

Economics 431
Homework 2
due Wed, July 11

Part I The economics of product lines (second degree price discrimination)

The common practice across producers is providing a high-end expensive product and a low-end cheap one. We ask how a company can increase its profits by offering this product line and whether it will always offer a line of two products

Suppose there are two types of buyers: high-end buyers and low-end buyers. The high-end buyers are willing to pay more for any given improvement in quality than low-end buyers. In particular, if a type i buyer ($i = h$ for high-end or $i = l$ for low-end) pays p for a product of quality q he receives utility (consumer surplus)

$$U_i = v_i q - p.$$

v_i is the maximum willingness to pay for an additional unit of quality which does not decrease with quality (this is the case of flat demand for quality with reservation price v_i). Then U_i is just consumer surplus of consumer type i from buying q units of quality at price p). Consumers of higher type are willing to pay more for quality increase:

$$v_h > v_l.$$

Suppose the monopolist knows that n_l of his customers are low-end and n_h are high-end but cannot distinguish the two types. He wants to arrange things so the low-end product appeals to low-end buyers while the high-end product appeals to high-end buyers. Let q_l denote the quality of the product targeted at the low-end buyer and q_h denote the quality of the product targeted at the high-end buyer.

a) Plot the indifference curves $U_i(q, p) = 0$ for consumer surplus for high and low types with q on the horizontal axis and p on the vertical (Hint: they are straight lines).

b) Using your plot, find the most profitable way to price a given product line (q_l, q_h) . Suppose that low-end product costs p_l . What is the high type buyer's utility from purchasing a low-end product (draw his indifference curve)? What is the maximum price (expressed through p_l, v_h, v_l) that a high-end buyer can be charged for high-end product before he switches to low-end product? Assume that the low-end customers will get a reservation utility of 0 if they do not purchase either type of product. What is the maximum price that can be charged for the low-end product before low-end customer refuses to buy it? Express p_l and p_h through q_l, q_h, v_h, v_l . (Answer: $p_l = v_l q_l, p_h = v_h q_h - q_l(v_h - v_l)$)

c) Now that we have determined the most profitable way to price any given product line, we turn to the question of choosing the optimal product line. Assume that

products of a given quality are produced at a constant per unit cost which depends on the quality chosen. Denote this per unit cost as $C(q)$. Write the monopolist's profit resulting from the sale of n_l products of quality q_l at price p_l and n_h products of quality q_h at price p_h . Substitute the profit-maximizing prices that you found in b) and express the profit as a function of the two quality levels .

Suppose that $C(q) = \frac{c}{2}q^2$

Questions:

c-1. What is the optimal quality for the high-end product? For the low-end product?

c-2. Suppose that n_h and n_l increase by the same factor (say, they both double). How does the product line change?

c-3. Suppose instead that $\frac{n_h}{n_l}$ increases. How does the product line change now? When is it optimal to offer just one product instead of two?

Part II Welfare effects of third degree price discrimination

Should we allow a monopolist to implement the third degree price discrimination? The monopolist will clearly be no worse off than under the uniform pricing (because he still has the choice of setting prices equal across all market segments). But what about social welfare? Does the monopolist gain more than consumers lose? Can they both gain? The answer depends on two opposing forces:

- Price discrimination facilitates access to the market for the consumers for whom the price was previously too high to make any purchase at all. These consumers are very price sensitive, so the monopolist targets them with a special low price.
- Price discrimination further segments the market, and transfers the goods away from the less elastic demand segment. The high price and low quantity in this elastic segment may lead to a net welfare loss.

Third degree price discrimination will be beneficial if some markets are not served under uniform pricing. When demand is linear, third degree price discrimination will be harmful when all markets are served under uniform pricing.

Suppose there are two groups of consumers: high demand (1) and low demand (2). Inverse demands are

$$p = 12 - q_1, 0 \leq p \leq 12$$

$$p = 8 - q_2, 0 \leq p \leq 8$$

a) Compute the inverse demand for the integrated market with two consumer groups. Plot it with p on the vertical and $q = q_1 + q_2$ on the horizontal (Hint: Follow these steps: express q_1 as a function of p and q_2 as a function of p . The aggregate

quantity demanded at price p will equal $q_1(p) + q_2(p)$. Note that if $8 \leq p \leq 12$, low demand consumers don't buy: $q_2 = 0$. The resulting inverse demand should have a kink at $q = 4$).

b) Suppose that the monopolist charges a uniform price on the integrated market and that his marginal cost is $c = 0$. Find price, quantity sold, consumer surplus and monopolist's profit. Compute the social welfare (sum of consumer surpluses and profits) (Hint: compute consumer surplus for high demand and low demand segments separately, then add them up. How much does the low demand group buy? How much does the high demand group buy?)

c) Now assume that the monopolist can price discriminate between the market segments. Find the prices and quantities for the low demand and the high demand market. Find the monopolist's profit and total consumer surplus. Show that welfare goes down.

d) Now assume that demands are the same but marginal cost is higher: $c = 7$. Show that under the uniform pricing the low demand group does not buy - some markets are not served. Compute the social welfare.

e) Allow the third degree price discrimination. Compute prices and quantities with $c = 7$. Show that now all markets are served and that welfare goes up relative to the uniform pricing case in part d).

Part III. Problems from the textbook

Practice problem 6.2

End of chapter 5.5, 6.5

Part IV Relevant problems from past exams (including review of theory of the firm and competitive equilibrium)

Winter 2002 first midterm

Questions 1-7

Fall 2003 first midterm

Questions 1-7