Economics 431
Fall 2003
1st midterm
Answer Key

1) (7 points) Consider an industry that consists of a large number of identical firms. In the long run competitive equilibrium, a firm's marginal cost must equal its average cost.

True. In the long-run competitive equilibrium, firms earn zero profit. Profit maximization implies $p = MC$, and zero profit condition says that $p = AC$. Therefore, $MC = AC$.

2) (5 points) Third degree price discrimination can lower monopolist's profits compared to uniform pricing, because the overall surplus from trade may decrease.

False. As part of third-degree price discrimination, monopolist is free to set prices equal across markets. Therefore, he has at least as much profit as under uniform pricing.

3) (8 points) Suppose that an industry has $CR_4 = 100$. Then its Herfindahl index H cannot be less than 2500.

Since $CR_4 = 100$, the total number of firms in the industry cannot exceed 4. Theoretically, $H \geq 10,000/n$. Since n is at most 4, H must be at least 2500.

4) (20 points) Suppose that initially the industry with many identical firms is in the long-run equilibrium. Each firm has a cost function $C_0(q) = 1 + q^2$. Discovery of a new production technique lowers the variable cost, and this makes each firm's total cost function $C_1(q) = 1 + \frac{1}{4}q^2$.

a) **(10 points)** Calculate the initial long-run equilibrium price (p_0) and the new long-run equilibrium price (p_1).

$$p_{LR} = \min(AC)$$

When the cost function is

$$C(q) = F + Zq^2,\ AC(q) = \frac{F}{q} + Zq$$

Minimum efficient scale

$$q_{MES} = \sqrt{\frac{F}{Z}},\ \min(AC) = AC(q_{MES}) = 2\sqrt{FZ}$$

$$p_0 = 2,\ p_1 = 1$$

b) **(10 points)** Suppose that market demand is linear and given by $p = 4 - 0.1 \cdot Q$, where p is the market price and Q is the total quantity demanded. Calculate the number of firms in the industry before and after the change in technology. (Hint: what is the quantity produced by each firm in the long-run equilibrium?)

Each firm produces the quantity equal to q_{MES}

$$q_0 = 1,\ q_1 = 2$$
$$Q_0 = 10\,(4 - p_0) = 20$$
$$N_0 = \frac{Q_0}{q_0} = 20$$
$$Q_1 = 10\,(4 - p_1) = 30$$
$$N_1 = \frac{Q_1}{q_1} = 15.$$

5) (15 points) A profit-maximizing electric utility (truthfully!) reported a profit of \$9 million. It charges a uniform price of \$0.14 per kilowatt. Assume that demand for electricity is *linear* with intercept $A = \$0.23$ per kilowatt

a) **(5 points)** Calculate the electric utility's marginal cost.

The profit-maximizing price is

$$p = \frac{A + c}{2},\ c = 2p - A = \$0.05$$

b) **(5 points)** Calculate the quantity (in kilowatts) sold on the market

$$\pi = (p - c)\,Q$$
$$Q = \frac{\pi}{p - c} = \frac{9 \cdot 10^6}{0.09} = 10^8 \text{ kilowatts (100 million)}$$

c) **(5 points)** Calculate the elasticity of demand for electricity at monopoly price

$$\frac{1}{\eta} = \frac{p - c}{p},\ \eta = \frac{14}{9}$$

6) (10 points) A software company sells a proprietary statistical package and charges a uniform price of \$200. At this price, there are no students among the buyers of the software, although some students are willing to pay above the software's marginal cost.

a) **(6 points)** Will the company sell more copies of software if it can charge students and other users different prices? Is third degree price discrimination in this case more efficient or less efficient than uniform pricing? Explain.

Student market is not served under uniform pricing. Therefore, offering a student discount increases the quantity sold to students. Offering such a discount does not change the price (and therefore, quantity sold) for the rest of the users, because under uniform pricing the manufacturer was selling *only* to non-student users anyway. Total quantity sold increases, so does total surplus from trade.

b) **(4 points)** The elasticity of demand for software among non-students is $-4/3$, and the price that the software company chooses to charge student users is \$100. Calculate the elasticity of demand for this software among students.

Prices at different markets are chosen to equate marginal revenues across markets.

$$200\left(1-\frac{1}{4/3}\right)=100\left(1-\frac{1}{\eta}\right)$$

$$1-\frac{1}{\eta}=\frac{1}{2},\ \eta=2.$$

7) (30 points) Big C cable is a monopoly that can offer cable packages with different number of channels at different monthly fees. The marginal cost of providing an additional TV channel is $c=0$. Let p be the customer's willingness to pay (in cents per month) for an additional TV channel when he already has q channels. There are two types of customers: high types whose demand for channels is given by

$$p=120-q$$

and low types whose demand for channels is given by

$$p=60-q$$

a) **(8 points)** Suppose all the high type customers live in Yuppieville and all the low type customers live in Sticksfield. What number of channels and at what monthly fee will Big C offer in each of these two areas?

It is optimal to use two-part tariff: offer the number of channels that maximizes total surplus from trade (120 and 60, respectively) and charge the price equal to consumer surplus

$$\begin{aligned}(q_H,F_H)&=(120,\$72)\\(q_L,F_L)&=(60,\$18)\end{aligned}$$

b) **(8 points)** Alternatively, assume that high types and low types live in the same area, and Big C cannot tell its customers apart. Now it has to design a Basic package with 60 channels targeted at low types and a Premium package with 120 channels targeted at high types. If Big C wants to sell to both types, what is the profit-maximizing price for the Basic and Premium packages?

Basic package: monthly fee equals consumer surplus for the low type, i.e. $(q_L,F_L)=(60,\$18)$

Premium package: high type must be indifferent between his package and the low type's package

$$CS_H(q_H)-F_H=CS_H(q_L)-F_L$$

$$CS_H(q_H)=CS_H(60)=\$72$$

$$CS_H(q_L)=\frac{1}{2}60(120+60)=\$54$$

Then

$$F_H = 18 + (72 - 54) = \$36$$

c) **(8 points)** If 20% of Big C's customers are high types and 80% are low types, will Big C benefit from including just 40 (instead of 60) channels in its Basic Package? (Credit given for explicit calculation of new prices for Basic and Premium packages)

The new price for the basic package is $F_L = CS_L(40) = \frac{1}{2}40(60+20) = \$16$

Now high type's consumer surplus from the low type's package is $CS_H(q_L) = CS_H(40) = \frac{1}{2}40(120+80) = \$40$. Then can charge a higher price for the Premium package Therefore, Big C can charge the high type

$$F_H = 16 + (72 - 40) = \$48$$

The monopolist will make an additional \$12 on each high type and lose an additional \$2 on each low type. Since there are 4 low types for each high type, monopolist will gain \$12 for each \$8 lost. He will benefit from offering a Basic package with 40 channels.

d) **(6 points)** Rate the packages in a), b) and c) in terms of their economic efficiency. Explain.

Both a) and b) are efficient, because the quantity of channels included in each package maximizes the total surplus from trade. c) is inefficient.

8) (20 points) An ice cream maker has to decides what types of ice cream to sell and at what prices. The customers who buy ice cream have different tastes x for fat content, with x ranging from $\frac{1}{4}$ to $\frac{5}{4}$. For each x, there is an equal number of customers with taste x. If x is the customer's most preferred fat content, and the ice cream has fat content z, then customer x is willing to pay

$$2 - 0.8|x - z|$$

for this ice cream. The marginal cost of producing an ice cream with any fat content is $c = 0.4$. Assume that the manufacturer always serves the whole market.

a) **(6 points)** If the manufacturer is free to offer any two types of ice cream, what will be their optimal "locations" in terms of fat content and what will be the profit maximizing prices?

Locations will be 1/2 and 1. For each of these products, a marginal consumer has to "go the distance" equal to 1/4 to the nearest product. Marginal consumer pays the full price equal to 2. Then each product is offered at price

$$p = 2 - 0.8\frac{1}{4} = 1.8$$

b) **(6 points)** Suppose that due to a fashion for low fat foods, customer tastes ("addresses") unexpectedly shift from the interval $\left[\frac{1}{4}, \frac{5}{4}\right]$ to the interval $[0, 1]$. For these new tastes, determine the profit-maximizing prices for the product line that you found in part a). Continue to assume that the manufacturer serves all the market.

Because the manufacturer serves all the market, the price for product $\frac{1}{2}$ has to be low enough to serve the customer with taste 0. Therefore $p(1/2) = 2 - 0.8\frac{1}{2} = 1.6$. But now the product $\frac{1}{2}$ can serve the entire market $[0, 1]$, and sales of product 1 are zero. Any price for product 1 will therefore be consistent with profit maximization.

c) **(8 points)** If it costs $F = 5$ per product (this fixed cost is measured in flow terms, same as profit) to adapt the product to the change in tastes (i.e. to re-locate each of the two products to its new optimal fat content), will the manufacturer choose to adapt or stay with his old product line? In doing profit calculations, assume that the total number of customers is 100.

Profit from the old product line: $[p(1/2) - c] \cdot 100 = 120$

The optimal line of two products will now have fat contents of 1/4 and 3/4, and, from part *a*), they will both sell for $p = 1.8$.

Profit from the optimal product line: $[1.8 - c] \cdot 100 = 140$

By spending an additional $2F = 10$, profit goes up by $140 - 120 = 20$. There is a net profit gain from adapting the product line to new tastes.

Reference guide

Derivatives of some functions

$$\frac{d}{dx}\left(x^2\right) = 2x$$
$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

Marginal revenue for a monopolist

$$\frac{d}{dq}\left(p(q)\cdot q\right) \equiv MR\left(q\right) = p(q)\cdot\left(1-\frac{1}{\eta\left(p\right)}\right),$$

where $p(q)$ is the market price when quantity sold is q and

$$\eta\left(p\right) = \left|\frac{dQ_D}{dp}\cdot\frac{p}{Q_D}\right|$$

is the positive of the elasticity of demand.

Profit maximization problem for the monopolist facing linear demand

$$p = A - BQ$$

and constant marginal cost c:

$$\pi = pQ - cQ = \left(p-c\right)Q = \left(A-c-BQ\right)Q$$

Condition for profit maximization

$$MR = A - 2BQ_M = c = MC$$

Monopoly quantity and monopoly price

$$Q_M = \frac{A-c}{2B};\ p_M = \frac{A+c}{2}.$$