Economics 431
Winter 2001
Optimal two-part tariff and
second degree price discrimination

If the monopolist charges a uniform price, we know he cannot get all the surplus he generates. Suppose the monopolist can charge a fixed fee *and* a uniform price. This way he can generate the maximum possible consumer surplus by setting the price equal to marginal cost, and then take this surplus away by charging the fixed access fee. For example, let the demand curve be

$$p = v - q.$$

Suppose that the monopolist has a marginal cost c, charges the access fee F and a per unit price p. He cannot set an arbitrarily high F, because the consumers would refuse to buy the good at all. In particular, consumers only buy the good for price p per unit and pay the access fee F if their utility is positive

$$U(q, F) = CS(q) - F \geq 0$$

At price p, consumers will demand $q = v - p$, and consumer surplus will equal to the area between price and demand:

$$CS(q) = \frac{1}{2}(v - p)\, q = \frac{q^2}{2}$$

Since consumer utility must be at least zero, the monopolist is free to set any price p and access fee F that satisfy

$$\frac{1}{2}q^2 - F \geq 0$$

Of course, for any given quantity, the monopolist would choose to charge the highest possible access fee, so that

$$F = \frac{1}{2}q^2 = CS(q)$$

The profit (per customer) from selling q units at price $p = v - q$ and charging the fixed fee $F = \frac{1}{2}q^2$ is

$$\pi(q) = p - c)q + F = (v - q - c)q + \frac{q^2}{2} = q\left(v - c - \frac{q}{2}\right).$$

This expression reaches its maximum at

$$q = v - c$$

which corresponds to the price equal to marginal cost. The (optimal) best two-part tariff is charge MC per unit and then make pay all the rest of CS as access fee.

If the monopolist knows demand of a particular customer group and if groups differ by an easily verifiable characteristics, then it is easy to price-discriminate, by charging different access fees for different customer groups.

For example, suppose that there are high-demand customers and low demand customers and the monopolist can distinguish between the two types:

$$\begin{aligned} H &: \ p = 16 - q \\ L &: \ p = 12 - q \end{aligned}$$

$c = 4$. The monopolist will offer two different packages (F_H, c, Q_H), and (F_L, c, Q_L) where

$$\begin{aligned} F_H &= CS_H(16 - c) = \frac{12^2}{2} = 72 \\ F_L &= CS_L(12 - c) = \frac{8^2}{2} = 32 \end{aligned}$$

This means "pay F up front and buy up to Q units for price c per unit". He will make available the package $(F_H, c, Q_H) = (72, 4, 12)$ *only* to type H consumers, and the package $(F_L, c, Q_L) = (32, 4, 8)$ *only* to type L consumers. This is still the *first-degree* price discrimination.

Efficiency: this scheme is always efficient because the monopolist gets to capture all the surplus from trade.

Now suppose that the monopolist cannot verify which customer is which type. Any customer is now free to say "I am type H" or "I am type L". What will happen? Now the monopolist cannot force the package on the customer, the customer gets to choose a package that gives him a higher utility. Will high demand customers choose the package (F_H, c, Q_H)? If customer H takes (F_H, c, Q_H), this gives him zero utility:

$$CS_H(Q_H) - F_H = 0.$$

However, if customer H takes the package intended for the other type, he gets positive utility:

$$\begin{aligned} U_H(Q_L, F_L) &= CS_H(Q_L) - F_L = \frac{1}{2}(v_H - c + v_H - Q_L - c)Q_L - F_L = \\ &= \frac{1}{2}(16 - 4 + 16 - 8 - 4)8 - 32 = 32 \end{aligned}$$

In general, if a consumer whose demand is $p = v - q$ is allowed to buy Q units of the good at price c per unit, his consumer surplus is (see figure 1

$$CS(Q) = \frac{1}{2}(v - c + v - c - Q)Q = \left(v - c - \frac{Q}{2}\right)Q.$$

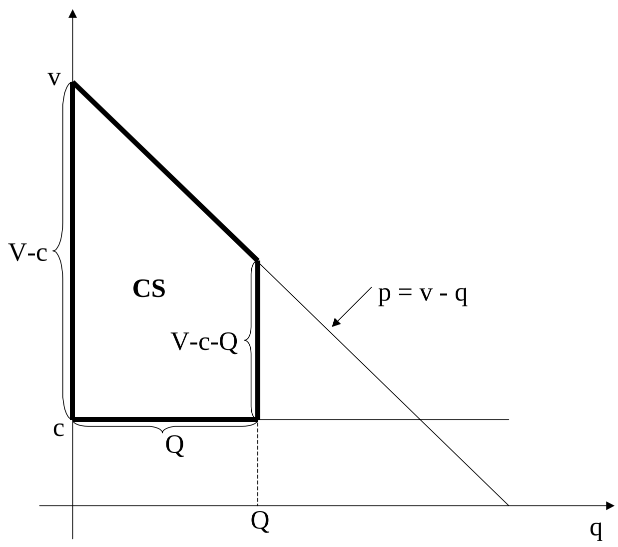

Figure 1:

Customer H will choose the package intended for customer L! The utility function of customer $i = \{H, L\}$ from having q units of the good at price c per unit and paying the access fee F is

$$U_i(q, F) = CS_i(q) - F = \left(v_i - c - \frac{Q}{2}\right) Q - F.$$

The indifference curves for the utility functions $U_H(q, F)$ and $U_L(q, F)$ are depicted on figure 2. Utility increases in the southeast direction, because consumers like more q and lower F. Also note that the optimal two part tariff must give each type exactly zero utility - all the consumer surplus is taken away from either type. However, type H has no incentive to take the package $(F_H, c, Q_H) = (72, 4, 12)$ intended for him, but would rather take type L's package.

Type H will choose the package intended for him, only if he gets a higher utility from it:

$$CS_H(Q_H) - F_H \geq CS_H(Q_L) - F_L.$$

On the figure, type H would prefer any package that is southeast of his indifference curve that goes through the type L's package (this is the indifference curve $U_H(q, F) = 32$). Since the monopolist wants to charge the *maximum* access fee that will still make type H choose his package, he will set F_H that will make type H just indifferent:

$$F_H = F_L + \underbrace{CS_H(Q_H) - CS_H(Q_L)}_{\text{Premium}}$$

The high type pays a higher access fee. The premium is exactly equal to additional consumer surplus that type H enjoys from his package compared to type L's package.

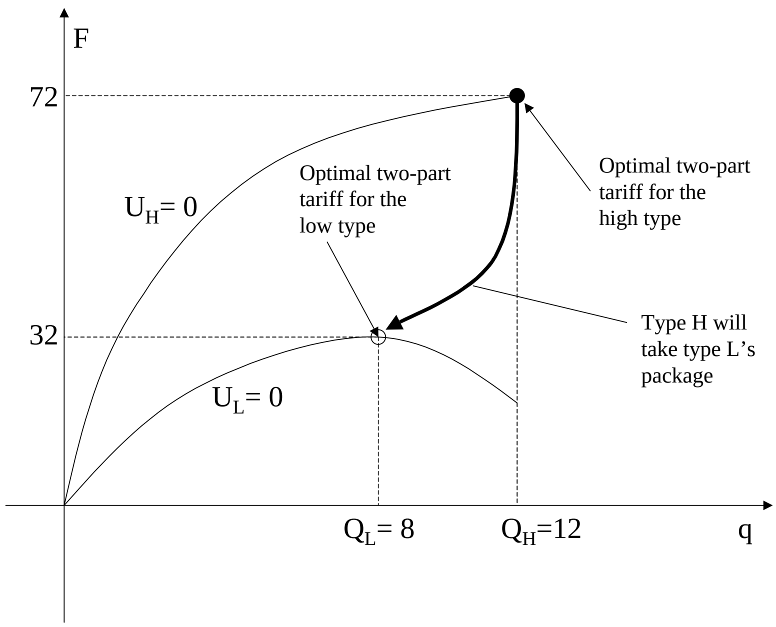

Figure 2:

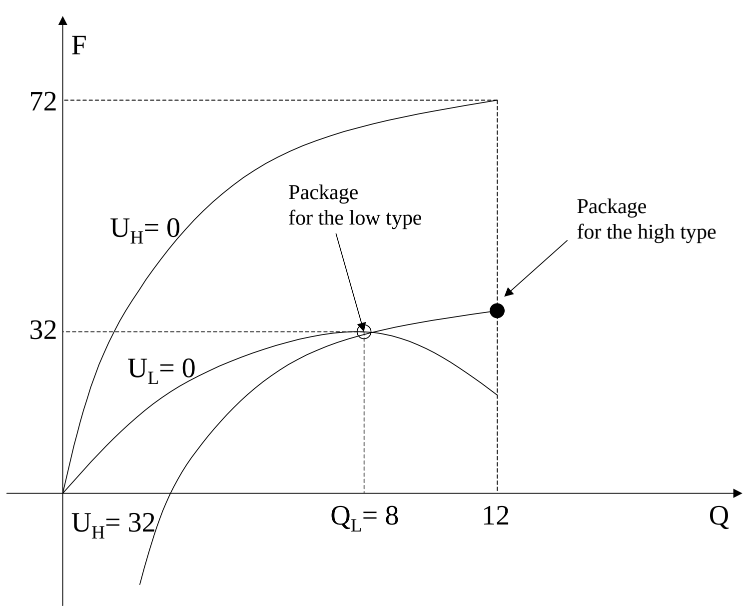

Figure 3:

Now, do we have to worry about the low type? Maybe he will want to take high type's package? No, because he always values extra quantity less. On figure 3, high type's package gives the low type negative utility, so he will prefer his own package.

$$\Delta CS_L < \Delta CS_H$$

Verify that.

$$\begin{aligned} \Delta CS_H &= CS_H(Q_H) - CS_H(Q_L) = \\ &= (16 - 4 - 6)\, 12 - (16 - 4 - 4)8 = 8 \end{aligned}$$

so that

$$F_H = F_L + 8.$$

But then, how is F_L determined? On the one hand, it cannot be more than $CS_L(Q_L) = 32$. But it makes no sense to make it less than 32 either. Suppose we set $F_L = 31$. Then F_H must be $F_H = F_L + 8 = 39$. Charging the low type 32 instead of 31will also allow the monopolist to charge the high type 40 instead of 39, so he will get more from *both* types by charging the low type his whole consumer surplus $CS_L(Q_L) = 32$.

To summarize, given package sizes $Q_L = 8$ and $Q_H = 12$, the best way to price these two packages is to set

$$F_L = 32$$

(the package intended for the low type takes away all the low type's surplus), and

$$F_H = F_L + 8 = 40$$

(the package intended for the high type comes at a premium exactly equal to high type's additional consumer surplus from his package.

Why maximizing the access fees means maximizing profit? Since price equals marginal cost, the monopolist makes no money on selling extra units of the good. His profit comes exclusively from access fees.

If n_H is the number of customers of type H and n_L is the number of customers of type L, monopolist's profit is simply.

$$\pi = n_H F_H + n_L F_L$$

Notice that the monopolist charges high type less than he could have: 40 instead of 72. Can't he instead offer just one package for 72 and not serve the low types at all? He can make more or less in profit, depending on n_H and n_L. This is because catering to low type requires offering a price cut to the high type. Sometimes, when the number of low customers is sufficiently low, it is not worth it. When

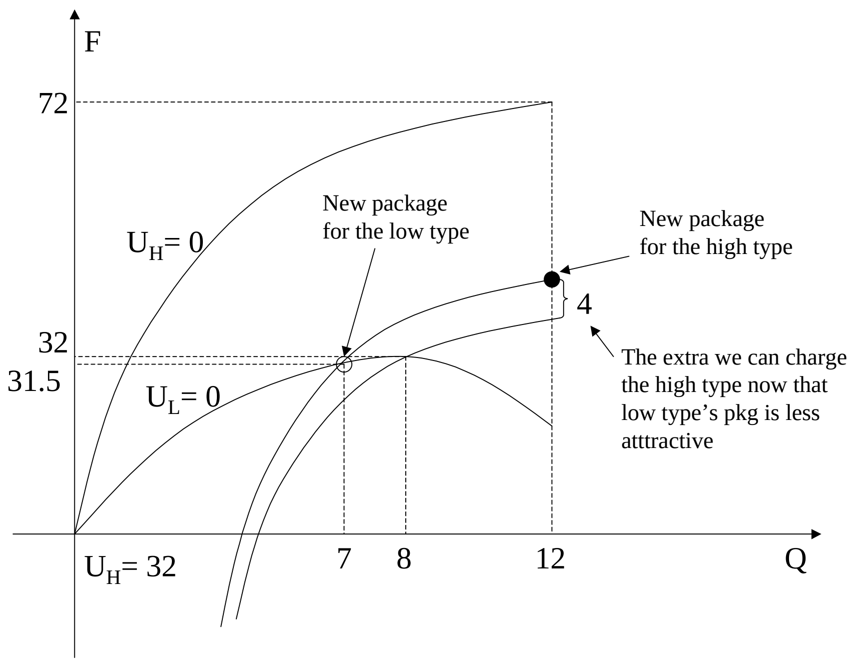

Figure 4:

$$n_H 72 > n_H 40 + n_L 32$$

or

$$n_H > n_L$$

the monopolist would rather serve only the high types and charge them 72.

In general, if

$$n_H CS_H(Q_H) > n_H(CS_L(Q_L) + CS_H(Q_H) - CS_H(Q_L)) + n_L CS_L(Q_L)$$

$$\underbrace{n_H(CS_H(Q_L) - CS_L(Q_L))}_{\text{Size of price cut to } H} > \underbrace{n_L CS_L(Q_L)}_{\text{Revenue from } L}$$

or the size of the price cut offered to high types exceeds the revenue from the low types, only high types are served.

The monopolist does not have to offer $Q_L = 8$ and $Q_H = 12$. He can do still better with other packages.

If he reduces Q_L to 7 in the low type's package, he of course loses money on now lower F_L, but this also makes the low package *less attractive* for the high type, so now he can charge the high type more instead!

Consider an alternative package for low - $Q_L = 7$ drinks at

$$F_L = CS_L(7) = (12 - 4 - 3.5)\,7 = 31.5$$

But now

$$\begin{aligned} F_H &= F_L + CS_H(12) - CS_H(7) = \\ &= 31.5 + (72 - 59.5) = 44 \end{aligned}$$

By offering this new package, the monopolist decreases the profit on every low customer by 32-31.5=0.50 but instead increases the profit from every high customer by 4 (see figure 4).

Welfare properties of the second degree price discrimination

At competitive price equal to marginal cost $p = c = 4$, high demand customers would buy 12 and low demand customers will buy 8. We know, however, that the monopolist can increase profit by offering low customers a $Q_L = 7$ - less than a competitive market would offer. Therefore, in general, the outcome of the second-degree price discrimination is *socially inefficient*.