Otherwise, never tips (NN) in the current period.

If you are to show that this strategy profile is a subgame perfect equilibrium for some values of p, how does your analysis in c) change? (Hint: now the waiter may have profitable deviations after some histories).

Part II Practice exams

Winter 2002 Second Midterm, Question 4

Fall 2003 Second Midterm, Question 4

Part III End of chapter and Practice problems

Practice problems to Chapter 14: 14.4, 14.5.

End of chapter problem 14.1, 14.2, 14.3

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Answer Key

Part I. The tipping game (due to Stephen Salant)

The customer comes to a restaurant for dinner every day and is served by the same waiter. The waiter can give either good or bad service. The customer *observes* the quality of the service and decides whether or not to tip.

Waiter's strategy is quality of service: good G or bad B.

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	TT	TN	NT	NN
G	3,5	3,5	-2,10	-2,10
B	5,-5	0,0	5,-5	0,0

a) What is the Nash equilibrium of this game if played only once?

	TT	TN	NT	NN
G	3,5	$\underline{3}$,5	-2,$\underline{10}$	-2,$\underline{10}$
B	$\underline{5}$,-5	0,$\underline{0}$	$\underline{5}$,-5	**$\underline{0}$,$\underline{0}$**

Suppose the customer comes to the restaurant every day and repeatedly plays the same game with the waiter.

b) If the customer knows he is permanently moving out of town in three weeks and tells this to the waiter, what is the subgame perfect equilibrium of the repeated game?

The game is then repeated $T = 21$ times. In period T both waiter and customer care only about their current payoffs, so the Nash equilibrium of the stage game (B, NN) is played. In period $T-1$ both customer and waiter know that their actions now will not affect their actions in period T: the Nash equilibrium of the stage game (B, NN) will be played in period T no matter what. Therefore, in period $T-1$ players act to maximize their current payoffs: again play (B, NN). By the same logic, since current actions cannot change future actions, the Nash equilibrium of the stage game is played in every period. This is the subgame perfect equilibrium.

c) Suppose that on any given day both customer and waiter know that the customer will return for dinner tomorrow with probability p and will never return with probability $1-p$. Also assume that the interest rate is zero (what is the discount factor?)

The discount factor

$$\delta = \frac{p}{1+r} = p$$

Consider the following agreement between the customer and the waiter

The waiter: Starts with giving good service. Continues to give good service in the current period if and only if has been giving good service in the past and has been always tipped in the past. Otherwise, gives bad service forever.

The customer: Starts with tipping only for good service (TN). Continues to tip only for good service provided that he had always tipped in the past and had been given only good service in the past. Otherwise, switches to never tipping (NN) forever.

This is a trigger strategy. There are two types of histories: the histories where (G, TN) was played every period and all the others. The strategy prescribes cooperation (G, TN) for the history that involved cooperating in all previous periods and punishment forever (B, NN) for all other histories.

For which values of p the strategies just described is a subgame perfect equilibrium of the repeated game?

The strategy profile is subgame perfect if a Nash equilibrium is played in every subgame (that is, after every history). That is, *every* player plays the best response to the opponent's strategy after *every* history. There are two players and two types of histories.

We need to check the following:

Does the waiter play the best response for the history involving past cooperation?

For this history, the strategy tells him to play G. He could deviate to B in the current period, but it gives him 0 payoff now and 0 payoff forever. Clearly, this is worse than continuing with (G, TN) and collecting 3 every period.

Does the waiter play the best response for all other histories?

Yes, because given the opponent's strategy NN after all other histories, playing B is the best response.

Does the customer play the best response for the history involving past cooperation?

The answer depends on p. The customer has an incentive to not tip in the current period (deviate to NT or NN) which gives him the payoff of 10, but this triggers the punishment, so the customer receives 0 thereafter.

$$(G, TN) \text{ forever gives the customer a payoff of } \frac{5}{1-p}$$

$$(G, NN), (B, NN), (B, NN)... \text{ gives the customer a payoff of } 10 + p\frac{0}{1-p}$$

For the customer to play the best response to the waiter's strategy, deviating must not be worth it:

$$10 + p\frac{0}{1-p} < \frac{5}{1-p}$$

or

$$p > \frac{1}{2}.$$

Does the customer play the best response for all other histories?
Yes, because NN is a best response to B.
Therefore, the strategy profile is subgame perfect for $p > \frac{1}{2}$.

d) Waiter's strategy: play G in the first period and play G in the current period for any history that involves only (G, TT) in the past. For *any* other history, play B in the current period.

Customer's strategy: play TT in the first period and play TT in the current period for any history that involves only (G, TT) in the past. For *any* other history, play NN in the current period.

Does the waiter have a profitable deviation after the history that involves only (G, TT) in the past? He can deviate to B (give bad service and collect a tip):

$$(B, TT), (B, NN), (B, NN)\text{... gives the waiter a payoff of } 5 + p\frac{0}{1-p} = 5$$

$$(G, TT) \text{ forever gives the waiter a payoff of } \frac{3}{1-p}$$

Clearly, the waiter has no incentive to deviate after any other history, because B is the best response to NN. Therefore, the waiter will have no profitable deviations if and only if the payoff from playing G every period is greater than the payoff from deviating to B and being punished forever:

$$\frac{3}{1-p} > 5$$

$$\frac{3}{5} > 1 - p,\ p > \frac{2}{5}$$

The analysis for the customer is similar to part c). The customer has an incentive to not tip in the current period (deviate to NT or NN) which gives him the payoff of 10, but this triggers the punishment, so the customer receives 0 thereafter:

$$(G, NN), (B, NN), (B, NN)\text{... gives the customer a payoff of } 10 + p\frac{0}{1-p}$$

$$(G, TT) \text{ forever gives the customer a payoff of } \frac{5}{1-p}$$

For the customer to play the best response to the waiter's strategy, deviating must not be worth it:

$$10 + p\frac{0}{1-p} < \frac{5}{1-p}$$

or

$$p > \frac{1}{2}.$$

The customer play the best response for all other histories, because NN is a best response to B.

Therefore, the strategy profile is subgame perfect for those p for which *both* waiter and customer have no profitable deviations. Waiter does not deviate for $p > \frac{2}{5}$, customer does not deviate for $p > \frac{1}{2}$. Since $\frac{1}{2} > \frac{2}{5}$, *both* do not deviate when

$$p > \frac{1}{2}.$$

Winter 2002 Second Midterm Answers

4) (30 points) Consider a senate race game. Player 1 (the incumbent senator) decides whether to launch an ad campaign or not (A, N). Player 2 (challenger) *simultaneously and independently* decides whether to enter the race or stay out (I, O). The payoffs from each action combination are given by (Player 1's payoff is the first entry in each cell).

		Player 2	
		I	O
Player 1	A	$1,1$	$3,3$
	N	$2,4$	$4,2$

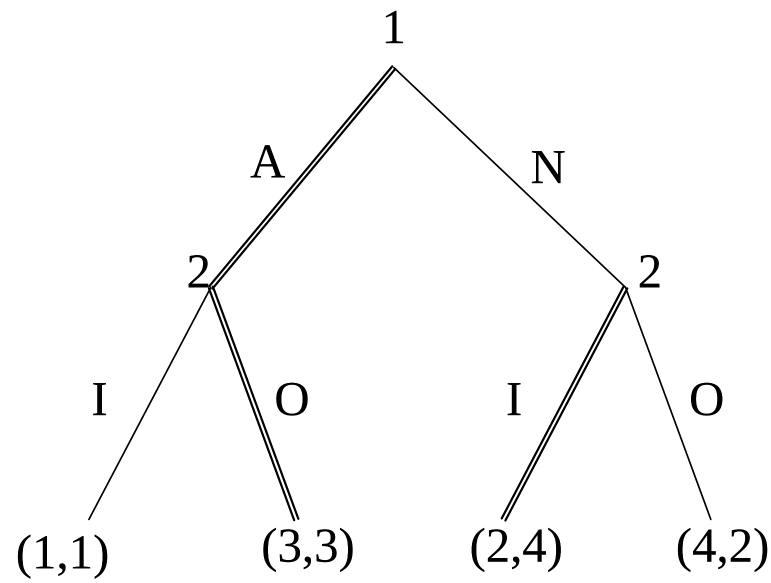
a) **(8 points)** Find all the Nash equilibria of this game. Which player(s), if any, have a dominant strategy?

		Player 2	
		I	O
Player 1	A	$1,1$	$3,\underline{3}$
	N	$\underline{2},\underline{4}$	$\underline{4},2$

The only Nash equilibrium is (N, I). Player 1 has a dominant strategy, player 2 does not.

b) **(10 points)** Now suppose that Player 1 moves first by choosing either A or N. Player 2 *observes* player 1's action and then chooses I or O. For every action combination, the players' payoffs are the same as in the above payoff matrix. Draw a tree of this new game. How many strategies does player 1 have and what are they? How many strategies does player 2 have and what are they? Find all the subgame perfect equilibria of this game.

$S_1 = \{A, N\}$
$S_2 = \{II, IO, OI, OO\}$



Subgame perfect equilibrium is (A, OI).

c) **(12 points)** Now find *all* the Nash equilibria of the game in part b). Are there any Nash equilibria that are not subgame perfect? Which Nash equilibrium is more believable? Explain. Does Player 1 receive a higher payoff in a static game of part a) or in a dynamic game of part b)? Why do you think this difference arises?

	II	IO	OI	OO
A	$1,1$	$1,1$	$\underline{3},\underline{3}$	$3,\underline{3}$
N	$\underline{2},\underline{4}$	$\underline{4},2$	$2,\underline{4}$	$\underline{4},2$

We can see that there are two Nash equilibria: (A, OI) and (N, II). In the first one, player 2's strategy prescribes him to always do what's best for him, no matter which node of the game he is in. The second strategy involves bluff, or non-credible threat. Player 2 says, "I will always enter, even if it hurts me". Knowing this, it is in player 1's best interest not to advertise. Player 2 expects player 1 not to advertise, and so he knows the game will never reach the point where his bluff is called, so II is a best response to N.

Which Nash equilibrium is more believable? In the second equilibrium, player 2 makes a threat that he has no incentive to carry out. So why should player 1 believe him? "I don't care what you say you'll do, I will just advertise, and we'll see what happens". Every time a player plays a strategy he himself has no incentive to stick to, we have this credibility problem.

Player 1 gets a higher payoff of 3 in the subgame perfect equilibrium of the dynamic game. By moving first, he is able to take advantage of the different responses of player 2 to his different actions, and therefore can better manipulate the outcome of the game.

Fall 2003 Second Midterm Answers

4) (35 points) Consider a game between two airlines. Players simultaneously choose whether to set a high price or a low price for the tickets. The payoffs are given by the following matrix

	L	H
L	$1,1$	$6,0$
H	$0,6$	$5,5$

a) **(10 points)** Find all the Nash equilibria of this game. Are equilibrium payoff allocations Pareto optimal? Suppose the game above is repeated a finite number of times. Is there a subgame-perfect equilibrium strategy that can lead to Pareto optimal payoffs at least in some rounds of the game? Explain.

The unique Nash equilibrium is (L, L). It is not Pareto optimal: the allocation that Pareto dominates $(1, 1)$ is $(5, 5)$. By Selten's theorem, all subgame perfect equilibria of the repeated game have players choose (L, L) action profile in every round. Therefore, none of the Pareto optimal payoff allocations $(0, 6)$, $(6, 0)$ or $(5, 5)$ can be realized in any of the rounds.

b) **(10 points)** Suppose instead that both airlines can choose among three prices $\{L, M, H\}$. The payoff matrix for the new game is

	L	M	H
L	$1,1$	$2,0$	$6,0$
M	$0,2$	$3,3$	$2,0$
H	$0,6$	$0,2$	$5,5$

Find all the Nash equilibria of this stage game. Suppose that the stage game is repeated twice, and the payoff in the second period is discounted by $\delta < 1$. Find the range of δ for which the following strategy profile is a subgame perfect equilibrium. Player 1 plays H in the first period. If the outcome of the first period is (H, H), then he plays M in the second period, otherwise he plays L in the second period. Player 2 has the same strategy.

The stage game has two Nash equilibria (M, M) and (L, L).

In the second period, the strategy tells the players to choose a Nash equilibrium action after *every* history, therefore, both players play their best response in any subgame that leads to the second period.

In the first period, a player can have a potential deviation to L instead of H. For this deviation to be unprofitable, we must have

$$5 + 3 \cdot \delta > 6 + 1 \cdot \delta$$

$$\delta > \frac{1}{2}$$

c) **(15 points)** Now suppose that the game in part *b*) is repeated three times. Write down the complete description of the strategies (that tell what action to take initially and after every possible history) that can sustain the outcome (H, H) in the first two periods as a subgame perfect equilibrium. Find the range of δ for which the strategy profile you described is a subgame perfect equilibrium.

Player 1. Play H in the first period. In subsequent periods, play M if the history has only (H, H) in the past. For all other histories, play L in the current period.

If the history in the first period is (H, H), from then on the game is identical to that of part *b*). The strategy profile is an equilibrium in this subgame for $\delta > \frac{1}{2}$. For any other history after the first period, the strategy prescribes to play a Nash equilibrium of the stage game, therefore, both players play their best response in any subsequent subgame.

In the first period, a player can also have a potential deviation to L instead of H. However, if it is unprofitable to deviate in the second period, it must also be unprofitable to deviate in the first period, because the punishment for the same deviation is then two periods long.

Problem 14.1

(a) $Q_1 = Q_2 = 40 \Rightarrow P = 260 - 2(80) = 100$

$\pi_1^{Cournot} = \pi_2^{Cournot} = (100 - 20)(40) = 3200$

(b) $Q^{Monopoly} = \dfrac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$

Therefore, profit of each firm in a cartel is

$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$

(c)

Without loss of generality, suppose Firm 2 cheats, but Firm 1 maintains its cartel quantity of 30. Then, the optimal choice for Firm 2 can be found from its best response function.

$Q_2^{Cheating} = \dfrac{1}{4}\left(260 - 20 - 2(30)\right) = 45$

Therefore, the market price is 260 – 2 (30+45) = 110. As a result, the profit of the cheating firm is:
$\pi_2^{Cheating} = (110 - 20)(45) = 4050$

If Firm 2 cheats, then it earns 4050 for one period, but earns its Cournot profit; 3200, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$3600 + \delta(3600) + \delta^2(3600) + ... \geq 4050 + \delta(3200) + \delta^2(3200) \Rightarrow \frac{3600}{1-\delta} \geq 4050 + \frac{3200\delta}{1-\delta}$$

$\Rightarrow \delta \geq 0.53$, where δ is the probability adjusted discount factor.

Problem 14.2

(a)
With Bertrand price competition $P_1 = P_2 = 20 \Rightarrow Q_1 = Q_2 = 60, \pi_1 = \pi_2 = 0$

(b) $Q^{Monopoly} = \dfrac{260 - 20}{2(2)} = 60 \Rightarrow P^{Monopoly} = 260 - 2(60) = 140$

Therefore, profit of each firm in a cartel is

$\pi_1^{Cartel} = \pi_2^{Cartel} = (140 - 20)(30) = 3600$

(c) Without loss of generality, let Firm 1 charges \$140, but Firm 2 cheat. Firm 2 needs to undercut Firm

1 only slightly to capture almost the entire monopoly profit. At the limit, Firm 2 captures the entire monopoly profit by cheating. Therefore, $\pi_2^{Cheating} = 7200$.

If Firm 2 cheats, then it earns 7200 for one period, but earns its Bertrand profit; 0, for all periods afterwards. On the other hand, if Firm 2 does not cheat, it can continue earning its cartel profit for ever. Hence, the collusive outcome can be sustained if

$$3600 + \delta(3600) + \delta^2(3600) + ... \geq 7200 + \delta(0) + \delta^2(0) \Rightarrow \frac{3600}{1-\delta} \geq 7200$$

$\Rightarrow \delta \geq \frac{1}{2}$, where δ is the probability adjusted discount factor.

Problem 14.3

Comparing the discount factors, it can be seen that it is more difficult to sustain a cartel under Cournot competition, since it requires a larger discount factor.