Print your name here _______________________________________

Your UM ID number\(^1\) _______________________________________

Instructions:

- Do not open the exam until you are told to do so.

- *Once the exam begins*, check that you have all the pages. There should be 11 pages including this one.

- *Once the exam begins*, print your name in capital letters on top of each page to receive credit for it.

- This is a closed book, closed notes exam.

- You have 75 minutes to take the exam.

- Answer the questions in the space provided. To get credit on word questions, you should provide a brief explanation of your answer. Please write concisely and to the point. Feel free to use diagrams, but label them properly. If your answer involves doing math, show all work (this way you will get partial credit in case your ideas are correct but your math is not).

- If you run out of space on a particular question, you may use the back side of the same page. Clearly indicate on the front of the page that your answer is on the back; and on the back, give the number of question you are answering.

Good Luck!

\(^1\)The underlined 8 digits on the face of your M-Card
1) **(20 points)** Big C cable company has a local monopoly in cable TV (good 1) and fast Internet (good 2). Assume that the marginal cost of producing either good is zero. There are three customers, A, B and C with different reservation prices for the two goods. Consumer utility from a product equals their reservation price minus the price of the product. Consumers do not buy unless they get positive utility. The reservation prices are as follows

<table>
<thead>
<tr>
<th>Customer</th>
<th>Reservation price for good 1</th>
<th>Reservation price for good 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

a) **(7 points)** Suppose the goods are offered separately. Compute the monopoly prices for good 1 and good 2.

b) **(5 points)** Now assume that both goods are offered only as a bundle, but not separately. Compute the profit-maximizing bundle price.
c) (8 points) Let the two goods be offered separately at prices you have found in $a)$, and also as a bundle at price you have found in $b)$. Determine which customers buys only individual goods or both as a bundle. Determine the monopolist’s profit. What is the most profitable way to bundle the goods?
2) (15 points) A local market for milk is served by several farms. Assume that you can use Cournot model with linear demand and constant marginal costs that may differ across farms to describe how this market operates. The price of milk is $2.50 a gallon. Farm A has marginal cost of $1.50 per gallon, has a 25% market share and makes a profit of $1000 per day.

a) (8 points) Farm B has a 20% share of this market. Calculate its marginal cost

b) (7 points) Calculate the profit of farm B (Hint: can the ratio of profits be expressed through market shares?)
3) **(15 points)** Assume that consumer tastes for soft drinks can be described by a location model of product differentiation. Let manufacturers have identical marginal costs and set drink prices strategically to maximize profits.

a) **(10 points)** Consider three drinks: Coke, Sprite (both made by Coca Cola Company) and Uncle Al’s Lemon Mixer. Suppose that consumers perceive Coke and Sprite as different products, but cannot tell the difference between Sprite and Uncle Al’s Lemon Mixer. Restaurant A serves only Coke and Sprite. Restaurant B serves only Coke and Uncle Al’s Lemon Mixer. Restaurant C serves only Sprite and Uncle Al’s Lemon Mixer. All three restaurants have consumers with identical distribution of tastes and willingness to pay. Which drinks at which restaurants will be priced above their marginal cost? Which restaurant will have a higher price for Coke and why?

b) **(5 points)** Using your results in part a), explain why we observe that restaurants serving Coke do not serve Pepsi, and vice versa.
4) **(35 points)** Consider a game between two airlines. Players simultaneously choose whether to set a high price or a low price for the tickets. The payoffs are given by the following matrix

\[
\begin{array}{c|cc}
   & L & H \\
\hline
L & 1,1 & 6,0 \\
H & 0,6 & 5,5 \\
\end{array}
\]

a) **(10 points)** Find all the Nash equilibria of this game. Are equilibrium payoff allocations Pareto optimal? Suppose the game above is repeated a finite number of times. Is there a subgame-perfect equilibrium strategy that can lead to Pareto optimal payoffs at least in some rounds of the game? Explain.
b) (10 points) Suppose instead that both airlines can choose among three prices \( \{L, M, H\} \). The payoff matrix for the new game is

\[
\begin{array}{ccc}
L & M & H \\
L & 1,1 & 2,0 & 6,0 \\
M & 0,2 & 3,3 & 2,0 \\
H & 0,6 & 0,2 & 5,5 \\
\end{array}
\]

Find all the Nash equilibria of this stage game. Suppose that the stage game is repeated twice, and the payoff in the second period is discounted by \( \delta < 1 \). Find the range of \( \delta \) for which the following strategy profile is a subgame perfect equilibrium. Player 1 plays \( H \) in the first period. If the outcome of the first period is \((H, H)\), then he plays \( M \) in the second period, otherwise he plays \( L \) in the second period. Player 2 has the same strategy.
c) **(15 points)** Now suppose that the game in part b) is repeated three times. Write down the complete description of the strategies (that tell what action to take initially and after every possible history) that can sustain the outcome \((H, H)\) in the first two periods as a subgame perfect equilibrium. Find the range of \(\delta\) for which the strategy profile you described is a subgame perfect equilibrium.
5) (20 points) Consider the following sequential game of entry deterrence on a market with linear demand given by

\[ p = 9 - Q \]

The incumbent (firm 1) first chooses whether or not to invest in new technology. The investment costs \( K \). If the incumbent invests, his marginal cost is \( c_L = 1 \), and if he does not invest, it is \( c_H = 6 \). Then the incumbent and the entrant (firm 2), whose marginal cost is \( c_H = 6 \) play a Cournot quantity game. The profit functions are

\[
\pi_1 = \begin{cases} 
(9 - c_L - (q_1 + q_2)) q_1 - K & \text{if invests} \\
(9 - c_H - (q_1 + q_2)) q_1 & \text{if does not invest} 
\end{cases}
\]

\[
\pi_2 = (9 - c_H - (q_1 + q_2)) q_2 
\]

a) (10 points) Show that if the incumbent invests, the Nash equilibrium of the subsequent Cournot game has the entrant produce zero. How much does the incumbent produce if the entrant produces zero? (Hint: will the entrant be better off producing zero if making \( q_2 > 0 \) drops the price below entrant’s marginal cost?)
b) **(10 points)** What are the equilibrium profits of the two firms if the incumbent does not invest? Find all values of $K$ for which the incumbent will choose to invest in new technology.
Reference Guide

Cournot Oligopoly with linear demand

\[ p = A - BQ \]

Elasticity

\[ \eta = \left| \frac{dQ}{dp} \frac{p}{Q} \right| = \frac{1}{BQ} \frac{p}{Q}. \]

Firm \( i \) chooses \( q_i \) to maximize its profit given the outputs of all other firms:

\[ \max_{q_i} (A - c_i - BQ_{-i} - Bq_i) q_i \]

In equilibrium, each firm’s output must satisfy the condition for profit maximization:

\[ A - c_i - BQ_{-i} - 2Bq_i = 0 \]

or

\[ p - c_i = Bq_i \]

or

\[ \frac{p - c_i}{p} = \frac{s_i}{\eta}, \]

where \( s_i \) is market share of firm \( i \).