Economics 431
Fall 2003
Final Exam
Answer Key

1) (25 points) Consider repeated labor disputes between the union (Player 1) and the employer (Player 2) that are modeled as an indefinitely repeated game. Each period, the players simultaneously choose whether to hire a lawyer (L) or negotiate without one (N). Hiring a lawyer is costly, and it improves the chances of winning the dispute only if the other side does not have a lawyer. Formally, the stage game is given by a payoff matrix

	L	N
L	$5, 5$	$8, 2$
N	$3, 7$	$6, 6$

a) **(5 points)** Find all the Nash equilibria of the stage game. Which players, if any, have a dominant strategy?

	L	N
L	$\underline{5}, \underline{5}$	$\underline{8}, 2$
N	$3, \underline{7}$	$6, 6$

Both players have a dominant strategy L.

b) **(15 points)** Now suppose that the discount factor for both players is equal to δ. Write down a complete description of the simplest possible strategy (for both players), such that the action profile (N, N) is played every period in a subgame perfect equilibrium. For what values of δ the strategy you described is a subgame perfect equilibrium? For full credit, you should check for all relevant deviations.

Player 1
Play N in the first period
For the history that had only (N, N) in all of the previous periods, play N this period.
For all other histories, play L this period

Player 2 - exactly the same strategy

A strategy profile is a subgame perfect equilibrium if neither player has a profitable deviation after any history.

Player 1
After $(NN, NN, ..., NN)$:

$$\underbrace{\frac{6}{1-\delta}}_{\text{Play } N} > \underbrace{8 + \delta\frac{5}{1-\delta}}_{\text{Play } L}$$

There are no profitable deviations if

$$6 > 8(1-\delta) + 5\delta,\ \delta > \frac{2}{3}$$

After all other histories, player 1 cannot improve his payoff in the current period (because L is the dominant strategy) and his current actions do not affect his future payoff.

Player 2
After $(NN, NN, ..., NN)$:

$$\underbrace{\frac{6}{1-\delta}}_{\text{Play } N} > \underbrace{7 + \delta\frac{5}{1-\delta}}_{\text{Play } L}$$

$$6 > 7(1-\delta) + 5\delta,\ \delta > \frac{1}{2}$$

The strategy profile is subgame perfect when $\delta > \frac{2}{3}$.

c) **(5 points)** Based on your results, when labor disputes arise frequently, is it more likely or less likely that they are resolved by negotiation? Explain.

Frequent disputes mean that δ is high, because the period between the two disputes is short. This makes it easier to sustain the outcome when labor disputes are negotiated.

2) (25 points) Consider an industry with more than two firms and assume that it can be described by a Cournot oligopoly game. True or false:

a) **(7 points)** If all firms have identical constant marginal costs and zero fixed costs, no two firms will have an incentive to merge.

True. In a Cournot industry with N firms

$$\pi(N) = \frac{(A-c)^2}{(N+1)^2 B}$$

For the merger to be profitable,

$$\pi(N-1) > \pi(N) + \pi(N)$$

This condition reads

$$\left(\frac{N+1}{N}\right)^2 > 2$$

This cannot hold for any $N > 2$.

b) **(5 points)** If the motive for merger is saving on the fixed costs, then consumers are worse off from this merger.

True. If production takes place at all, fixed costs are irrelevant for output choice. After merger concentration increases, as so does the price.

c) **(7 points)** Suppose that some firms in the industry have high marginal costs (c_H) and others have low marginal costs $c_L < c_H$. Then a merger between two low-cost firms is always more profitable than the merger between a low-cost and a high-cost firm.

False. A merger between two low-cost firms creates another low cost firm. But so does the merger between a high-cost and a low-cost firm, because no output is produced at cost c_H if same output can be now produced at cost c_L. So post-merger profits are the same in either merger. Because pre-merger the c_H firm was making less profit than a c_L firm, it is cheaper to buy a c_H firm, which is why merger between c_H and c_L is more profitable.

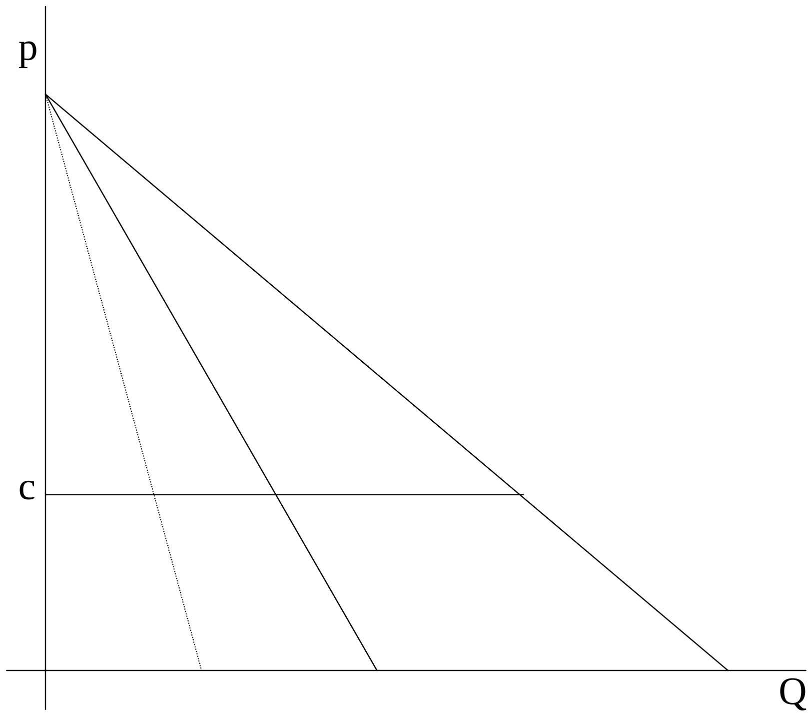
d) **(6 points)** Now assume that the industry is no longer described by the Cournot model. Initially, there are several big firms (leaders, as in Stackelberg game) and several small firms (followers). Two follower firms decide to merge in order to become another leader firm. Describe under what circumstances this merger benefits consumers.

There are two effects that move the price in the opposite directions: the quantity effect and the concentration effect. Quantity increases with the number of leader firms, pushing price down. However, mergers decrease the total number of firms, pushing price up. The merger benefits consumers if there are not too many leaders already. The very first leader firms make a large impact on the total quantity, price goes down after merger. When there are many leaders already, an extra leader makes less of an impact on quantity, so the concentration effect becomes more important.

3) (35 points) There is a monopoly manufacturer and a monopoly retailer. Demand for the retail product is

$$p = 12 - Q.$$

This demand and some other lines are shown on the diagram. Marginal cost of the manufacturer is constant and equal to $c = 4$. The manufacturer charges the retailer a uniform wholesale price r.



a) **(15 points)** Clearly show the following on the diagram and mark it as instructed:
(2 points) Retailer's demand for the manufacturer's product and label it $r(Q)$;
(2 points) Manufacturer's marginal revenue line and label it $mr(Q)$;
(2 points) Profit-maximizing quantity for the manufacturer and label it Q^*;
(2 points) Wholesale price and label it r^*;
(2 points) Retail price and label it p^*;
(5 points) If retailer and manufacturer were maximizing their joint profits, would they still choose to produce Q^* or some other quantity? Calculate this quantity, show it on the diagram and label it Q_M.

They would produce monopoly quantity

$$Q_M = \frac{A - c}{2B} = 4$$

b) **(3 points)** There are various ways to force the retailer to sell Q_M rather than Q^*. One way is a contract that forbids the retailer to charge more than $\bar{p}$ (a price ceiling). Calculate $\bar{p}$.

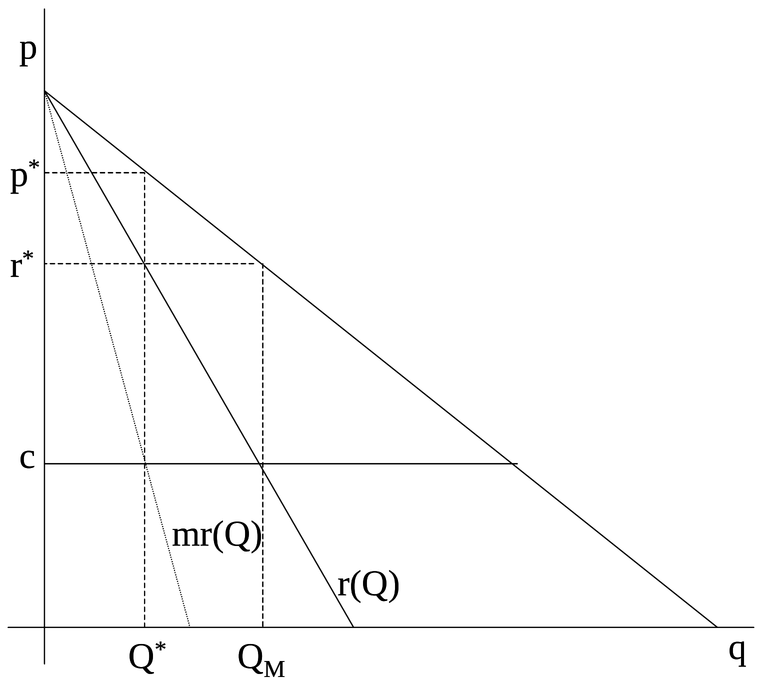


Figure 1:

$$\bar{p} = 12 - 4 = 8$$

c) **(8 points)** What other contracts between the manufacturer and the retailer can achieve a bigger joint profit for them? Describe at least three other arrangements.

Franchise agreement - the manufacturer charges a flat fee and sells for marginal cost. It is the equivalent of a two-part tariff.

Royalty agreement - the manufacturer sells at marginal cost, but the retailer pays back a fraction of his profits

Vertical merger

Minimum sales quota

d) **(9 points)** In part b), a price ceiling was used to improve the joint profits of manufacturer and retailer. Another very common agreement between manufacturers and retailers is a price *floor* (also known as resale price maintenance agreement). Explain when this restriction is used and why it may improve the manufacturer's profits from a franchise contract. (Hint: you do not have to assume monopolistic retailer).

Suppose that there are several retailers that provide a promotional service for which consumer is willing to pay extra. This service is costly and non-excludable, that is, if one retailer provides the service, the others enjoy its benefits for free. Without a price floor, retailers' profits are low due to competition, and retailers cannot afford to pay for the promotional service. As a result, consumer demand is low, and retailers' profits are low, too. With price floor, retailers' profits go up. This may just be enough to induce retailers to pay for the promotional service. Then consumer demand and retailers' profits are high, and the manufacturer can extract a bigger franchise fee from the retailers.

4) (15 points)

a) **(6 points)** Suppose that the monopolist maximizes profit by choosing the price for the good and the amount of "persuasive" advertising. Then, at the profit maximum, which of the following will have a larger impact on quantity demanded: a 1% cut in price or a 1% increase in advertising? Explain. (Hint: how does the price elasticity of demand η_P compare to the advertising elasticity of demand η_S?)

We know that any profit maximum satisfies

$$\frac{\tau S}{pQ} = \frac{\eta_S}{\eta_P}$$

Also, because the monopolist makes at least zero profit, $\tau S < pQ$. Then it must be that $\eta_S < \eta_P$.

A price cut must always have a stronger effect on quantity.

b) **(9 points)** Assume that the monopolist has total sales (i.e. revenue) of \$ 60 million, spends \$ 5 million on advertising and makes a profit (net of advertising expenditures) of \$ 15 million. Calculate the price elasticity of demand, η_P, and advertising elasticity of demand, η_S.

$$\pi = (p - c)Q - \tau S$$

$$\frac{(p-c)\,Q}{pQ} = \frac{1}{\eta_P}$$

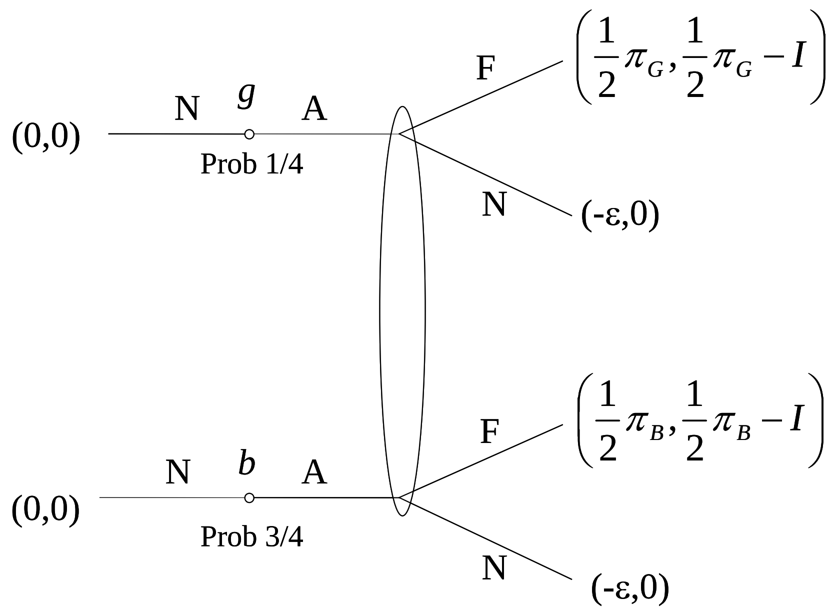
$$\frac{\tau S}{pQ} = \frac{\eta_S}{\eta_P}$$

$$(p - c)Q = \pi + \tau S = 15 + 5 = 20$$

$$\frac{1}{\eta_P} = \frac{(p-c)\,Q}{pQ} = \frac{20}{60} = \frac{1}{3},\ \eta_P = 3$$

$$\eta_S = \eta_P \frac{\tau S}{pQ} = 3\frac{5}{60} = 0.25.$$

5) (35 points) Consider a game between the entrepreneur (Player 1) and the investor (Player 2). The entrepreneur needs an investment of $I = 90$ to implement a project and promises to repay the investor with one half of whatever profits the project generates (this is called equity financing). The project can be either good or bad. It is good with probability $\frac{1}{4}$ and bad with probability $\frac{3}{4}$. A good project generates a payoff $\pi_G = 200$, and a bad project generates a payoff $\pi_B = 80$. The entrepreneur knows whether the project is good or bad, but the investor does not. The investor has to decide whether to finance the project or not (F or N), based on his beliefs about the quality of the project. The entrepreneur decides whether to apply for financing (A or N), depending on the quality of his project. There is a small application cost $\varepsilon > 0$ that the entrepreneur pays if he applies but is not funded.



a) **(3 points)** In terms of the sum of payoffs, which project(s) is it efficient to fund? Explain.

It is efficient to fund only the good project, funding the bad project reduces the sum of payoffs.

b) **(6 points)** Is there an equilibrium where only the good project is funded? Explain

This can only be the case if the bad type does not apply. But if the project is funded, the bad entrepreneur has an incentive to apply as well, because doing so increases his payoff. There cannot be an equilibrium where only the good type applies.

c) **(6 points)** Is there an equilibrium where both projects are funded? (Hint: if both entrepreneur types apply, what is the investor's expected payoff from funding the project?)

Both entrepreneurs must apply if both projects are to be funded. The investor's payoff is negative.

$$\frac{3}{4}\left(\frac{1}{2}\pi_B\right)+\frac{1}{4}\left(\frac{1}{2}\pi_G\right)-I=-35<0$$

If both types apply, the investor is better off not funding the project.

d) **(8 points)** What is the equilibrium of this game?

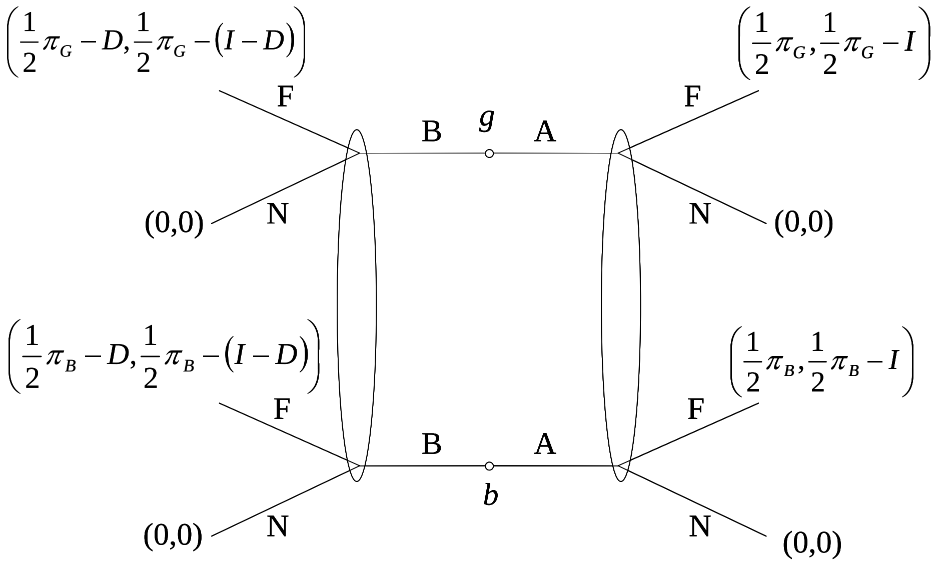
The investor does not fund the project, and neither type applies for funding. For the investor to prefer not to fund the project, he has to believe that the payoff from funding is negative. That is, the investor has to believe that the project is bad with sufficiently high probability. For example, if the investor believes that the project is bad with probability $\frac{3}{4}$, not funding the project will be a best response.

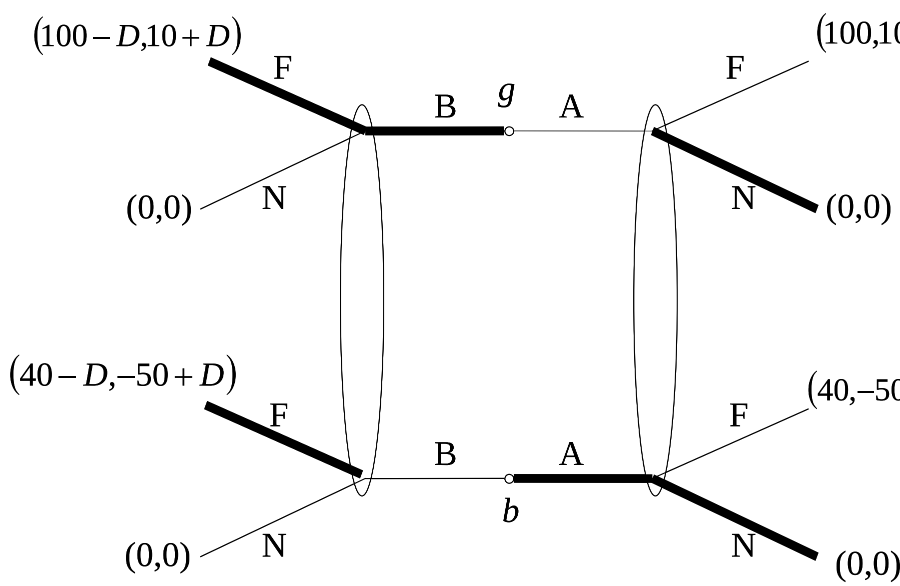
Any belief ρ that gives the investor a negative payoff from funding the project is consistent with equilibrium:

$$\rho\left(\frac{1}{2}\pi_B\right)+(1-\rho)\left(\frac{1}{2}\pi_G\right)-I=40\rho+100\,(1-\rho)-90=10-60\rho<0,\text{ i.e. }\rho>\frac{1}{6}$$

Finally, both entrepreneur types play the best response by not applying, because if either applies, he is denied funding and pays the application cost.

e) **(12 points)** Now suppose that the entrepreneur has two sources of funding. As before, he can ask the investor for $I=90$ and promise $\frac{1}{2}\pi$ in return (i.e. choose action A). Alternatively, the entrepreneur can take action B - borrow an amount $D<90$ from the bank (a debt which he has to repay regardless of the profitability of his project), and then ask the investor to put up the rest of the funds, $I-D$, in exchange for a promise to pay $\frac{1}{2}\pi$. The investor does not observe the quality of the project, but he does observe what action is taken by the entrepreneur. Formally, the actions and payoffs are depicted on the figure below. Assume that the application cost ε is zero. Find an equilibrium where the good type borrows (takes action B) and the bad type does not (takes action A). How much must the good type borrow in this equilibrium (that is, find all the values of D consistent with this separating equilibrium)? Explain why funding part of the project with debt is a good signal for the investor.





If only the good type takes action B, investor has to believe that a type who took action B is good for sure. Therefore, the best response to B is F.

If only the bad type takes action A, investor has to believe that a type who took action A is bad for sure. Therefore, the best response to B is N.

Is B the best response for the good type?. If he chooses A his project is not funded and he gets 0, but if he chooses B, his payoff is $100 - D > 0$. B is the best response for the good type.

Is A the best response for the bad type?. If he instead chooses B his project is funded and he gets $40 - D$, but if he chooses A, his payoff is 0. A is the best response for the bad type when $0 > 40 - D$, that is when

$$D > 40.$$

Funding part of the project by debt is a good signal for the investor, because the entrepreneur puts his money where his mouth is - a bad type cannot afford this much debt.

Reference Guide

Present value calculations

$$1 + \delta + \delta^2 + ... = \frac{1}{1-\delta}$$

$$\delta + \delta^2 + \delta^3 + ... = \frac{\delta}{1-\delta}$$

Cournot game between N identical firms with marginal cost c
Demand

$$p = A - BQ$$

Equilibrium profit

$$\pi(N) = \frac{(A-c)^2}{B(N+1)^2}$$

Monopolist in a market with linear demand

$$Q = \frac{A-c}{2B}, \, p = \frac{A+c}{2}$$

Monopolist that chooses price (p) and advertising (S)

$$\pi = \max_{p,S} [(p-c)\, Q(S,p) - \tau S]$$

Inverse elasticity rule

$$\frac{p-c}{p} = \frac{1}{\eta_P}$$

Dorfman-Steiner condition

$$\frac{\tau S}{pQ} = \frac{\eta_S}{\eta_P}$$