Economics 431
Homework 3
Answer Key

Part I. Bundling

GigaTech is *the only* manufacturer of the cutting-edge new generation phone handsets that sell for \$ 200. SneakyCom is one among *many* providers of prepaid calling cards that sell for \$ 20. The customers who buy GigaTech phones and/or prepaid calling cards have the following reservation prices:

Customer's name	Reservation price for a phone, R_1	Reservation price for a card, R_2
A	205	10
B	210	15
C	180	25
D	190	40

Consumer utility from a product equals their reservation price minus the price of the product. Consumers do not buy unless they get positive utility.

a) Suppose that the calling card and the phone are offered separately at prices $p_1 = 200$ for the phone and $p_2 = 20$ for the card. Which consumer buys which product(s).

A and B buy phone only, C and D buy card only.

b) SneakyCom CEO approaches GigaTech CEO with the following plan. Instead of selling phones and cards separately, they can offer the phone and the card *only* as a package for the price of \$220 per package. Then consumers who like calling cards a lot may also buy phones, and GigaTech will be able to sell more phones. Which consumers buy which product(s) if the phone and the card are bundled? (Hint: calling cards are still available separately from other providers) What happens to the sales of phones?

A buys nothing
B buys a phone and card package
C buys card only
D buys card only:

$$190 + 40 - 220 = 10 < 20 = 40 - 20$$

Phone sales go down.

c) Reconcile your result in b) with the fact that most new cars are offered only together with tires as one package. Will the dealers sell more cars if they offer them without tires and let the customers buy their most preferred tires elsewhere? Explain.

Cars and tires are *complementary* products. Tires are useless without cars, and cars are useless without tires. If tires and cars are not bundled, tire and car manufacturers eat away at each other's demand curve, which results in a higher price for a car plus tires, and necessarily lower sales. If cars are bundled with tires, setting the price for the bundle is under one party's control. This makes the price of car plus tires lower, and the quantity sold greater.

Part II. Tie in sales of complementary products

a) Solve for the monopoly price for operating systems and quantity sold.

Microsoft maximizes its profits from sales of operating systems taking price of browser $p_2 \geq c_2$ as given

$$MR(Q) = V - p_2 - 2Q = c_1$$

$$Q_a = \frac{V - c_1 - p_2}{2}$$

$$p_1 = V - p_2 - \frac{V - c_1 - p_2}{2} = \frac{V - p_2 + c_1}{2}$$

Profit

$$\pi_a = (p_1 - c_1)Q_a = \frac{(V - p_2 - c_1)^2}{4}$$

b) Now suppose that Microsoft can tie in the operating system and the browser, i.e. sell them as a bundle for a common price $p_b = p_1 + p_2$. What is the demand for this bundle?

$$Q = V - (p_1 + p_2) = V - p_b$$

What is the quantity produced?

$$MR(Q) = V - 2Q = c_1 + c_2$$

$$Q_b = \frac{V - c_1 - c_2}{2}$$

$$p_b = V - \frac{V - c_1 - c_2}{2} = \frac{V + c_2 + c_1}{2}$$

Profit

$$\pi_b = (p_b - (c_1 + c_2))Q_b = \frac{(V - c_2 - c_1)^2}{4} \geq \pi_a, \text{ since } p_2 \geq c_2$$

Note that

$$Q_b \geq Q_a$$

and $Q_b = Q_a$ when $p_2 = c_2$.

c) Show that consumer surplus in b) is always no less than in a).

$$p_a = p_1 + p_2 = \frac{V - p_2 + c_1}{2} + p_2 = \frac{V + c_1 + p_2}{2} = p_b + \frac{p_2 - c_2}{2} \geq p_b$$

In b) consumers buy more quantity *and* at a lower price than in a), so

$$CS_b \geq CS_a$$

The only case when the two are equal is when $p_2 = c_2$.

d) Does welfare (sum of producers' profits and consumer surplus) increase when we allow tying in the operating system and the browser?

$$W_a = CS_a + \pi_a = \underbrace{\frac{(2V - Q_a)Q_a}{2} - p_a Q_a}_{CS} + \underbrace{(p_1 - c_1)\,Q_a + (p_2 - c_2)\,Q_a}_{\text{Producers' profits}} =$$

$$= \frac{(2V - Q_a)Q_a}{2} - (c_1 + c_2)Q_a$$

$$W_b = CS_b + \pi_b = \underbrace{\frac{(2V - Q_b)Q_b}{2} - p_b Q_b}_{CS} + \underbrace{p_b Q_b - (c_1 + c_2)\,Q_b}_{\text{Microsoft's profit}} =$$

$$= \frac{(2V - Q_b)Q_b}{2} - (c_1 + c_2)Q_b$$

Welfare is more where quantity supplied is more (draw a diagram to convince yourself). Since $Q_b \geq Q_a$, $W_b \geq W_a$.

e) Microsoft's profit from not selling the browser:

$$\pi_a = \frac{(V - p_2 - c_1)^2}{4}$$

If Microsoft "cannot compete" on the browser market, then it must be unprofitable for it to enter this market alone:

$$c_2 > p_2$$

But then, it has no incentive to bundle the browser and produces only operating systems, because

$$\pi_a = \frac{(V - p_2 - c_1)^2}{4} > \pi_b = \frac{(V - c_2 - c_1)^2}{4}.$$

The fact that Microsoft chooses to bundle the browser is implicit evidence that it CAN compete on the browser market.

Part III A three-player game

Consider a game between three major car producers, C (Player 1), F (Player 2) and G (player 3). Each producer can produce either large cars, or small cars, but not both. That is, the action set of each producer i ($i = C, F, G$), is $A_i = \{S, L\}$. Let

$$\pi_i(a_C, a_F, a_G)$$

be the payoff producer i when the action combination is (a_C, a_F, a_G)

The payoff function is defined as follows:

If all three producers make only large cars, each gets the payoff of γ

If all three producers make only small cars, each gets the payoff of γ

Whenever any two producers make the same type of car and the other one makes a different type of car, these two producers get a payoff of β, and the other producer gets a payoff of α

For example:

$$\begin{aligned} \pi_C(S, L, L) &= \alpha \\ \pi_F(S, L, L) &= \beta \\ \pi_G(S, L, L) &= \beta \end{aligned}$$

Here F and G make the same type of car (L), so they get β each. C makes a different type of car, so he gets α.

a) Let C be the row player, F be the column player and G be the page player. Write the payoff matrix for this game. (it will consist of two matrices - one will correspond to all strategy combinations where G plays S (i.e. chooses the left matrix) and the other will correspond to all strategy combinations where G plays L (i.e. he chooses the right matrix)

Before putting the payoffs in a payoff matrix, it is convenient to represent the players' payoff functions in the following table

a_C	a_F	a_G	π_C	π_F	π_G
L	L	L	γ	γ	γ
L	L	S	β	β	α
L	S	L	β	α	β
L	S	S	α	β	β
S	L	L	α	β	β
S	L	S	β	α	β
S	S	L	β	β	α
S	S	S	γ	γ	γ

Now we can take each payoff triple and put it in the appropriate cell of the payoff matrix. For example, which cell will correspond to the action combination (L, S, S)? If G chooses S, we are in the left matrix. If F chooses S, we are in the first column of the left matrix. If C chooses L, we are in the second row of the left matrix. Therefore, (L, S, S) is the action combination that corresponds to the second row and the first column cell of the left matrix. This cell should contain the payoff (α, β, β) corresponding to the action combination (L, S, S).

G plays S		F plays		G plays L		F plays	
		S	L			S	L
C plays	S	γ,γ,γ	β,α,β	C plays	S	β,β,α	α,β,β
	L	α,β,β	β,β,α		L	β,α,β	γ,γ,γ

b) Find all the Nash equilibria when $\gamma > \alpha > \beta$. Are any of these equilibria Pareto optimal? Is the sum of profits (social welfare) maximized in these Nash equilibria?

G plays S		F plays		G plays L		F plays	
		S	L			S	L
C plays	S	$\underline{\gamma},\underline{\gamma},\underline{\gamma}$	$\underline{\beta},\alpha,\underline{\beta}$	C plays	S	$\underline{\beta},\underline{\beta},\alpha$	$\alpha,\underline{\beta},\underline{\beta}$
	L	$\alpha,\underline{\beta},\underline{\beta}$	$\underline{\beta},\underline{\beta},\alpha$		L	$\underline{\beta},\alpha,\underline{\beta}$	$\underline{\gamma},\underline{\gamma},\underline{\gamma}$

To find the Nash equilibria, we must find all the action combinations that are best responses to each other. For example, to find G's best response to (S_C, S_F), we must compare his payoff when he plays S (in the (S, S) cell of the left matrix) to his payoff from playing L (in the (S, S) cell of the right matrix). Since $\gamma > \alpha$, $BR_G(S_C, S_F) = L$. Another example: C's best response to (L_F, S_G). Compare C's payoff from playing S (in the (S, L) cell of the left matrix) to C's payoff from playing L (in the (L, L) cell of the left matrix). Both are equal to β. Both actions are best response to (L_F, S_G): $BR_C(L_F, S_G) = \{S, L\}$.

There are two Nash equilibria: (S, S, S) and (L, L, L). They are Pareto optimal (γ is the highest possible payoff for *every* player). Both payoff allocations are social welfare maxima (any allocation that involves α and β reduces the sum of payoffs)

c) Find all the Nash equilibria when $\alpha > \gamma > \beta$. Are any of these equilibria Pareto optimal? Is the sum of profits (social welfare) maximized in these Nash equilibria?

G plays S		F plays		G plays L		F plays	
		S	L			S	L
C plays	S	γ,γ,γ	$\underline{\beta},\underline{\alpha},\underline{\beta}$	C plays	S	$\underline{\beta},\underline{\beta},\underline{\alpha}$	$\underline{\alpha},\underline{\beta},\underline{\beta}$
	L	$\underline{\alpha},\underline{\beta},\underline{\beta}$	$\underline{\beta},\underline{\beta},\underline{\alpha}$		L	$\underline{\beta},\underline{\alpha},\underline{\beta}$	γ,γ,γ

All of these equilibria are Pareto optimal. We cannot find a Pareto improvement from any of them. Suppose we would like to find a Pareto improvement from (α, β, β). Since $\alpha > \gamma > \beta$, C's payoff will be reduced in any alternative allocation. Same can be shown for any other Nash equilibrium. We cannot the Nash equilibrium allocations maximize the social welfare: the sum of payoffs in any Nash equilibrium is $\alpha + 2\beta$. The sum of payoffs in (γ, γ, γ) is 3γ. The answer depends on which of these two numbers is greater.

Part IV Bertrand duopoly

Two firms, A and B, engage in Bertrand price competition in a market with inverse demand given by $p = 12 - Q$. Firm A has a higher marginal cost: $c_A > c_B$. Whenever a firm undercuts the rival's price, it has all the market. If a firm charges the same price as the rival, it has half of the market. If a firm charges more than the rival, it has zero market share.

a) Suppose that $c_A = 8$ and $c_B = 3$. Find a Nash equilibrium of this game (p_A, p_B) where one of the firms charges its marginal cost.

Monopoly price for firm B would be

$$p_M^B = \frac{A + c_B}{2} = 7.5 < c_A = 8$$

This is why firm B will not charge the rival's marginal cost, but will charge its monopoly price. The Nash equilibrium is

$$(p_A, p_B) = (8, 7.5)$$

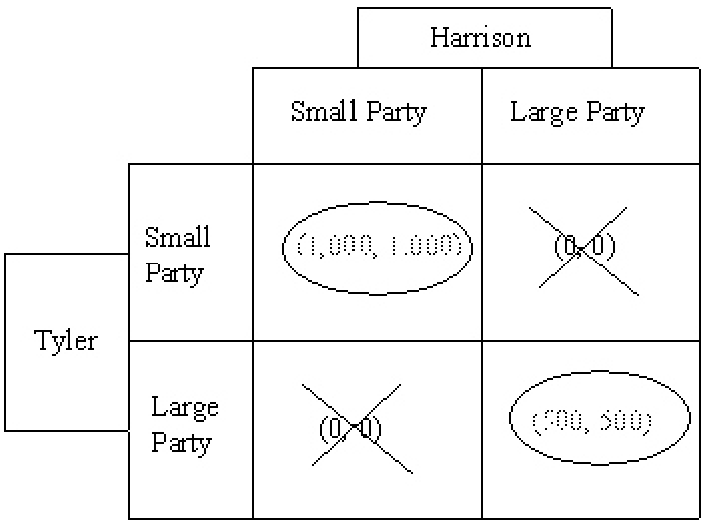
b) Suppose that there are not 2, but n firms with different marginal costs. Any number of firms may also have equal marginal costs. Can we have a marginal cost structure where one firm earns a positive profit? Can we have a marginal cost structure where more than one firm earns a positive profit? Explain.

If someone's monopoly price is below everybody else's marginal cost, this firm will earn a positive profit, and everybody else will earn zero profit.

If two firms are supposed to earn positive profit, this can only happen if they charge the same price (otherwise, the firm with a lower price has all the market). But any two firms who charge the same price will set it equal to marginal cost, because of the incentive to undercut the rival. We can have at most one firm with positive profit in a Bertrand setting.

Problem 9.1

(a) This is a classic matching problem. The easiest way to find the Nash equilibrium is to first eliminate from each row the dominated strategies for Harrison. Harrison has the second payoff in each pair. Looking at the first row, if Tyler chooses small (S), Harrison should also choose small. Thus the point (S,L) in the upper right-hand corner can be eliminated. Looking at the second row, if Tyler chooses large (L), then Harrison should also choose large. Thus the point (L,S) in the lower left-hand corner can be eliminated. Now we move to the dominated strategies for Tyler. If Harrison chooses the first column (S), then Tyler should also choose small. This is already removed and so we gain no information. Unfortunately checking the second column also yields no new information and we are left with the two Nash Equilibria (S,S) and (L,L).



(b) The Pareto optimal outcome is (1,000, 1,000) which results from the strategy pair (S,S). At this point there is no way to make either party better off.

Problem 9.5

Assume that I am firm 1. To determine my best response function, I equate my marginal revenue with my marginal cost.

$$290 - \frac{2}{3}Q_1 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right) = 50 \Rightarrow Q_1 = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q_i\right)\right]$$

Since my rivals and I are identical, $Q_i^* = Q^*$ for all *i*. Therefore,

$$Q^* = \frac{3}{2}\left[240 - \frac{1}{3}\left(\sum_{i=2}^{14} Q^*\right)\right] \Rightarrow Q^* = 48 \Rightarrow P^* = 290 - \frac{1}{3}(14)(48) = 66$$

My profit is

$$\pi = (66 - 50)(48) - 200 = 568$$

Problem 9.6

(a)
To determine firm 1's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_1 - 2Q_2 = 40 \Rightarrow Q_1 = \frac{1}{4}[360 - 2Q_2]$$

Since the firms are identical,

$$Q_1^* = Q_2^* = Q^* \Rightarrow \frac{1}{4}[360 - 2Q^*] = Q^* \Rightarrow Q^* = 60 \Rightarrow P^* = 160$$

Firm 1's profit is

$$\pi_1 = (160 - 40)60 = 7200$$

(b)
The monopoly output is $Q_M = \frac{1}{4}[360] = 90$
(45, 45) is a not a solution, because if one firm produces 45, then the other produces $\frac{1}{4}[360 - 2(45)] = 67.5$ to maximize its profit.

(c)

To determine firm 1's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_1 - 2Q_2 = 25 \Rightarrow Q_1 = \frac{1}{4}[375 - 2Q_2]$$

To determine firm 1's best response function, equate its marginal revenue with its marginal cost

$$400 - 4Q_2 - 2Q_1 = 40 \Rightarrow Q_2 = \frac{1}{4}[360 - 2Q_1]$$

Equate $4Q_1 = \frac{1}{4}[375 - 2Q_2]$ and $Q_2 = \frac{1}{4}[360 - 2Q_1]$ to determine the equilibrium quantity for each firm.

Verify that $Q_1^* = 65,$ and $Q_2^* = 57.5$