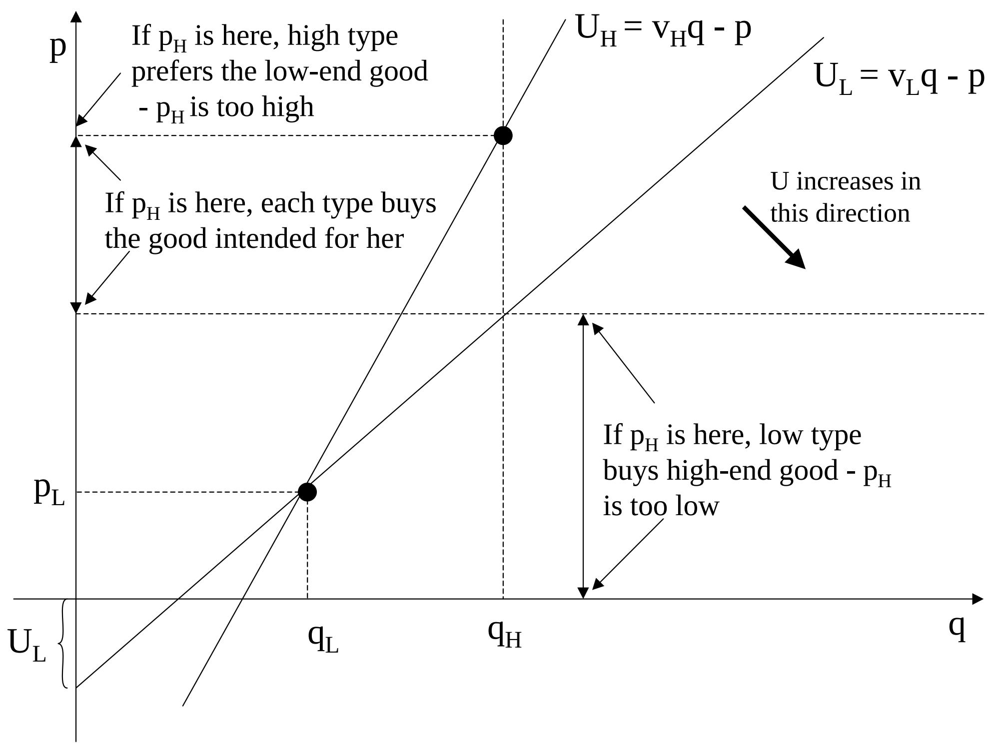
# Economics 431
# Homework 2
# Answer Key

**Part I The Economics of Product lines**

a) Plot the indifference curves $U_i(q,p) = 0$ for consumer surplus for high and low types with $q$ on the horizontal axis and $p$ on the vertical (Hint: they are straight lines).




The indifference curve for consumer $i$ is all the combinations of $p$ and $q$ that give him constant utility equal to $U_i$. The equation for these curves is

$$v_h q - p = U_h \text{ or } p = -U_h + v_h q$$

$$v_l q - p = U_l \text{or } p = -U_l + v_l q$$

If we draw $p$ on the vertical, these are upward sloping straight lines, the IC for high type is steeper. The intercept equals negative CS (whenever intercept is negative, CS is positive). Utility increases in south-east direction, because everybody likes to pay less and to have higher quality. Hence the area that corresponds to higher utility is below the indifference curve.

b) Using your plot, find the most profitable way to price a given product line $(q_l, q_h)$. Suppose that low-end product costs $p_l$. What is the high type buyer's utility from purchasing a low-end product (draw his indifference curve)?

$$U_h(p_l, q_l) = v_h q_l - p_l$$

What is the maximum price (expressed through $p_l$, $v_h$, $v_l$) that a high-end buyer can be charged for high-end product before he switches to low-end product? If the price of high-end product is too high, the high-end buyer's utility from buying it is too low, so he prefers low-end product. The maximum price for the high end product makes him just indifferent between the two:

$$U_h(p_h, q_h) = v_h q_h - p_h = v_h q_l - p_l = U_h(p_l, q_l)$$

This price is

$$p_h = p_l + v_h(q_h - q_l)$$

This pricing formula has an intuitive interpretation: the premium that high-end buyer pays for high-end product exactly equals to his willingness to pay for the increase in quality that this product offers. Now observe that if both $p_h$ and $p_l$ are increased by the same amount, high-end buyer will still buy the high-end product, but the profit of the monopolist from both buyers will grow. What is the maximum price that can be charged for the low-end product before low-end customer refuses to buy it?

We need not worry about low-end buyer wanting the high-end product (why? see the figure). Therefore, we simply choose the price that gives the low-end buyer exactly zero utility

$$p_l = v_l q_l$$

Therefore,

$$\begin{aligned} p_l &= v_l q_l, \\ p_h &= p_l + v_h(q_h - q_l) = \\ &= v_h q_h - q_l(v_h - v_l) \end{aligned}$$

c) Monopolist's profit is his total revenue minus total cost

$$\Pi = n_l p_l + n_h p_h - n_l C(q_l) - n_h C(q_h) =$$

$$= n_l v_l q_l + n_h[v_h q_h - q_l(v_h - v_l)] - n_h C(q_h) - n_l C(q_l)$$

Use the marginal revenue equals marginal cost condition for each of the two quality levels (which is the same as differentiating the profit expression with respect to $q_h$ or $q_l$ and then setting the derivative to zero).

$$\text{Low: } \underbrace{n_l v_l - n_h(v_h - v_l)}_{MR} = \underbrace{n_l c q_l}_{MC}$$

$$\text{High: } \underbrace{n_h v_h}_{MR} = \underbrace{n_h c q_h}_{MC}$$

to find

$$\begin{aligned} q_l &= \frac{1}{c}\left(v_l - \frac{n_h}{n_l}(v_h - v_l)\right) \\ q_h &= \frac{1}{c}v_h \end{aligned}$$

2. Monopolist faces higher total demand but the same type mix. He will just sell more units of the same product line.

3. Products become more differentiated. The quality of low-end product falls as $\frac{n_h}{n_l}$ rises. If there are more high-end customers around, the monopolist cares more about charging a high price for the high-end product to increase revenue. However, any price increase for the high-end product must be accompanied by making a low-end one less attractive for high-end buyers - hence decrease in quality at low end.

If $\frac{n_h}{n_l}$ is high enough, making a low quality product does not pay and the monopolist caters exclusively to the high-end market. Suppose that the monopolist contemplates entering the market with a low-end product of quality $q_l$ This will earn him $n_l v_l q_l$ in revenues from low-end customers, but will make him cut the price for high-end customers pay by $q_l(v_h - v_l)$ to maintain the attractiveness of the high-end product. This results in a loss of revenue equal to $n_h q_l(v_h - v_l)$. The total revenue from introducing a low end product is

$$\underbrace{n_l v_l q_l}_{\text{Gain from low-end sales}} - \underbrace{n_h q_l(v_h - v_l)}_{\text{Loss from price cut at high end}}$$

When there is a lot of high end customers,

$$\frac{n_h}{n_l} > \frac{v_l}{v_h - v_l}$$

this revenue is simply negative, so *any* low-end product will earn negative profit (you also can regroup the terms in the expression for profit to get the same result). So only one (high end) is offered.

**Part II Welfare effects of third degree price discrimination**

a) Compute the inverse demand for the integrated market with two consumer groups. Plot it with $p$ on the vertical and $q = q_1 + q_2$ on the horizontal.

$$\begin{aligned} p &= 12 - q_1, \; 0 \le p \le 12 \\ p &= 8 - q_2, \; 0 \le p \le 8 \end{aligned}$$

Demand curves for individual groups of consumers:

$$q_1(p) = \begin{cases} 12 - p, & p \le 12 \\ 0, & p > 12 \end{cases}$$

$$q_2(p) = \begin{cases} 8 - p, & p \le 8 \\ 0, & p > 8 \end{cases}$$

Total quantity demanded:

$$q(p) = \begin{cases} 20 - 2p = q_1(p) + q_2(p), & 0 \le p \le 8 \\ 12 - p = q_1(p) + 0, & 8 < p \le 12 \\ 0, & p > 12 \end{cases}$$

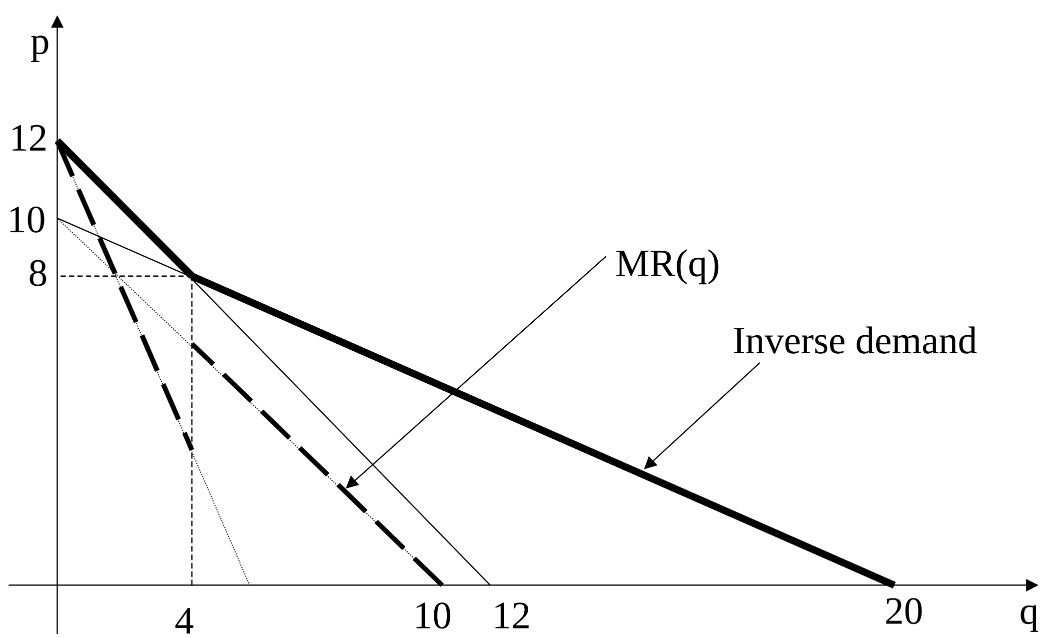
Inverse demand - a separate inverse demand curve for each "piece":

When $0 \le p \le 8$, $q$ is between 20 and 4, respectively (both groups buy)

When $8 \le p \le 12$, $q$ is between 4 and 0, respectively (only high group buys)

$$p(q) = \begin{cases} 12 - q, & 0 \le q \le 4 \\ 10 - \frac{q}{2} & 4 < q \le 20 \\ 0, & q > 20 \end{cases}$$

b) Suppose that the monopolist charges a uniform price on the integrated market and that his marginal cost is $c = 0$. Find price, quantity sold, consumer surplus and monopolist's profit. Compute the social welfare (sum of consumer surpluses and profits) (Hint: compute consumer surplus for high demand and low demand segments separately, then add them up. How much does the low demand group buy? How much does the high demand group buy?)



$$MR(q) = \begin{cases} 12 - 2q, & 0 \le q \le 4 \\ 10 - q & 4 < q \le 20 \\ 0, & q > 20 \end{cases}$$

Notice that $MR$ curve has a *gap*. At $q = 4$ the price is sufficiently low for the low type consumers to start buying, so $MR$ jumps up because of these additional consumers.

Equating $MR$ to $MC$

$$MR(q) = 10 - q = 0 = MC$$

If pricing is uniform,

$$q_u = 10 \text{ and } p_u = 5$$

Consumer surplus is the area between the inverse demand and the price. You can compute it as a sum of the areas of a triangle (above $p = 8$) and a trapezoid (between $p = 5$ and $p = 8$).

$$CS = \frac{1}{2}(12 - 8) \cdot 4 + \frac{10 + 4}{2} \cdot 3 = 29$$

Or, you can compute $CS$ by type and then add them up. At price $p_u = 5$, high types buy $12 - 5 = 7$ units, and low types buy $8 - 5 = 3$ units.

Consumer surplus of either group is the area between *their* demand curve and price:

$$\begin{aligned} CS_1 &= \frac{1}{2}q(12 - p) = \frac{1}{2} \cdot 7 \cdot (12 - 5) = \frac{49}{2} \\ CS_2 &= \frac{1}{2}q(8 - p) = \frac{9}{2} \end{aligned}$$

$$CS = CS_1 + CS_2 = 29.$$

Firm's profit equals

$$\pi = (p_u - MC) \cdot q_u = 50$$

The total surplus

$$W = \pi + CS = 50 + 29 = 79.$$

c) Now assume that the monopolist can price discriminate between the market segments. Find the prices and quantities for the low demand and the high demand market. Find the monopolist's profit and total consumer surplus. Show that welfare goes down.

With price discrimination, the producer acts as if she has two separate monopolies. Profit maximization conditions determine the quantities produced at either market:

$$\begin{aligned} MR_1 &= 12 - 2q_1 = 0,\ q_1 = 6, p_1 = 6 \\ MR_2 &= 8 - 2q_2 = 0,\ q_2 = 4, p_2 = 4. \end{aligned}$$

$$CS_1 = 18,\, CS_2 = 8$$

High types are worse off under price discrimination than under uniform pricing, because their price goes up. Low types are better off with price discrimination. The firm is, of course, better of, because it could have chosen $p_1 = p_2 = 5$, but instead chose a price combination $p_1 = 6$ and $p_2 = 4$ that gives it an even larger profit

$$\pi = (p_1 - MC)\, q_1 + (p_2 - MC)\, q_2 = 52$$

$$W = \pi + CS_1 + CS_2 = 52 + 18 + 8 = 78.$$

Overall welfare is lower - high type consumers lose more than low type and the monopolist gain.

d) Now assume that demands are the same but marginal cost is higher: $c = 7$. Show that under the uniform pricing the low demand group does not buy - some markets are not served. Compute the social welfare.

Profit maximization implies

$$MR = 12 - 2q_u = 7 = MC$$
$$q_u = \frac{5}{2},\, p_u = \frac{19}{2}$$

Note that since $p_u = \frac{19}{2} > 8$, low types do not buy at all.

$$\begin{aligned} CS_1 &= \frac{1}{2} q_u \cdot (12 - p_u) = \frac{25}{8} \\ CS_2 &= 0 \end{aligned}$$

$$\pi = (p_u - c) \cdot q_u = \frac{50}{8} = \frac{25}{4}$$
$$W = \pi + CS = \frac{75}{8}$$

e) Allow the third degree price discrimination. Compute prices and quantities with $c = 7$. Show that now all markets are served and that welfare goes up relative to the uniform pricing case in part d).

$$\begin{aligned} MR_1 &= 12 - 2q_1 = 7,\, q_1 = \frac{5}{2}, p_1 = \frac{19}{2} \\ MR_2 &= 8 - 2q_2 = 7,\, q_2 = \frac{1}{2}, p_2 = \frac{15}{2}. \end{aligned}$$

Notice that only high types were served under uniform pricing. Under price discrimination, high types receive the *same* price. In addition, monopolist also sells $q_2 = \frac{1}{2}$ to the low types, which, of course, increases the surplus from trade above its level in part $d$).

$$CS_1 = \frac{25}{8}, \; CS_2 = \frac{1}{8}$$

$$\pi = (p_1 - c)\, q_1 + (p_2 - c) q_2 = \frac{25}{4} + \frac{1}{4}$$

Compared to part d), total consumer surplus goes up by $\frac{15}{8}$, and monopolists profit goes up by $\frac{1}{4}$. Both monopolist and consumers are better off from price discrimination.

**End of Chapter Problem 5.5**

Profits if it serves only North America

$$Q_N = 100 - P_N \;\Rightarrow\; P_N = 100 - P_N \;\Rightarrow\; MR_N = 100 - 2Q_N$$

Now, equate marginal revenue in North America with marginal cost.

$$100 - 2Q_N = 20 \;\Rightarrow\; Q_N = 40 \Rightarrow\; P_N = 100 - (40) = 60$$

Hence, $\pi_N = (P_N - 20)Q_N = (60 - 20)(40) = 1600$

Profits if it serves both Sub-Saharan Africa and North America

Aggregate Demand is $Q_A = (1+\alpha)100 - 2P$

In that case, marginal revenue is $MR_A = (1+\alpha)50 - Q_A$

Equate marginal cost with marginal revenue

$$(1+\alpha)50 - Q_A = 20 \;\Rightarrow\; Q_A = (1+\alpha)50 - 20$$

Hence, $P_A = (1+\alpha)50 - \frac{1}{2}\left[(1+\alpha)50 - 20\right] = (1+\alpha)25 + 10$

Therefore, $\pi_A = (P_A - 20)Q_A = 2\left[(1+\alpha)25 - 10\right]^2$

Now, equate $\pi_A = \pi_N$, or equivalently

$$2\left[(1+\alpha)25 - 10\right]^2 = 1600 \;\Rightarrow\; \alpha = \left(\left(\frac{40}{\sqrt{2}} + 10\right)/25\right) - 1 = 0.531$$

**End of Chapter Problem 6.5**

Suppose the monopolist offers two entry fees, unit price combinations. One is targeted towards the low demanders and the other is targeted towards the high demanders. Let $(F_1, p_1)$ be the entry fee and unit price combination paid by the low demanders and $(F_2, p_2)$ be the entry fee and unit price combinations paid by the high demanders.

Now, from the discussions in the text, it follows that $F_1 = \frac{1}{2}(12 - p_1)^2$. Also, the monopolist needs to adjust $p_2$ so that the high demanders do not buy the combination intended for the low demanders. Therefore, it follows that

$$\frac{1}{2}(16 - p_2)^2 - F_2 = \frac{1}{2}(16 - p_1)^2 - \frac{1}{2}(12 - p_1)^2$$

$$\Rightarrow F_2 = \frac{1}{2}(16 - p_2)^2 - \frac{1}{2}(16 - p_1)^2 + \frac{1}{2}(12 - p_1)^2$$

Thus, the profit of the monopolist is given by

$$\pi_1 = N_h[(p_2 - 4)(16 - p_2) + F_2] + N_l\left[(p_1 - 4)(12 - p_1) + \frac{1}{2}(12 - p_1)^2\right]$$

Differentiate $\pi$ with respect to $p_1$ and $p_2$, and equate each expression to zero to obtain

$$p_1 = 4\left[1 + \frac{N_h}{N_l}\right], p_2 = 4$$

Substituting the optimal prices in to the profit expression, we obtain the maximum profit the monopolist can earn by serving both groups. Observe that

$$\pi_1 = \left[72 - \frac{1}{2}\left(12 - 4\frac{N_h}{N_l}\right)^2 + \frac{1}{2}\left(8 - 4\frac{N_h}{N_l}\right)^2\right]N_h + \left[4\frac{N_h}{N_l}\left(8 - 4\frac{N_h}{N_l}\right) + \frac{1}{2}\left(8 - 4\frac{N_h}{N_l}\right)^2\right]N_l$$

On the other hand, if the monopolist serves the high demanders only, its profit will be

$$\pi_2 = N_h\frac{1}{2}(16 - 4)^2 = 72N_h$$

Therefore, the monopolist serves both groups if and only if $\frac{\pi_1}{\pi_2} \geq 1$.

It can be verified that (remark: use a program such as MAPLE)

$\frac{\pi_1}{\pi_2} \geq 1$ if and only if $\frac{N_h}{N_l} \geq 1$

**Economics 431**
**Winter 2002**
**1st Midterm exam**
**Answer Key**

**1) (8 points)** Consider a competitive industry with identical firms (in equilibrium). If the fixed costs of all firms go up, the concentration ratio $CR_4$ has to go up in the new long-run equilibrium.

*True.* The higher is the fixed cost, the larger is the minimum efficient scale. Less firms will stay, so the 4 biggest firms will have to account for a larger fraction of the total market share. In particular, if all firms are identical,

$$CR_4 = \frac{4}{N} \cdot 100\%.$$

**2) (7 points)** As long as consumers are willing to pay a positive price for the good, the more is the quantity produced, the greater is the total surplus from trade.

*False* When quantity exceeds the point where supply crosses demand, consumers value goods at less than the marginal cost of production, and surplus from trade is less than that at the crossing point.

**3) (7 points)** It costs the local telephone company \$2 per month to provide caller ID service to a household. Elasticity of demand for caller ID service equals $-\frac{4}{3}$ (at any price). Then the telephone company will make more money if it offers caller ID at \$5 a month than if it offers this service at \$8 a month.

*False.*

$$p\left(1 - \frac{1}{\eta}\right) = c$$

$$p\left(1 - \frac{1}{4/3}\right) = \frac{p}{4} = 2$$

The profit maximizing price is \$8.

**4) (8 points)** Grocery stores typically have a 10% senior citizen discount, but some tanning salons offer as much as 25%. This must be because the pressure for companies to look charitable is much higher in the services industry than in retail.

*False.* Senior discounts have nothing to do with firms being charitable. Senior discounts are a form of price discrimination, and as such they raise the firms' profits. Elasticity of demand for groceries by seniors is not much higher than anyone else's, hence a modest discount. But elasticity of demand for tanning among seniors must be *a lot* higher than in the rest of the population - on average, seniors have much less to gain from looking tanned. Hence price discrimination results in a deeper discount.

**5) (20 points)** An industry consists of many identical firms. Each firm's total cost function is

$$C(q) = \frac{1}{8} + \frac{1}{2}q^2$$

and each firm's supply curve is

$$p = q$$

where $p$ is the market price and $q$ is quantity supplied.

a) **(10 points)** What is the market price in the *long-run* equilibrium?

Long run equilibrium obtains after all entry and exit decisions have been made. Price equals the minimum average cost

$$p^* = \min_q \left( \frac{1}{8q} + \frac{1}{2}q \right)$$

The minimum is reached where

$$q^2 = \frac{1}{4}, q = \frac{1}{2}$$

$$p^* = \frac{1}{2}.$$

b) **(6 points)** Suppose that $Q^* = 100$ units of the good are sold in this long-run equilibrium. What is the equilibrium number of firms in the industry?

Each firm supplies $q = \frac{1}{2}$. Then there must be a total of $N = 200$ firms who supply $Q^* = 100$.

c) **(4 points)** What is the Herfindahl index for this industry? (If you did not get part $b$, simply assume that the number of firms is equal to $N$ and leave your answer in letters)

$$H = \sum_{i=1}^{N} s_i^2 = \sum_{i=1}^{N} \left( \frac{100}{N} \right)^2 = \frac{10,000}{N} = 50.$$

**6) (20 points)** The same textbook that sells for \$ 70 in the US sells for \$ 5 in India. Suppose you know that if the publisher were to offer this book for the same (uniform) price in the two countries, *no one* in India would buy the book. Assume that the textbook is not offered outside of US and India.

a) **(10 points)** If the publisher had to charge the same price in both countries, what would it be? Are the US consumers harmed by price discrimination in this case? Explain.

The publisher's decision problem on the US market is *identical* under uniform pricing and under price discrimination. If Indian consumers do not buy at the common uniform price, the publisher acts as if she were only dealing with one market - the US. Hence the US price under third degree price discrimination is the same as that under uniform pricing - \$ 70. The US consumers are not harmed by price discrimination, but the Indian consumers and the publisher are made better off.

b) **(10 points)** Assume that the publisher's costs are exactly the same in either country. Suppose that the elasticity of demand for the textbook in the US is $-\frac{35}{34}$. What is the elasticity of demand for this textbook in India?

$$p_{US}\left(1-\frac{1}{\eta_{US}}\right)=p_I\left(1-\frac{1}{\eta_I}\right)$$

$$70\cdot\left(1-\frac{34}{35}\right)=5\cdot\left(1-\frac{1}{\eta_I}\right)$$

$$\frac{2}{5}=1-\frac{1}{\eta_I}$$

$$\eta_I=\frac{5}{3}$$

**7) (20 points)** VoiceAce is a manufacturer of unique voice recognition software sold to home and business users. The software is offered in two versions, the Regular with the vocabulary of 20,000 words and the Deluxe with the vocabulary of 50,000 words.

Business customers ($B$) value each word in the software's vocabulary at $v_B = 1.5$ cents (\$0.015) whereas home use customers ($H$) value each word at 0.8 cents (\$0.008). That is, if customer of type $i$ ($i$ is either $B$ or $H$) has a voice recognition software with a vocabulary of $q$ words and pays a price $p$, her utility is

$$U_i = v_i q - p.$$

a) **(10 points)** Suppose that VoiceAce wants to sell the Regular version to the home use customers and the Deluxe version to the business customers, but cannot tell the customer types apart. What prices should VoiceAce charge for the two versions of software?

Low type ($H$) must be indifferent between buying the Regular and not buying

$$p_R = v_H \cdot 20,000 = 0.008 \cdot 20,000 = \$160$$

High type ($B$) must be indifferent between buying Deluxe and Regular at \$160

$$v_B \cdot 50,000 - p_D = v_B \cdot 20,000 - 160$$

$$p_D = 160 + 0.015 \cdot 30,000 = 610$$

b) **(10 points)** Suppose VoiceAce can make either one version or two versions of software at the *same* cost. Will VoiceAce always find it profitable to offer two versions of the software? Explain. What fraction of VoiceAce total sales (by quantity) must be to home use customers in order for it to offer two versions of the software?

VoiceAce can sell the Deluxe version to business customers only at

$$p_D = 0.015 \cdot 50,000 = \$750$$

For VoiceAce to offer two versions, it must be more profitable than selling to just the business customers for \$750 apiece and selling nothing to home use customers.

$$750n_B \leq 610n_B + 160n_H$$

$$140n_B \leq 160n_H$$

$$n_H \geq \frac{7}{8}n_B$$

$$n_H + \frac{7}{8}n_H \geq \frac{7}{8}(n_B + n_H)$$

$$\frac{n_H}{n_H + n_B} \geq \frac{7}{15} \text{ or } 46.7\%$$

**Economics 431**
**Fall 2003**
**1st midterm**
**Answer Key**

**1) (7 points)** Consider an industry that consists of a large number of identical firms. In the long run competitive equilibrium, a firm's marginal cost must equal its average cost.

*True.* In the long-run competitive equilibrium, firms earn zero profit. Profit maximization implies $p = MC$, and zero profit condition says that $p = AC$. Therefore, $MC = AC$.

**2) (5 points)** Third degree price discrimination can lower monopolist's profits compared to uniform pricing, because the overall surplus from trade may decrease.

*False.* As part of third-degree price discrimination, monopolist is free to set prices equal across markets. Therefore, he has at least as much profit as under uniform pricing.

**3) (8 points)** Suppose that an industry has $CR_4 = 100$. Then its Herfindahl index $H$ cannot be less than 2500.

Since $CR_4 = 100$, the total number of firms in the industry cannot exceed 4. Theoretically, $H \geq 10,000/n$. Since $n$ is at most 4, $H$ must be at least 2500.

**4) (20 points)** Suppose that initially the industry with many identical firms is in the long-run equilibrium. Each firm has a cost function $C_0(q) = 1 + q^2$. Discovery of a new production technique lowers the variable cost, and this makes each firm's total cost function $C_1(q) = 1 + \frac{1}{4}q^2$.

a) **(10 points)** Calculate the initial long-run equilibrium price ($p_0$) and the new long-run equilibrium price ($p_1$).

$$p_{LR} = \min(AC)$$

When the cost function is

$$C(q) = F + Zq^2,\ AC(q) = \frac{F}{q} + Zq$$

Minimum efficient scale

$$q_{MES} = \sqrt{\frac{F}{Z}},\ \min(AC) = AC(q_{MES}) = 2\sqrt{FZ}$$

$$p_0 = 2,\ p_1 = 1$$

b) **(10 points)** Suppose that market demand is linear and given by $p = 4 - 0.1 \cdot Q$, where $p$ is the market price and $Q$ is the total quantity demanded. Calculate the number of firms in the industry before and after the change in technology. (Hint: what is the quantity produced by each firm in the long-run equilibrium?)

Each firm produces the quantity equal to $q_{MES}$

$$q_0 = 1,\ q_1 = 2$$

$$Q_0 = 10\,(4 - p_0) = 20$$

$$N_0 = \frac{Q_0}{q_0} = 20$$

$$Q_1 = 10\,(4 - p_1) = 30$$

$$N_1 = \frac{Q_1}{q_1} = 15.$$

**5) (15 points)** A profit-maximizing electric utility (truthfully!) reported a profit of \$9 million. It charges a uniform price of \$0.14 per kilowatt. Assume that demand for electricity is *linear* with intercept $A = \$0.23$ per kilowatt

a) **(5 points)** Calculate the electric utility's marginal cost.

The profit-maximizing price is

$$p = \frac{A + c}{2},\ c = 2p - A = \$0.05$$

b) **(5 points)** Calculate the quantity (in kilowatts) sold on the market

$$\pi = (p - c)\,Q$$

$$Q = \frac{\pi}{p - c} = \frac{9 \cdot 10^6}{0.09} = 10^8 \text{ kilowatts (100 million)}$$

c) **(5 points)** Calculate the elasticity of demand for electricity at monopoly price

$$\frac{1}{\eta} = \frac{p - c}{p},\ \eta = \frac{14}{9}$$

**6) (10 points)** A software company sells a proprietary statistical package and charges a uniform price of \$200. At this price, there are no students among the buyers of the software, although some students are willing to pay above the software's marginal cost.

a) **(6 points)** Will the company sell more copies of software if it can charge students and other users different prices? Is third degree price discrimination in this case more efficient or less efficient than uniform pricing? Explain.

Student market is not served under uniform pricing. Therefore, offering a student discount increases the quantity sold to students. Offering such a discount does not change the price (and therefore, quantity sold) for the rest of the users, because under uniform pricing the manufacturer was selling *only* to non-student users anyway. Total quantity sold increases, so does total surplus from trade.

b) **(4 points)** The elasticity of demand for software among non-students is $-4/3$, and the price that the software company chooses to charge student users is \$100. Calculate the elasticity of demand for this software among students.

Prices at different markets are chosen to equate marginal revenues across markets.

$$200\left(1-\frac{1}{4/3}\right) = 100\left(1-\frac{1}{\eta}\right)$$

$$1-\frac{1}{\eta}=\frac{1}{2},\ \eta = 2.$$

**7) (30 points)** Big C cable is a monopoly that can offer cable packages with different number of channels at different monthly fees. The marginal cost of providing an additional TV channel is $c = 0$. Let $p$ be the customer's willingness to pay (in cents per month) for an additional TV channel when he already has $q$ channels. There are two types of customers: high types whose demand for channels is given by

$$p = 120 - q$$

and low types whose demand for channels is given by

$$p = 60 - q$$

a) **(8 points)** Suppose all the high type customers live in Yuppieville and all the low type customers live in Sticksfield. What number of channels and at what monthly fee will Big C offer in each of these two areas?

It is optimal to use two-part tariff: offer the number of channels that maximizes total surplus from trade (120 and 60, respectively) and charge the price equal to consumer surplus

$$\begin{aligned}(q_H, F_H) &= (120, \$72)\\ (q_L, F_L) &= (60, \$18)\end{aligned}$$

b) **(8 points)** Alternatively, assume that high types and low types live in the same area, and Big C cannot tell its customers apart. Now it has to design a Basic package with 60 channels targeted at low types and a Premium package with 120 channels targeted at high types. If Big C wants to sell to both types, what is the profit-maximizing price for the Basic and Premium packages?

Basic package: monthly fee equals consumer surplus for the low type, i.e. $(q_L, F_L) = (60, \$18)$

Premium package: high type must be indifferent between his package and the low type's package

$$CS_H(q_H) - F_H = CS_H(q_L) - F_L$$

$$CS_H(q_H) = CS_H(60) = \$72$$

$$CS_H(q_L) = \frac{1}{2}60(120+60) = \$54$$

Then

$$F_H = 18 + (72 - 54) = \$36$$

c) **(8 points)** If 20% of Big C's customers are high types and 80% are low types, will Big C benefit from including just 40 (instead of 60) channels in its Basic Package? (Credit given for explicit calculation of new prices for Basic and Premium packages)

The new price for the basic package is $F_L = CS_L(40) = \frac{1}{2}40(60+20) = \$16$

Now high type's consumer surplus from the low type's package is $CS_H(q_L) = CS_H(40) = \frac{1}{2}40(120+80) = \$40$. Then can charge a higher price for the Premium package Therefore, Big C can charge the high type

$$F_H = 16 + (72 - 40) = \$48$$

The monopolist will make an additional \$12 on each high type and lose an additional \$2 on each low type. Since there are 4 low types for each high type, monopolist will gain \$12 for each \$8 lost. He will benefit from offering a Basic package with 40 channels.

d) **(6 points)** Rate the packages in a), b) and c) in terms of their economic efficiency. Explain.

Both a) and b) are efficient, because the quantity of channels included in each package maximizes the total surplus from trade. c) is inefficient.