

Phil 413: Problem set #1

For problems (1)–(4b), if the sentence is as it stands false or senseless, change it to a true sentence by supplying quotes and/or corner quotes, or explain why no such alteration is possible. Give a “best” solution, i.e. one that involves as few additional symbols as possible. (CARTWRIGHT 1987)

1. For every sentence ϕ , the last word of the last word of ϕ is polysyllabic is polysyllabic.
2. The first letter of the Greek alphabet is α is satisfied by an object β only if β is identical with α .
3. For every sentence ϕ , ϕ implies $\neg\phi$ implies ϕ implies ϕ and $\neg\phi$.
4. a. The last word of #4a is obscene.
b. The last word of #4a is obscene.
5. Say that a sentence is *incorrigible* just in case there is no way of supplying quotes and/or corners such that the result is neither false nor senseless. It appears that:
 - (1) #4a is incorrigible.
 - (2) #4b is not incorrigible.
 - (3) #4a is identical with #4b.

But at least one of (1), (2), and (3) must be false. Which is false, and why? (CARTWRIGHT 1987)

6. Explain why the following are true or false, using everyday English as much as possible in your explanation. (HEIM & KRATZER 1998)
 - a. $\{x : x \in \{y : y \in B\}\} = B$
 - b. $\{x : \{y : y \text{ likes } x\} = \emptyset\} = \{x : \{x : x \text{ likes } x\} = \emptyset\}$
7. Consider the tree structure on p.33 of HEIM & KRATZER 1998. For each of the following units of the tree, identify the syntactic category to which they belong.
 - a. **introduces**
 - b. the mother of **introduces**
 - c. the daughters of the NP nodes
 - d. the sisters of the NP nodes
 - e. the daughter of the sister of the VP node
 - f. the daughter(s) of the mother of the VP node
 - g. the terminal nodes
8. Consider the tree structures (a)–(c) on p.19 of McCawley 1993 and say whether they conform to the grammar given in 1.5.8b on p.18. If you give a negative answer, indicate exactly where the structure fails to conform to the grammar.

9. For each of the following, say whether the relation is reflexive, whether it is symmetric, and whether it is transitive. Provide a counterexample in order to demonstrate each negative answer.
 - a. the relation of being adjacent
 - b. the relation defined on pairs of integers that holds just in case both integers are even
 - c. the relation defined on natural numbers of having a common factor greater than 3

10. Specify the denotation $\llbracket \mathbf{and} \rrbracket$ using lambda notation.

References

- CARTWRIGHT, RICHARD. 1987. *Philosophical Essays*. MIT Press, Cambridge.
- HEIM, IRENE & ANGELIKA KRATZER. 1998. *Semantics in Generative Grammar*. Blackwell Publishers, Ltd., Malden, MA.
- MCCAWLEY, JAMES D. 1993. *Everything That Linguists Have Always Wanted to Know about Logic*. University of Chicago Press, Chicago.

Phil 413: Problem set #2

For problems (1)–(6), draw a syntactic tree diagram representing the sentence, record the semantic values of each terminal node of the tree, and derive the truth conditions for the sentence.

1. **Some young philosopher smokes.**

2. **Every day is Christmas.**

For (2) and (6): you are free to stipulate any reasonable semantic value for **is**, as long as your stipulation yields the correct truth conditions.

3. **Every man corrected himself.**

4. **The house that John abandoned is nice.**

For (4) and (5): you may treat **is** as semantically vacuous in these sentences.

5. **Coakley is such that most Democrats like her and most Republicans hate her.**

6. **Lois hopes that Clark Kent is Superman.**

7. Compute the truth conditions of **In the world of Sherlock Holmes, Holmes is quick and Watson is slow** using the extensional meaning for **and** introduced on p.5 of VON FINTEL & HEIM 2005, taking **in the world of Sherlock Holmes** as a primitive constituent of the sentence. Then compute the truth conditions using the intensional meaning introduced on p.11. Comment on your results. (VON FINTEL & HEIM 2005)

8. Develop a hypothesis about the semantic value of **or** as it is used in the following sentence: **John or Mary smokes.**

9. State an alternative hypothesis about the semantic value of **or**, and give at least one argument for the claim that your proposed theory in (8) is better than the alternative.

10. Develop a hypothesis about the semantic value of **doubts**. Use your hypothesis to derive the correct truth conditions for the following sentence: **Lois doubts that Clark Kent is Superman.** Comment on any property you observe that distinguishes the semantic value of **doubts** from the semantic values of attitude verbs such as **believes, hopes, wants, desires, perceives, remembers** and **knows**.

References

VON FINTEL, KAI & IRENE HEIM. 2005. "Intensional Semantics Lecture Notes." Ms., MIT. Available online at <http://semantics-online.org/advsem/IntensionalSemantics.pdf>.

Phil 413: Problem set #3

For problems (1)–(6): if the formula is S-valid for the given formal system S, then prove its S-validity. Otherwise, construct a countermodel to demonstrate its S-invalidity.

1. in the formal system **K**:

$$\vDash \Box p \supset \Box \Box p$$

1. in the formal system **T**:

$$\vDash \Box p \supset p$$

3. in the formal system **S5**:

$$\vDash \Box p \supset p$$

4. in the formal system **D**:

$$\vDash \Box p \supset \Box \Diamond p$$

5. in the formal system **D**:

$$\vDash (\Box p \wedge \Box(\neg p \vee q)) \supset \Diamond q$$

6. in the formal system **S4**:

$$\vDash \Diamond \Diamond (p \wedge q) \supset \Diamond q$$

7. Prove that the formula $\lceil \Box \Diamond \alpha \rceil$ is not a theorem of **K**, where α is any formula of propositional modal logic. You may assume that **K** is sound with respect to the class of all models, which entails that if the negation of a given formula is true in some world in some model, then that formula is not a theorem of **K**.

8. Show that the rule 'if $\vdash \Box \alpha \supset \Box \beta$, then $\vdash \alpha \supset \beta$ ' preserves validity in **K**.

9. Show that the rule 'if $\vdash \Box \alpha \supset \Box \beta$, then $\vdash \alpha \supset \beta$ ' does not preserve validity in **T**.

10. Which of the formal systems we have studied—**K**, **D**, **T**, **B**, **S4**, or **S5**—seems best suited to model deontic necessity, i.e. moral obligation? Which seems best suited to model epistemic necessity? In both cases, justify your answers. If none of the systems we have studied seems adequate, explain what axioms are missing.

References

HUGHES, GEORGE & MAX CRESSWELL. 1968. *A New Introduction to Modal Logic*. Routledge, New York.

SIDER, THEODORE. 2010. *Logic for Philosophy*. Oxford University Press, Oxford.

Phil 413: Problem set #4

1. State the two main challenges for the material conditional analysis of the indicative that we discussed in class, and give an example of an indicative conditional that simultaneously illustrates both of them.
2. Construct a model that demonstrates that Stalnaker's semantics for the subjunctive conditional does not validate Antecedent Strengthening.
3. BENNETT 2003 says that intuitively, the subjunctive conditional is not transitive. In class, we discussed how the variably strict conditional analysis predicts this result. Consider the following rule of inference: from 'if A would B' and 'if A and B, would C', infer 'if A, would C'. Can one simply extend the intuitive counterexamples to transitivity to demonstrate that this rule of inference is also intuitively invalid? Explain your answer.
4. Do variably strict conditional analyses validate the rule of inference introduced in (3)? If not, construct a model that demonstrates that the semantics in STALNAKER 1968 or the semantics in LEWIS 1973 does not validate this inference rule. If so, give a proof that one of these analyses does validate the rule.
5. Consider the following sentence, read as a *material* conditional with embedded subjunctive conditionals: 'if it is the case that if A then would B or C, then either if A would B or if A would C'. Intuitively, is this sentence valid? Explain your answer.
6. Do variably strict conditional analyses make the sentence introduced in (5) valid? If not, construct a countermodel. If so, give a proof that the sentence is valid, given either the semantics in STALNAKER 1968 or the semantics in LEWIS 1973.
7. On the variably strict semantics for the subjunctive conditional, what is the relationship between the subjunctive conditional and the necessitated material conditional, i.e. are they equivalent, independent, or does one strictly entail the other? Give examples and/or arguments to demonstrate your answer.
8. WILLIAMS 2008 observes that the indicative analog of (2) is felicitous:

(2a') If Sophie went to the parade, she saw Pedro.

(2b') But if Sophie went to the parade and got stuck behind a tall person, she did not see Pedro.

Meanwhile, the indicative analog of (3) is infelicitous:

(3a') If Sophie went to the parade and got stuck behind a tall person, she did not see Pedro.

(3b') #But if Sophie went to the parade, she saw Pedro.

MOSS 2012 says: "Williams accounts for these data by adopting a variant of the strict conditional analysis for indicative conditionals, according to which the domain of the necessity modal is the *context set*: the set of worlds compatible with what is treated as true for purposes of conversation. He says that 'if p , q ' is true just in case all p worlds in the context set are q worlds. Like von Stechow and Gillies, Williams then adds another component to the meaning of a conditional: Williams says that 'if p , q ' presupposes that the context set contains some p worlds" (6). Fill in the details of this argument: why do the two assumptions introduced by Williams suffice to explain the above judgments?

9. Given Conditional Excluded Middle for 'would' counterfactuals and Lewis's duality thesis about 'might' counterfactuals, prove that 'if p , might q ' entails 'if p , would q '.
10. Comment on the result you obtained in (9). If the result you obtained seems unpalatable, which premise should we reject? Give some argument to support your answer.

References

- BENNETT, JONATHAN. 2003. *A Philosophical Guide to Conditionals*. Oxford University Press, Oxford.
- LEWIS, DAVID K. 1973. *Counterfactuals*. Basil Blackwell Ltd., Malden, MA.
- MOSS, SARAH. 2012. "On the Pragmatics of Counterfactuals." *Notus*, vol. 46 (3): 561–86.
- STALNAKER, ROBERT. 1968. "A Theory of Conditionals." In *Ifs: Conditionals, Belief, Decision, Chance, and Time*, WILLIAM L. HARPER, ROBERT STALNAKER & GLENN PEARCE, editors, 41–55. D. Reidel Publishing Company, Dordrecht.
- WILLIAMS, J. ROBERT G. 2008. "Conversation and Conditionals." *Philosophical Studies*, vol. 138 (2): 211–223.

Phil 413: Problem set #5

1. Can your credence in a proposition that is compatible with your new information decrease when you update by conditionalization? In other words, can we have $f(q|p) < f(q)$ when p and q are not mutually exclusive? If so, give an example. If not, prove that this never happens.
2. Can your credence in a proposition that entails your new information decrease when you update by conditionalization? In other words, can we have $f(q|p) < f(q)$ when q entails p ? If so, give an example. If not, prove that this never happens.
3. Prove that conditionalizing on new information preserves ratios of credences between propositions that entail that information. In other words, prove that if q and r each entail p , $f(q|p)/f(r|p) = f(q)/f(r)$.
4. Prove that conditionalizing on some evidence E preserves the conditional probabilities that you assign to propositions conditional on that evidence, i.e. $f(p|E)$.
5. Derive $f(p \wedge q)$ as a function of $f(p)$ and $f(q)$, where f is a probability function with respect to which p and q are independent propositions.
6. "Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?" (WHITAKER 1990)

Say what credences you should have before and after the host opens one of the doors you do not open. Explain your answer.

7. Create an example similar to the example in (6) except with one million doors instead of three, and set up the example so that the correct answer to your revised example is more intuitive than the correct answer to the original problem.
8. "A jailer is in charge of three prisoners, called A, B, and C. They learn that two of the prisoners have been chosen at random to be executed the following morning; the jailer knows which two they are, but will not tell the prisoners. Prisoner A realizes that his chance of survival is $\frac{1}{3}$. But he says to the jailer, "Tell me the name of one of the other two who will be executed; this will not give me any information, because I know that at least one of them must die, and I know neither of them." The jailer agrees and tells A that B is going to be executed." (KELLY 1994)

Say what credences A should have before and after the jailer says that B is going to be executed. Explain your answer.

9. Give an example that demonstrates that if you compute the arithmetic mean of two probability measures and then conditionalize them on a proposition, you do not always get the same result as if you had first conditionalized each measure on that same proposition and then computed their arithmetic mean.

10. Create a pair of bets that constitute a synchronic Dutch book for an agent who has .3 credence in a proposition and .3 credence in its negation. Hint: recall that “Bayesians say that if $f(p)$ is your credence that p is true, then you should be willing to pay up to $f(p)$ dollars for a ticket that is worth one dollar if p is true and nothing if p is false.”

References

KELLY, D. G. 1994. *Introduction to Probability*. Macmillan, New York.

WHITAKER, CRAIG. 1990. “Ask Marilyn.” *Parade Magazine*, vol. September 9 1990: 16.

Phil 413: Problem set #6

1. Pascal argues in *Pensées*, §233 that it is better to act so as to cultivate the belief in yourself that God exists, since if God exists, you will thereby gain an “eternity of life and happiness” rather than misery, whereas if God does not exist, your decision makes no difference. Explain what principle is supporting Pascal’s reasoning, using a decision table to explain your answer.
2. If you disagree with Pascal’s conclusion, explain what principle supports your own reasoning about the wager. Or if you agree with Pascal, introduce and respond to the strongest objection that you can imagine someone bringing against his argument.
3. “Set up a decision table for the following decision situation. Jack, who is now twenty, must decide whether to marry his true love Jill immediately or not see her again until he is twenty-one. If he marries her now then he will lose the million dollars his uncle has left him in trust. If he waits to see her until he is twenty-one, he will receive the money and can marry Jill at that time—if she still loves him. (Part of your problem is selecting an appropriate set of acts, states, and outcomes.)” (RESNIK 1987)
4. “Formulate the following decision problem using a decision tree. Danny, who has been injured by Manny in an automobile accident, has applied to Manny’s insurance company for compensation. The company has responded with an offer of \$10,000. Danny is considering hiring a lawyer to demand \$50,000. If Danny hires a lawyer to demand \$50,000, Manny’s insurance company will respond by either offering \$10,000 again or offering \$25,000. If they offer \$25,000, Danny plans to take it. If they offer \$10,000, Danny will decide whether or not to sue. If he decides not to sue, he will get \$10,000. If he decides to sue, he will win or lose. If he wins, he can expect \$50,000. If he loses, he will get nothing.” (RESNIK 1987)
5. Reformulate the decision problem in (4) using a decision table rather than a decision tree.
6. Suppose I offer you the following game: we toss a fair coin. If it lands heads, I pay you \$2. If it lands tails, I toss the coin again. If it lands heads then, I pay you \$4. If it lands tails, I toss the coin again. I keep tossing the coin until it lands heads, doubling the amount that I would pay you for a heads result each time. How much should you be willing to pay me to play this game? Explain your answer.
7. The game of “chicken” consists of two players riding bikes straight at each other. Suppose that each player would most prefer to continue riding while the other swerves, would next prefer that both players swerve, would next prefer that he himself “chickens out” while the other player is brave, and would last prefer that neither swerves and both players die. Is the resulting game a variant of the prisoner’s dilemma? Explain your answer.
8. Suppose players in “chicken” have the preferences in (7) except that each player actually prefers that neither swerves and both die heroically, than that he himself “chickens out” and lives in humiliation as the other player is praised for being brave. Is the resulting game a variant of the prisoner’s dilemma? Explain your answer.

9. Formulate the “location game” on p.18 of OSBORNE & RUBINSTEIN 1994 as a strategic game.
10. Find the set of Nash equilibria when there are two players in the location game. Prove that if there are three players in the location game, then the game has no Nash equilibrium.

References

OSBORNE, MARTIN & ARIEL RUBINSTEIN. 1994. *A Course in Game Theory*. MIT Press, Cambridge.

RESNIK, MICHAEL. 1987. *Choices: An Introduction to Decision Theory*. University of Minnesota Press, Minneapolis.