Economics 431
Fall 2003
2nd midterm
Answer Key

1) (20 points) Big C cable company has a local monopoly in cable TV (good 1) and fast Internet (good 2). Assume that the marginal cost of producing either good is zero. There are three customers, A, B and C with different reservation prices for the two goods. Consumer utility from a product equals their reservation price minus the price of the product. Consumers do not buy unless they get positive utility. The reservation prices are as follows

Customer	Reservation price for good 1	Reservation price for good 2
A	2	5
B	4	4
C	6	2

a) **(7 points)** Suppose the goods are offered separately. Compute the monopoly prices for good 1 and good 2.

Good 1:

Price	Quantity	Profit
2	3	6
4	2	**8**
6	1	6

, Good 2:

Price	Quantity	Profit
2	3	6
4	2	**8**
5	1	5

Monopoly prices $(4, 4)$.

b) **(5 points)** Now assume that both goods are offered only as a bundle, but not separately. Compute the profit-maximizing bundle price.

	Price	Quantity	Profit
Good 1:	**7**	3	**21**
	8	2	16

The bundle is offered for price 7

c) **(8 points)** Let the two goods be offered separately at prices you have found in a), and also as a bundle at price you have found in b). Determine which customers buys only individual goods or both as a bundle. Determine the monopolist's profit. What is the most profitable way to bundle the goods?

Customer	Utility from bundle	Utility from good 1	Utility from good 2	Decision
A	0	-2	1	buy good 2
B	1	0	0	buy bundle
C	1	2	-2	buy good 1

Profit: $4+4+7=15$
Pure bundling is the most profitable.

2) (15 points) A local market for milk is served by several farms. Assume that you can use Cournot model with linear demand and constant marginal costs that may differ across farms to describe how this market operates. The price of milk is $2.50 a gallon. Farm A has marginal cost of $1.50 per gallon, has a 25% market share and makes a profit of $1000 per day.

a) **(8 points)** Farm B has a 20% share of this market. Calculate its marginal cost

$$\frac{p-c_i}{p}=\frac{s_i}{\eta}$$

$$\frac{p-c_B}{p-c_A}=\frac{s_B}{s_A}$$

$$\frac{2.50-c}{2.50-1.50}=\frac{0.2}{0.25}$$

$$2.50-c=0.8,\ c=1.70$$

b) **(7 points)** Calculate the profit of farm B (Hint: can the ratio of profits be expressed through market shares?)

$$p-c_i=Bq_i$$

$$\pi_i=(p-c_i)\,q_i=Bq_i^2$$

$$\frac{\pi_B}{\pi_A}=\frac{Bq_B^2}{Bq_A^2}=\frac{s_B^2}{s_A^2}$$

$$\pi_B=\$640\text{ per day.}$$

3) (15 points) Assume that consumer tastes for soft drinks can be described by a location model of product differentiation. Let manufacturers have identical marginal costs and set drink prices strategically to maximize profits.

a) **(10 points)** Consider three drinks: Coke, Sprite (both made by Coca Cola Company) and Uncle Al's Lemon Mixer. Suppose that consumers perceive Coke and Sprite as different products, but cannot tell the difference between Sprite and Uncle Al's Lemon Mixer. Restaurant A serves only Coke and Sprite. Restaurant B serves only Coke and Uncle Al's Lemon Mixer. Restaurant C serves only Sprite and Uncle Al's Lemon Mixer. All three restaurants have consumers with identical distribution of tastes and willingness to pay. Which drinks at which restaurants will be priced above their marginal cost? Which restaurant will have a higher price for Coke and why?

Drinks at A and B will be priced above marginal cost, drinks at C will be priced at marginal cost. Spatial Bertrand vs. Bertrand.

Restaurant A will have a higher price for Coke, because Coca Cola is a monopolist in restaurant A.

b) **(5 points)** Using your results in part *a*), explain why we observe that restaurants serving Coke do not serve Pepsi, and vice versa.

If consumers make little difference between Coke and Pepsi, and they are both offered at the same restaurant, they will have to be priced close to marginal cost, and profit is low. Being a monopolist in some locations is more profitable than being a Bertrand duopolist in all locations.

4) (35 points) Consider a game between two airlines. Players simultaneously choose whether to set a high price or a low price for the tickets. The payoffs are given by the following matrix

	L	H
L	$1,1$	$6,0$
H	$0,6$	$5,5$

a) **(10 points)** Find all the Nash equilibria of this game. Are equilibrium payoff allocations Pareto optimal? Suppose the game above is repeated a finite number of times. Is there a subgame-perfect equilibrium strategy that can lead to Pareto optimal payoffs at least in some rounds of the game? Explain.

The unique Nash equilibrium is (L, L). It is not Pareto optimal: the allocation that Pareto dominates $(1, 1)$ is $(5, 5)$. By Selten's theorem, all subgame perfect equilibria of the repeated game have players choose (L, L) action profile in every round. Therefore, none of the Pareto optimal payoff allocations $(0, 6)$, $(6, 0)$ or $(5, 5)$ can be realized in any of the rounds.

b) **(10 points)** Suppose instead that both airlines can choose among three prices $\{L, M, H\}$. The payoff matrix for the new game is

	L	M	H
L	$1,1$	$2,0$	$6,0$
M	$0,2$	$3,3$	$2,0$
H	$0,6$	$0,2$	$5,5$

Find all the Nash equilibria of this stage game. Suppose that the stage game is repeated twice, and the payoff in the second period is discounted by $\delta < 1$. Find the range of δ for which the following strategy profile is a subgame perfect equilibrium. Player 1 plays H in the first period. If the outcome of the first period is (H, H), then he plays M in the second period, otherwise he plays L in the second period. Player 2 has the same strategy.

The stage game has two Nash equilibria (M, M) and (L, L).

In the second period, the strategy tells the players to choose a Nash equilibrium action after *every* history, therefore, both players play their best response in any subgame that leads to the second period.

In the first period, a player can have a potential deviation to L instead of H. For this deviation to be unprofitable, we must have

$$5 + 3 \cdot \delta > 6 + 1 \cdot \delta$$

$$\delta > \frac{1}{2}$$

c) **(15 points)** Now suppose that the game in part *b*) is repeated three times. Write down the complete description of the strategies (that tell what action to take initially and after every possible history) that can sustain the outcome (H, H) in the first two periods as a subgame perfect equilibrium. Find the range of δ for which the strategy profile you described is a subgame perfect equilibrium.

Player 1. Play H in the first period. In subsequent periods, play M if the history has only (H, H) in the past. For all other histories, play L in the current period.

If the history in the first period is (H, H), from then on the game is identical to that of part b). The strategy profile is an equilibrium in this subgame for $\delta > \frac{1}{2}$. For any other history after the first period, the strategy prescribes to play a Nash equilibrium of the stage game, therefore, both players play their best response in any subsequent subgame.

In the first period, a player can also have a potential deviation to L instead of H. However, if it is unprofitable to deviate in the second period, it must also be unprofitable to deviate in the first period, because the punishment for the same deviation is then two periods long.

5) (20 points) Consider the following sequential game of entry deterrence on a market with linear demand given by

$$p = 9 - Q$$

The incumbent (firm 1) first chooses whether or not to invest in new technology. The investment costs K. If the incumbent invests, his marginal cost is $c_L = 1$, and if he does not invest, it is $c_H = 6$. Then the incumbent and the entrant (firm 2), whose marginal cost is $c_H = 6$ play a Cournot quantity game. The profit functions are

$$\pi_1 = \begin{cases} (A - c_L - B(q_1 + q_2))\, q_1 - K & \text{if invests} \\ (A - c_H - B(q_1 + q_2))\, q_1 & \text{if does not invest} \end{cases}$$

$$\pi_2 = (A - c - B(q_1 + q_2))\, q_2$$

a) **(10 points)** Show that if the incumbent invests, the Nash equilibrium of the subsequent Cournot game has the entrant produce zero. How much does the incumbent produce if the entrant produces zero? (Hint: will the entrant be better off producing zero if making $q_2 > 0$ drops the price below his marginal cost?)

If the entrant produces zero, the incumbent's best response is to produce monopoly output that corresponds to his marginal cost c_L

$$q_M = \frac{A - c_L}{2B} = 4$$

If at this output level the market price is below c_H,

$$p = \frac{A + c_L}{2} = 5 < 6 = c_H,$$

then the entrant will have a negative profit from any $q_2 > 0$. Therefore, entrant chooses to produce zero as a best response. The strategies that are best responses to each other constitute a Nash equilibrium.

b) **(10 points)** What are the equilibrium profits of the two firms if the incumbent does not invest? Find all values of K for which the incumbent will choose to invest in new technology.

If the incumbent does not invest, both firms have equal marginal costs c_H, and they each get

$$\pi_1 = \pi_2 = \frac{1}{9}\frac{(A - c_H)^2}{B} = 1$$

If the incumbent invests, he gets

$$\frac{1}{4}\frac{(A - c_L)^2}{B} - K = 16 - K$$

The incumbent invests if $16 - K > 1$, i.e.

$$K < 15.$$

Reference Guide

Cournot Oligopoly with linear demand

$$p = A - BQ$$

Elasticity

$$\eta = \left| \frac{dQ}{dp} \frac{p}{Q} \right| = \frac{1}{B} \frac{p}{Q}.$$

Firm i chooses q_i to maximize its profit given the outputs of all other firms:

$$\max_{q_i} \left(A - c_i - BQ_{-i} - Bq_i \right) q_i$$

In equilibrium, each firm's output must satisfy the condition for profit maximization:

$$A - c_i - BQ_{-i} - 2Bq_i = 0$$

or

$$p - c_i = Bq_i$$

or

$$\frac{p - c_i}{p} = \frac{s_i}{\eta},$$

where s_i is market share of firm i.