PROBLEM 1

Do the following problems from the course textbook (D.C. Karnopp, D.L. Margolis and R.C. Rosenberg. SYSTEM DYNAMICS: Modeling and Simulation of Mechatronic, Third Edition)

Chapter 5: Problems 1.a, 2.c, 4, 5, 7, 9, 12, 14, 15*

- * This problem requires a lot of algebra to solve. Therefore, it is sufficient to simple set up the algebraic equations. and comment on how you would solve them. If you are shooting for an A+, you might want to consider using a symbolic manipulation software package (Maple or Mathematica) to solve the system of algebraic equations.

PROBLEM 2

The chemical processing system is shown in Figure 1. The purpose of the device is to maintain the correct height of fluid in the tank. Fluid height is affected by the disturbance flow entering the tank at the top and by drainage through the outlet pipe at the bottom. Replacement fluid is provided by the pump through a long thin inlet pipe. In the final system design the fluid height will be measured and a controller will determine the command signal to the pump motor. We wish to determine a model for the system shown in Figure 1 so that the controller can be properly designed.
For our preliminary analysis of this system, let us make the following assumptions:

i) Assume that the dynamics of the motor are fast compared to the rest of the system and therefore the inductance of the motor windings can be neglected and only the winding resistance, $R_m$, needs to be included. The motor’s mechanical properties, bearing losses and rotor inertia, are also small and can be neglected.

ii) The positive displacement pump pressure/flow curves are shown in Figure 2 for various pump speeds. The pump is driven by the motor such that the motor torque, $\tau_m$, and the pump pressure, $P_0$, are given by a constant. The constant or modulus is called $n$. $P_0$ is the pressure of the pump at zero flow (i.e. $Q = 0$)

The pump output pressure can be characterized by:

$$P = P_0 - KQ$$

where: $P_0$ is the pressure drop across the pump at $Q = 0$ for a given motor torque $\tau_m$, $K$ is a constant.

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**Figure 1:** Pump/Tank System

**Figure 2:** Static pressure flow characteristics of the pump
iii) Assume the inlet pipe has linear fluid inductance (inertia), $I_L$, but no fluid resistance (i.e. the flow in the pipe, $Q_i$, is related to the pressure drop in the pipe, $P_i$).

iv) Assume that the outlet pipe has only fluid resistance but that it is characterized by the following nonlinear function:

$$P_b - P_a = R_1 Q + R_3 Q^3$$

v) Assume the tank can be characterized as a linear fluid capacitor, $C_T$.

1) Make a model (bond graph) for each component and then combine them in order to generate a bond graph for this system.

2) Augment the bond graph with causal strokes.

3) Augment the graph to include the controller and determine what type of signal the controller should produce?

4) What is the order of this system without the controller? Determine a suitable minimum set of state variables required to represent this system.

5) Determine the state equations.

6) Assume that for small flows the constitutive law for the outlet fluid resistance is given by:

$$P_b - P_a = R_1 Q$$

6i. Determine the state equations in state matrix form.

6ii. Determine an expression for the system natural frequency and damping ratio (optional).

7) If the inlet pipe has resistance $R_i$, will this affect the effective pump characteristics as seen by the tank? Explain by deriving a new set of state equations.

Use 20Sim for parts 8) & 9)

8) Verify your explanation in part 7 by comparing the simulated response of the model with and without $R_i$. (See parameters values given below).
9) For high flows the cubic term of the outlet pipe resistance cannot be neglected. Compare the behavior of the system by simulating the linear versus nonlinear\(^1\) output pipe resistance. In both cases neglect the inlet pipe resistance, \(R_i\).

For both simulation studies using the given parameters run the simulation for 5 minutes and for each variable listed below plot the time response of both cases in a single graph (e.g., \(R_i\) included in the model versus \(R_i\) neglected). Plot the following variables and comment on the results:

- flow through the outlet pipe
- flow through inlet pipe
- water height in tank

**System Parameters**

1. **DC-Motor**
   
   Motor Constant, \(K_t\) = 7 N-m/Amp
   
   Motor Resistance, \(R_m\) = 2 Ohms

2. **Pump**
   
   Pump Modulus, \(n\) = 0.1 m\(^3\)/rad
   
   Energy losses, \(K\) = 15,000 N-s/m\(^5\)

3. **Inlet Pipe**
   
   Area, \(A_p\) = 0.1 m\(^2\)
   
   Length, \(L\) = 4 m
   
   Water Density, \(\rho\) = 1000 Kg/m\(^3\)
   
   Resistance, \(R_i\) = 5000 N-s/m\(^5\)

4. **Tank**
   
   Area, \(A_t\) = 50 m\(^2\)
   
   Water Density, \(\rho\) = 1000 Kg/m\(^3\)

5. **Outlet Pipe**
   
   Linear resistance, \(R_1\) = 10,000 N-s/m\(^5\)

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\(^1\) See instructions at the end of this assignment on how to implement the nonlinear resistor in 20SIM.
Nonlinear coefficient, $R_3 = 25,000$

**Initial Conditions**

- Tank water volume, $v_0 = 22.0 \text{ m}^3$
- Inlet pipe fluid momentum, $P_o = 0 \text{ N/s-m}^2$

**Inputs**

- Motor current, $i(t) = 200 \text{ Amp}$
- Disturbance Flow, $Q_d(t) = 0 \text{ m}^3/s$
Implementation of the nonlinear R element in 20SIM:

Here is the 20SIM instructions about the HW 5. It explains how to implement the nonlinear R element, which has the following constitutive law.

\[ P_B - P_A = R_1 \cdot Q + R_3 \cdot Q^3 \]  

The difficulty is that the R element has the following causality.

0 \longrightarrow | R

**Figure 1:** Causality of the nonlinear R element

As can be seen from the causality on Figure 1, the output of the R element is flow variable. 20SIM cannot solve for the flow from this implicit relation. So the trick is to separate the R element into 2 elements as in Figure 2.

0 \longrightarrow | 1

\[ R_{\text{linear}} \]

\[ R_{\text{nonlinear}} \]

**Figure 2:** Modification of the bond graph (2 separate R elements)

For Rlinear, there is no need to make any changes. For Rnonlinear, change the constitutive law. You can do that by simply selecting the R element and pressing the “Go Down” button. Now change the constitutive equation as follows:

\[ p.e = R_3 \cdot p.f^3 \]  

\[ (2) \]

Note that nonlinear R element has causality such that effort is the output. So 20SIM can easily calculate effort from equation (2).