Economics 431
Homework 1
Answer key

Part II
a) $C(q) = 1 + q + q^2$
$AC(q) = \frac{1}{q} + q + 1$
$MC(q) = 1 + 2q$
b) Supply decision

$$\max_q (pq - C(q))$$

Marginal revenue equals marginal cost condition reads

$$p = 1 + 2q$$

It describes an individual farmer'sss supply curve.

c) To determine the market equilibrium, must compute the industry supply. If there are 10 identical farmers and industry supplies quantity Q, then each farmer must supply

$$q = \frac{Q}{10}$$

Each farmer must find it optimal to supply $\frac{Q}{n}$ gallons at price p. This implies that

$$p = 1 + 2\frac{Q}{10}.$$

We have found a relationship between industry supply Q and price p, which is the industry supply curve.

The equilibrium price must be such that supply equals demand. Demand is $Q = 13 - p$ or inverse demand $p = 13 - Q$.

Equilibrium condition:

$$13 - Q = 1 + 2\frac{Q}{10}$$

$$\frac{6}{5}Q = 12,\ Q = 10,\ p = 13 - 10 = 3$$

At price $p = 3$ each farmer produces $q = 1$ and earns $3 \cdot 1 - (1 + 1 + 1^2) = 0$. Each farmer just breaks even. Note that this same price corresponds to minimum average cost

$$\min_q AC(q) = AC(1) = 3.$$

No one will want to enter. This is a long-run equilibrium.

d) When demand for honey goes up, the SR equilibrium is where new demand intersects the 10-farmer industry supply:

$$19 - Q = 1 + 2\frac{Q}{10}$$

$$\frac{6}{5}Q = 18,\ Q = 15,\ p = 19 - 15 = 4.$$

This cannot be a long run equilibrium, since the price is above the break-even point. More farmers will enter and supply curve will pivot. Entry will continue until the equilibrium price is back to break-even level of $p = 3$.

Since we know that in the long run price must be back to $p = 3$, the long-run quantity is going to be $Q_{LR} = 19 - 3 = 16$. In the long-run, each individual farmer must be at his break-even point, which is to produce $q = 1$ at price 3. Since total quantity supplied is16, there must be 16 farmers in the long run, so 6 more will enter.

Elasticity of residual demand and price-taking behavior

a) What is $AC(q_i)$, $MC(q_i)$ and firm i's supply as a function of market price p?

$AC(q_i) = cq_i$

$MC(q_i) = 2cq_i$

Profit maximization implies that

$$p = MC(q_i)$$
$$p = 2cq_i$$
$$q_i(p) = \frac{p}{2c}$$

The total quantity supplied by this n-firm industry at price p (denote it $Q(p)$) is the sum of quantities supplied by each individual firm.

$$Q^S(p) = \sum_{i=1}^{n} q_i(p) = \sum_{i=1}^{n} \frac{p}{2c} = \underbrace{\frac{p}{2c} + ... + \frac{p}{2c}}_{\text{n times}} = n\frac{p}{2c}$$

b) Show that the elasticity of supply equals

$$\eta_S = \frac{\Delta Q^S}{\Delta p}\frac{p}{Q} = \frac{n}{2c}\frac{p}{Q}.$$

This is because

$$\frac{\Delta Q^S}{\Delta p} = \frac{n}{2c}$$

Let $Q^D(p)$ be quantity demanded at price p. Let us compute the portion of this demand that is satisfied by firm i. This portion is called *residual demand of firm i*. Let $Q^S_{-i}(p)$ be the quantity supplied by all firms except firm i:

$$Q^S_{-i}(p) = Q^D(p) - q_i(p)$$

Then the residual demand of firm i equals to the total market demand minus what was supplied by all other firms:

$$q_i^D(p) = Q^D(p) - Q_{-i}^S(p)$$

b) Show that the price elasticity of residual demand

$$\varepsilon_i(p) = \frac{\Delta q_i^D}{\Delta p}\frac{p}{q_i} = n\varepsilon(p) - (n-1)\eta_S$$

where p is market price Q is the total quantity sold and $\varepsilon(p)$ is the point elasticity of market demand.

$$\frac{\Delta q_i^D}{\Delta p} = \frac{\Delta Q^D}{\Delta p} - \frac{\Delta Q_{-i}^S}{\Delta p}$$

this says that the slope of residual demand equals the slope of market demand minus the slope of the rivals' supply. Since all firms are identical,

$$Q^S(p) = nq_i(p)$$

Therefore

$$Q_{-i}^S(p) = Q^S(p) - q_i(p) = (n-1)q_i(p) = (n-1)\frac{p}{2c}$$

$$\frac{\Delta q_i^D}{\Delta p}\frac{p}{q_i} = \left(\frac{\Delta Q^D}{\Delta p} - \frac{\Delta Q_{-i}^S}{\Delta p}\right)\frac{p}{q_i} = \left(\frac{\Delta Q^D}{\Delta p} - (n-1)\frac{1}{2c}\right)\frac{p}{q_i} =$$

$$= \left(\frac{\Delta Q^D}{\Delta p} - (n-1)\frac{1}{2c}\right)\frac{p}{Q/n} = \frac{\Delta Q^D}{\Delta p}\frac{p}{Q}n - (n-1)\frac{1}{2c}\frac{p}{Q}n = \varepsilon(p)n - (n-1)\eta_S$$

c) Show that as n becomes larger residual demand becomes more and more elastic.

$\varepsilon(p)$ is a negative number, η_S is a positive number.. As n becomes larger, $\varepsilon_i(p)$ becomes a large negative number. That is to say, residual demand becomes more and more elastic.

End of chapter Problem 2.4

(a) First find the inverse demand function by solving the demand equation for P as a function of Q

$$Q = 1{,}000 - 50P$$
$$\Rightarrow 50P = 1{,}000 - Q$$
$$\rightarrow P = 20 - \frac{Q}{50}$$

Then set this equal to marginal cost to find the competitive solution. This will give

$$P = 20 - \frac{Q}{50} = 10 = MC$$
$$\rightarrow \frac{Q}{50} = 10$$
$$\Rightarrow Q = 500$$
$$\Rightarrow P = 20 - \frac{Q}{50}$$
$$= 20 - \frac{500}{50}$$
$$= 20 - 10$$
$$= 10$$

Under monopoly we set marginal revenue equal to marginal cost. We find marginal revenue by finding total revenue first and taking the derivative with respect to Q or by applying the same intercept - twice the slope rule to the inverse demand. Using the same intercept - twice the slope rule we obtain

$$P = 20 - \frac{Q}{50}$$
$$MR = 20 - 2\left(\frac{Q}{50}\right)$$
$$= 20 - \frac{Q}{25}$$

If we derive an equation for revenue we obtain

$$R = PQ = \left(20 - \frac{Q}{50}\right)Q$$
$$= 20Q - \frac{Q^2}{50}$$

Taking the derivative we obtain

$$R = 20Q - \frac{Q^2}{50}$$

$$MR = \frac{dR}{dQ} = 20 - \frac{Q}{25}$$

Setting this equal to marginal cost we obtain

$$MR = 20 - \frac{Q}{25} = 10 = MC$$

$$\Rightarrow \frac{Q}{25} = 10$$

$$\Rightarrow Q = 250$$

$$\Rightarrow P = 20 - \frac{Q}{50}$$

$$= 20 - \frac{250}{50}$$

$$= 20 - 5$$

$$= 15$$

(b)

First compute the elasticity for the competitive case where Q = 500 and P = 10.

$$\varepsilon_D(competitive) = -\frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$= -\frac{10}{500}(-50)$$

$$= \frac{500}{500} = 1$$

Then compute the elasticity for the monopoly case where Q = 250 and P = 15.

$$\varepsilon_D(monopoly) = -\frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

$$= -\frac{15}{250}(-50)$$

$$= \frac{750}{500} = \frac{3}{2}$$

(c) The monopoly price is P = 15. Marginal cost for this firm is MC = 10. So we obtain

$$\frac{P - MC}{P} = \frac{15 - 10}{15} = \frac{5}{15} = \frac{2}{3}$$

$$\epsilon_D = \frac{3}{2}$$

$$\frac{1}{\epsilon_D} = \frac{1}{\left(\frac{3}{2}\right)} = \frac{2}{3}$$