Finding the Proper Model: Deduce or Reduce?

by

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Problem –1: Model Deduction via MODA

1. Introduction

At this point in your engineering career, you are probably well-versed in the skill of analyzing various physical phenomena. During your undergraduate education, you learned how to analyze (i.e., calculate, estimate, predict, etc.) stresses, heat flows, fluid flows, etc. In ME360, or a similar course, you learned how to analyze the time-response and frequency-response of mechanical, electrical, and hydraulic systems. While all of this knowledge is useful, and, depending on your particular task, perhaps indispensable, you should recognize that the skill and art of synthesizing (i.e., choosing and assembling) a collection of components to perform a given task is also useful - and can be quite lucrative.

As engineers we synthesize a collection of parts, let’s call it a system-configuration description, to do something. Unless this something is limited to holding down the floor, we may want to model our configuration, and we use this model to analyze our system.

We have at our disposal many types of models; the state-determined system model is only one type. We can model a system using partial differential equations, finite-elements, a set of algebraic equations, a set of difference equations, etc. In ME560, we’ve been using one particular modeling abstraction, state-determined systems. We’ve taken some continuous, spatially-distributed system and abstracted (i.e., modeled) it with a finite number of masses, springs, electrical resistors, valves, etc. With bond graphs, we go one step further with this abstraction. We can represent the energy storage, dissipation, and transfer between these masses, springs, resistors, etc. using bond graph elements. This two-step abstraction, illustrated in Figure 1 is quite an accomplishment, because we’ve simplified the real-world behavior that we want to model into a very small set of bond graph elements, which has a well-defined theory and methodology for synthesizing state equations.

![Figure 1: Modeling of Physical, State-Determined Systems](image)

If the bond graph is to represent some aspect of real-world phenomena, a valid question is How complex should the bond graph (model) be? The simple answer (to paraphrase Einstein) is As complex as necessary, but no more so. Since modeling, as you should realize, is so case-dependent, it’s impossible to give a firm set of rules regarding the required model complexity. However, an approach to specifying the required model complexity for one application can be given. The approach for this application will give you one way of looking at this problem, and you may be able to adapt this approach to other situations, as they arise.

Suppose we need to model a machine-tool drive-train, such as the one shown in Figure 2. What kinds of lumped-parameter models of this drive-train can be envision? Some choices include:
Rigid-Body Model
where all components in a drive-train as assumed inextensible, i.e., rigid and the viscous losses due to sliding (as opposed to deformation) are included. This type of model help you to calculate the power requirements for a system.

Flexible-Body Model
where all components in a drive-train are assumed flexible and the inertia set to zero. This type of model help determine the static stiffness of a system.

First-Torsional-Resonance Model
where all components in a drive-train are assumed rigid, except he component whose compliance causes the fundamental resonance in a drive-train

Frequency-Range-of-Interest Model
where only the component compliances that cause poles within some specified frequency range of interest (FROI) are included in the drive-train model

Order-N Model
which is similar to the FROI model, except model order, not frequency, is the model criterion.

Each of these models has its niche. The rigid-body model enables engineers to estimate the torque required to accelerate the drive-train or to overcome a specific load torque, the amount of power the motor requires, and the equivalent inertia of the drive-train components reflected back to the motor. Younkin [2] notes that an approximately equal ratio between the motor armature inertia and the reflected inertia of the other elements of a drive-train is desirable; the rigid-body model also facilitates this analysis. The flexible-body model can be used to determine the static compliance of a drive-train. As for the first-torsional-resonance model, given that the frequency of the first torsional resonance places fundamental limits on the achievable closed-loop performance of a machine-tool drive-train, a model that predicts this resonance is frequently necessary. The FROI model is useful for studying system response to an input with a known frequency content [1] and for designing conventional controllers based on some “plant model”. The order-N model helps engineers to determine if higher-frequency eigenvalues are of sufficiently large magnitudes such that a lower-order model is adequate to describe system behavior. This model can also be used to determine the required frequencies of controller notch filters.

Figure 2: Representative Drive Train
2. Rigid and Flexible-Body Models

Ok, enough reading for awhile. Let’s create some bond-graph models of the system configuration shown in Figure 3. The parameters for each of the components in Figure 3 follow.

![Figure 3: System Configuration to be Modeled](image)

1. DC-Motor
   - Armature Resistance, $R$ = 9 Ohms
   - Inductance, $L$ = 0.002 Henries
   - Motor Constant, $K_t$ = 0.06 N-m/Amp
   - Armature Inertia, $J_m$ = $1.0 \times 10^{-4}$ kg-m$^2$
   - Viscous Friction, $R_m$ = $1.0 \times 10^{-5}$ N-m/rad/sec

2. First Belt-Drive
   - Viscous Friction of First Sheave, $R_{v1}$ = 0.0001 N-m/rad/sec
   - Diameter of First Sheave, $d_1$ = 0.1 m
   - Thickness of First Sheave, $t_1$ = 0.01 m
   - Diameter of Second Sheave, $d_2$ = 0.2 m
   - Thickness of Second Sheave, $t_2$ = 0.01 m
   - Viscous Friction of Second Sheave, $R_{v2}$ = 0.0001 N-m/rad/sec
   - Belt Stiffness, $K_{b12}$ = 1000 N/m
   - Sheave Density, $\rho_{12}$ = $2.71 \times 10^3$ kg/m$^3$

3. Second Belt-Drive
   - Viscous Friction of First Sheave, $R_{v3}$ = 0.0001 N-m/rad/sec
   - Diameter of First Sheave, $d_3$ = 0.1 m
   - Thickness of First Sheave, $t_3$ = 0.01 m
   - Diameter of Second Sheave, $d_4$ = 0.3 m
   - Thickness of Second Sheave, $t_4$ = 0.01 m
   - Viscous Friction of Second Sheave, $R_{v4}$ = 0.0001 N-m/rad/sec
   - Belt Stiffness, $K_{b34}$ = 1000 N/m
   - Sheave Density, $\rho_{34}$ = $2.71 \times 10^3$ kg/m$^3$

4. Shaft
   - Diameter, $d_s$ = 0.0127 m
   - Length, $L_s$ = 0.5 m
   - Density, $\rho_s$ = 7755 kg/m$^3$
   - Shear Modulus, $G_s$ = $9.31 \times 10^{10}$ N/m$^2$
5. Flywheel
- Diameter, $d_f = 0.3$ m
- Thickness, $t_f = 0.01$ m
- Viscous friction, $R_{vf} = 0.0001$ N-m/rad/sec
- Density, $\rho_f = 2.71 \times 10^3$ kg/m$^3$

Task 1
*Synthesize the rigid-body model in bond graph form of the drive-train in Figure 3.*

Task 2
*Synthesize the flexible-body model of the drive-train in Figure 3. Include all possible compliant elements in the model.*

3. Models that Predict Torsional Resonances

We’ve mentioned that the first torsional resonance of a system places some fundamental limitations on the achievable performance; let’s examine this in more detail. Suppose a tachometer, a summing junction, and an amplifier are added to the drive-train in Figure 3. The block diagram of the control system is shown in Figure 4, where $G(s)$ is the transfer function of the plant and is given by:

$$G(s) = \frac{\omega_{out}}{V_{in}}$$

The Characteristic equation of the closed-loop system in Figure 4 is

$$1 + K_{prop} \cdot G(s) \cdot K_{tach}$$  \hspace{1cm} (1)

If we assume that the drive-train in Figure 3 is rigid, $G(s)$ will have the form

$$\frac{K}{s}$$  \hspace{1cm} (2)

and the characteristic equation of the closed-loop system will be

$$1 + K_{prop} \cdot \frac{K}{s} \cdot K_{tach}$$  \hspace{1cm} (3)

The pole predicted by Equation 3 is stable (in the left-hand plane) regardless of the magnitudes of any of the gain terms, and Equation 3 also indicates that we can make the response of the system as fast as we want. Both of these predictions go against our intuition, and they are not true in practice.
Task 3

Explain the discrepancy between our intuition about the controlled-system of Figure 3 and the prediction given by Equation 3.

A more detailed model of the drive-train in Figure 3 will allow us to make more accurate and valid predictions about the open and closed-loop system behavior. A reasonable way to add more tail to the plant model is to drop the assumption that all the components are rigid, and to add the first torsional resonance to the plant model. We do this by making a more complicated submodel of the component. The new plant transfer function will have the form

\[
\frac{K}{s \cdot \left( \frac{s^2}{\omega_n^2} + \frac{2 \cdot \zeta \cdot s}{\omega_n} + 1 \right)}
\]

(4)

Where \( \omega_n \) = the natural frequency of the first resonance
\( \zeta \) = the damping ratio (assume \( 0 < \zeta < 0.2 \))

Task 4

Write out the characteristic equation of the block diagram in Figure 4 using the plant transfer function in Equation 4.

Task 5

Determine the stability of this system as a function of the gain \( K_{prop} \). You can use the Routh-Hurwitz criterion to do this.

The task you just did should convince you that it’s important that a model contain sufficient detail for the task at hand, e.g., we could get an unhappy surprise if we relied on the plant transfer function given by Equation 2 instead of the one given by Equation 4 to design our control loop. The disadvantage of using a model with extra detail is that it takes more effort to synthesize such a model; whereas it’s relatively easy to synthesize a model that assumes all components are rigid. However, most engineers (and their employers) find it more worthwhile to spend an hour (or a week) to synthesize a better model and predict system performance up front, than to spend two years and $500K to develop a new machine and realize (too late) that it will not perform as anticipated.

The first-torsional-resonance plant model includes the compliance of the component that causes the first (lowest-frequency) torsional resonance in the system. The next two tasks require you to synthesize this model.

Task 6

Identify the component that causes the first torsional resonance of the drive-train in Figure 3, when the compliance of this component is included in the overall system model. You will need to test the effect of including the individual compliances of the first belt-drive, the second belt-drive, and the shaft in the model. This can be done either by hand or 20SIM. To use 20SIM just go into the Simulation window, and hit Tools->Model Linearization.

Task 7

Draw a bond graph of the model that includes the compliance of component that causes the first torsional resonance.
In Task 6 you found the component whose compliance causes the first torsional resonance of the drive-train in Figure 3. At this point you have several options. You can modify the component to make it stiffer and, hence, increase the frequency of the resonance. You can substitute one or more components for this component - a synthesis problem. Both of these are viable options when the first resonance frequency is too low; however, let’s assume that the frequency is sufficiently high for our desired system performance. Your job at this point is to design a controller, but, before doing this, you need more information about the dynamic response of the drive-train.

By identifying the component that causes the first torsional resonance of the drive-train, you have a rough idea of how the drive-train will respond to excitation within a frequency band between 0 to \( \omega_{n1} \). Recognize that in the real system there are in infinite number of these resonant (natural) frequencies, and, even with our state-determined-system abstraction there are several more unmodelled natural frequencies in the system. To check the validity of your model over a frequency-range-of-interest (FROI) \( 0 < \omega < \omega_{\text{required}} \), you need to check for the existence of system resonances over this entire frequency band. If such resonances exist, they must be included in the model. One way to synthesize a model that predicts all the resonances with a FROI is to use the following procedure:

1. Identify the component that causes the next pole (pair) when the compliance of this component is included in the model.¹

2. Calculate the spectral radius (the largest eigenvalue) of the model’s state matrix.

3. If the spectral radius that you just calculated is less than \( \omega_{\text{required}} \), use the more complicated component submodel, and go back to Step 1. If the spectral radius is greater than \( \omega_{\text{required}} \), do not include the compliance of this component in the model.

**Task 8 (Optional)**

*Synthesize a model of the drive-train in Figure 3 that is valid up to a frequency of 200 radians/second. Draw the bond graph and comment on the individual component submodels.*

**References**


¹ In the case of a DC motor, test the effect on the system model of including the inductance of the motor in the model.
PROBLEM 2 – Automated Proper Model Production via Deduction

Introduction:

In the previous part of this assignment you were asked to synthesize several models of a drive-train. In this assignment you’ll use automated-modeling software programs to obtain some of these models. Synthesizing the correct model for Task 6 of your previous assignment required you to test a number of models, a repetitive process that has been automated. The goal of Task 6 was to synthesize a model that illustrates the first torsional resonance of the drive-train in Figure 3 of your previous assignment. To find this model you needed to test the effect on the overall system model of including the compliances of individual components. Only the component that - when its compliance is included in the system model - results in the system model with the lowest natural frequency (eigenvalue) should have its compliance modeled in the first-torsional-resonance model. Finding this component is a repetitive systematic process, and, as such, should be automated. Two computer programs Model-Building Assistant (MBA) [1 and 4] and Computer Aided Model Building Automation System (CAMBAS) [5 and 6] have been written and are able to perform the task of identifying the component causing the first torsional resonance in a drive-train. In addition, each program is capable of generating other types of proper models. In this problem set you will use CAMBAS to generate the proper models. In addition you will be able to compare proper models deduced using MODA with proper models reduced using MORA (Modeling Order Reducing Algorithm based on activity).

CAMBAS uses a graphical interface to define a component level description of the system, i.e., the system is decomposed into components. You can build this component level description of the system using the component library. CAMBAS is capable of synthesizing the models described in your previous assignment. CAMBAS was developed under the UNIX environment and you can run it only on the SUN workstations. For the drive-train in Figure 3 and the set of parameters given in your previous assignment do the following.

Task 1

Using the CAMBAS component library (Configuration on the menu bar) import all the components into the system representation, and define the inputs (voltage for the DC Motor and torque for the Flywheel) of the system. Use the Connect tool to interconnect the components and inputs. Draw the CAMBAS component level description of the system.

Note: You can find the DC Motor, Belt-drive, and Flywheel under the Bounded component library and the Shaft under the Unbounded component library. Select the component (double click), click on Import and then click at a point in the drawing area (dark blue area) of the main window. Each dot on the component corresponds to an I/O port where you can connect it to another component. To define a connection between two components first select the Connect tool and then click on the two dots, which you want to connect. For the Belt-drive the dots on the top of the block correspond to the first sheave and the dots on the bottom to the second sheave.

Task 2

Select the Define parameters tool and then click to a component, to define the parameters of each component (check the given units in order to define the correct values). For the shaft use a finite segment representation. Save the model (File on the menu bar).

Task 3

Select the Model deduction option (Analysis on the menu bar). Synthesize the rigid body model (Rigid model) and then choose Increase rank to synthesize the first torsional resonance model. Go back to the main window and use the Expand tool to check the complexity of each component. Which element is causing the first torsional resonance? What
is the first natural frequency of the system? Draw the bond graph of each component and note their rank.

Task 4

From the model reduction window select the FROI model and define the FROI to be 200 rad/sec to generate the proper model. Record all the steps taken (iterations) until the Proper Model is generated. What are the system eigenvalues (natural frequencies). Again, go back to the main window and use the Expand tool to check the complexity of each component. Note the rank of each component and draw the system bond graph model of the system.

PROBLEM 3 – Proper Models via Model Reduction

Objective

Another approach to address the complexity and proper modeling of dynamic systems is model reduction [9]. This approach starts with the most complicated model of the system, and the “importance” of each ideal element in the model is defined using an energy-based metric (activity). Elements with low activity are eliminated from model in order to generate a reduced proper model. The activity metric is implemented in a reduction algorithm called MORA – Model Order Reduction Algorithm. The objective of this problem is to apply MORA to the drive-train system used in Problem 1 and to generate a series of reduced models. The calculations are performed in 20SIM. The equations of all energy elements in 20SIM are modified in order to calculate activity. For example, the equations of a modified I element are:

\[
\text{equations}
\begin{align*}
\text{state} &= \text{int}(p.e); \\
p.f &= \text{state} / i; \\
power &= p.e*p.f; \\
\text{activity} &= \text{int}(\text{abs(power)}, 0);
\end{align*}
\]

The first two equations are the standard equations of a generalized inductance. The third equation calculates the element power as the product of effort and flow. The fourth equation determines the activity by computing integral of the absolute value of power.

The full model is given in Figure 1. Note that some elements are lumped into a single element, e.g., \(J_m + J_{s1}, B_{s2} + B_{s3}, \) etc. Consider these as single elements when calculating their activity. The model is already in the 20SIM library under the ME560 folder. All the parameters are set to match the values from the previous assignment and the model is excited with a pulse voltage from 0 to 60 seconds.

![Figure 1: Drive-train System – Full model](image)

Task 1

Open the full system model from the ME560 folder of the 20SIM library. Start the 20SIM simulator and select the motor current, flywheel angular velocity and first belt force as outputs. Run the simulation for 60 seconds and print the time response of the outputs.
Task 2

In a new experiment, add the activity of each one of the 13 energy elements as output variables (change the label of the variable so you can keep track to which element it corresponds). Run the simulation for 60 seconds. Plot the time history of the activities and record their final value \((t = 60 \text{ seconds})\). Use the numerical value tool to get the numerical values. Comment on the time response of the activities.

Task 2

Sort the activities such that \(A_1\) and \(A_{13}\) are the elements with the highest and lowest activities, respectively. Create a four-column table with element name, activity \((A)\), activity index \((AI)\), and cumulative activity index \((CAI)\) for each of the energy elements.

\[
AI_i = \frac{A_i}{\sum_{j=1}^{13} A_j}
\]

\[
CAI_j = \frac{\sum_{j=1}^{13} A_j}{\sum_{i=1}^{13} A_i}
\]

Which is the most and least active element?

Task 3

Based on the element sorting identify the elements that account for at least 98% of the total activity. Generate a reduced model by eliminating the remaining energy elements. Compare the outputs (same as in Task 1) of the full Vs the reduced model. Comment on the frequency content of the responses.1

Note: Zoom in to the response in order to get a better picture of the frequency content.

Task 4

Draw a bond graph of the reduced model according to MORA that includes only the first torsional resonance. In other words, based on the activities levels determined for all the elements eliminate the elements starting with the lowest activity until the bond graph has one compliant element that is the smallest model where energy is exchanged between/among storage elements”. Which elements does MORA suggest to eliminate? How much of the total activity is included in this reduced model? How is this reduced model compared with the first torsional resonance model generated by MODA? Compare the outputs (same as in Task 1) of the full Vs the reduced model. Comment on the frequency content of the responses.

1 To calculate eigenvalues in 20SIM, Linearize the system (as you did in HW 7) using the Linear System Editor in 20SIM, 20SIM displays the system in different forms. It gives A,B,C,D matrices in the State Matrix form. It also gives the poles and zeros of the system. So if you press the "poles button", you will see the system poles which correspond to the eigenvalues.
REFERENCES


PROBLEM 4 - Proper Models via Deduction – the effect of finite segment versus modal expansion representations

Objective

The objective of this problem is to compare the performance of the search algorithms MODA and Extended-MODA using CAMBAS with respect to a problem containing finite segment and finite mode representations of distributed parameter components.

The System

The schematic in Figure 2 shows a system consisting of 4 components. Components #1 and #3 are rods of uniform cross section. Component #2 is a shaker. This component excites the system. Component #4 is a mechanical isolator. The specifications for these components are given in Table 1. Components #1 and #3 are unbounded components. Component #1 is to be represented by lumped parameter (finite segment) model while component #3 is to be represented by a modal expansion. Components #2 and #4 are bounded components. Component #2 includes its inertia and loses in the rank zero model. The rank 1 model includes the winding inductance. Component #3 has a rank zero model which includes the inertia or a rank 1 model that also includes the spring.

Figure 2: Schematic of a hybrid system
Figure 3: Component models
Table 1: Component specifications

<table>
<thead>
<tr>
<th>Component</th>
<th>Model type</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rod 1</td>
<td>Unbounded, finite segment</td>
<td>density ((\rho)) = 7755 kg/m(^3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>elastic modulus ((E)) = 2.1 (10^{10}) N/m(^2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>diameter ((d)) = 0.05 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>length ((l)) = 2 m</td>
</tr>
<tr>
<td>2 Shaker</td>
<td>Bounded, lumped parameter</td>
<td>gyrator modulus ((K_s)) = 1 N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>resistance ((R)) = 1 Ohm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>inductance ((L)) = 0.0005 H</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mass of base ((M_{base})) = 1 kg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mass of armature ((M_{arm})) = 1 kg</td>
</tr>
<tr>
<td>3 Rod 2</td>
<td>Unbounded, modal expansion</td>
<td>density ((\rho)) = 7755 kg/m(^3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>elastic modulus ((E)) = 2.1 (10^{10}) N/m(^2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>diameter ((d)) = 0.05 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>length ((l)) = 2 m</td>
</tr>
<tr>
<td>4 Isolator</td>
<td>Bounded, lumped parameter</td>
<td>mass 1 ((M_1)) = 0.5 kg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>spring stiffness ((K)) = 10000 N/m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mass 2 ((M_2)) = 0.5 kg</td>
</tr>
</tbody>
</table>

Task 1

*Using CAMBAS component library (Configuration on the menu bar) import all the components into the system representation, and define the inputs (velocity for the first Rod, voltage for the shaker and force for the Isolator) of the system. Use the Connect tool to interconnect the components and inputs. Draw the CAMBAS component level description of the system.*

Note: You can find the Shaker and Isolator under the Bounded component library and the Rod under the Unbounded component library.

Task 2

*Find the proper model of this system with a Frequency Range Of Interest of 1000 rad/sec using extended MODA. Use a 1% tolerance for the eigenvalue convergence accuracy.*

- Show the bond graph model of the system clearly identifying each component.
- What is the rank required of each component model?
- What is the system rank?
- Make a graph showing the rank of each component at each iteration.
- Which component requires the largest rank? Why?

Note: To find the proper model, first use MODA for a FROI of 1000 rad/s and then use Extended MODA with 1% tolerance.

Task 3

*Find the proper model of this system using MODA. Start with the rigid model and then increase the rank until the system rank is equal to the answer in Task 2.*

- Show a bond graph model of the system clearly identifying each component.
- What is the rank required of each component model?
- Make a graph showing the rank of each component at each iteration.
- Compare the component ranks to the component ranks in Task 2. How are they the same or different? Why?
Task 4

_Compare and contrast the results in Tasks 2 and 3._

- _How are they different?_
- _Why are they different?_

**Note:** Be as specific as possible.