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Global constraints

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A lot of conventional work in formal epistemology proceeds under the assumption that subjects have precise credences. Traditional statements of the requirement of coherence presuppose that you have a precise credence function, for instance, and say that this function must satisfy the probability axioms. The traditional rule for updating says that you must update your precise credence function by conditionalizing it on the information that you learn. Meanwhile, fans of imprecise credences challenge the assumption behind these rules. They argue that your partial beliefs are best represented not by a single function, but by a set of functions, or *representor*.²

The move to imprecise credences leaves many traditional requirements of rationality surprisingly intact. Fans of imprecise credences often simply reinterpret these rules as applying to the individual functions in your representor. For instance, they assume that in order for you to be rational, each member of your representor must satisfy the probability axioms. And it is often assumed that in order for you to update rationally, your later representor must contain just those functions that result from conditionalizing each member of your representor on the information you learn.³

When it comes to agents with imprecise credences, though, the requirements of rationality needn't take this form. Whether you are rational might just as easily depend on *global* features of your representor, features that can't be reduced to each

1. I am grateful to Eric Swanson for several insightful comments that prompted the writing of this paper, and for many subsequent conversations about the central ideas in it. Thanks also to Jim Joyce for helpful discussion of several formal details.

2. The term 'representor' is from VAN FRAASSEN 1990. For early discussions of imprecise credence models, see SMITH 1961, LEVI 1974, and WILLIAMS 1976.

3. This updating rule is part of the definition of an *imprecise probability model* in the sense of JOYCE 2010. For further discussion, see GROVE & HALPERN 1998 and PIRES 2002.

member of your representor having a certain property. Global features of your representor are like the properties attributed by collective readings of predicates such as ‘lift the piano’.⁴ What it takes for a group of people to lift a piano is not the same as what it takes for each individual member of the group to lift it. Similarly, what it takes for an imprecise agent to be rational might not be for each member of her representor to satisfy familiar constraints on precise credence functions. To take the point further, imagine a band director commanding a marching band to spread out to fill a football field. This command is global in an especially strong sense: no individual could possibly satisfy it. Similarly, for fans of imprecise credences, the requirements of rationality could include rules that no precise agent could possibly satisfy.

This paper is an extended investigation of global rules of rationality. Some rules surveyed in this paper are rules analogous to the command to lift a piano, and some are analogous to the command to spread out to fill a football field. In section 1, I state formal definitions for both of these kinds of global constraints. I also spell out some foundational claims about how we should interpret the formalism of imprecise credences. In the remainder of the paper, I discuss three applications of my ideas, using them to address serious challenges that have been raised for fans of imprecise credences. Sections 2 and 3 discuss cases in which it seems like imprecise agents are forced to make bad choices about whether to gather evidence. Section 4 discusses the “problem of belief inertia,” according to which certain imprecise agents are unable to engage in inductive learning.⁵ Finally, section 5 addresses the objection that many imprecise agents are doomed to violate the rational principle of Reflection.⁶

A note of clarification: in discussing global requirements of rationality, I am playing a defensive game on behalf of fans of imprecise credences. I am not aiming to prove that imprecise credences are sometimes rationally required, or even that they are rationally permissible. Rather, I am aiming to demonstrate that fans of imprecise credences have more argumentative resources at their disposal than previously thought, resources brought out by the observation that the rules of rationality could be global in character. Global constraints on imprecise credences demand our attention partly because there are many significant theoretical roles for them to play.

4. For classic discussions of the semantics of collective predication, see LINK 1983 and LANDMAN 1989.

5. For an introductory discussion of the problem of belief inertia, see §3.2 of BRADLEY & STEELE 2014.

6. For a prominent statement of the objection that imprecise agents violate Reflection, see WHITE 2009.

1 Two notions of globalness

Let a *representor* be a set of probability measures, and let a *constraint* be a set of representors.⁷ We define the notion of a *pointwise* constraint as follows:

C is *pointwise* if and only if: there is some set of probability measures S such that for every representor R , $R \in C$ if and only if $R \subseteq S$.

When a constraint is pointwise, we can figure out whether it contains a representor just by testing to see whether every probability measure contained in that representor has a certain property. For example, say that your friend is about to toss a fair coin. Although you may have imprecise credences in many propositions, say you have exactly .5 credence that the fair coin will land heads. Then your representor will be a member of a certain constraint, namely the set of representors whose members agree that it is .5 likely that the coin will land heads. This is a pointwise constraint, satisfied by your representor in virtue of the fact that every one of its members assigns .5 probability to the coin landing heads.

With this notion in hand, we can define our first notion of globalness:

C is *global* if and only if: C is not pointwise.

For example, consider the set of representors that contain *at least one* probability measure that assigns .5 to the coin landing heads. This constraint does not correspond to a test on the individual members of a representor, as evidenced by the fact that a representor can satisfy this constraint, while a proper subset of that same representor fails to satisfy it. In short, pointwise constraints are like distributive readings of predicates, which are satisfied in virtue of every member of a group having a certain property, while global constraints are more like collective readings.

In order to spell out another useful characterization of this first notion of globalness, we must make a small detour and say more about how to interpret the formalism of representors and constraints. What exactly does your representor represent? This question is best answered by analogy. According to one traditional model of your full beliefs, you believe a proposition just in case it contains every world that is doxastically possible for you.⁸ Analogously, your representor can be used to model your *probabilistic beliefs*—that is, your credences, conditional credences, comparative probability judgments, and so on. We can think of these probabilistic beliefs as attitudes

7. Let us assume for simplicity that the credence functions of rational precise agents are probability measures, and that the same goes for the members of the representors of rational imprecise agents.

8. Classic discussions of this model of belief include HINTIKKA 1962, STALNAKER 1984, and LEWIS 1986.

towards sets of probability spaces, or *probabilistic contents*.⁹ For instance, you have .6 credence that Jones smokes in virtue of standing in the belief relation to a certain set of probability spaces, namely those that assign .6 probability to the proposition that Jones smokes. Just as you believe a proposition if and only if it contains every one of your doxastic possibilities, you believe a probabilistic content if and only if it contains every member of your representor. For instance, you have .6 credence that Jones smokes if and only if every member of your representor assigns .6 probability to the proposition that Jones smokes.¹⁰

Just as a set of worlds can represent your full beliefs, and your representor can represent your probabilistic beliefs, these same models can also represent another doxastic attitude. Given any content such that you could believe it or believe its complement, there is also a third attitude that you can hold toward that content—namely, the attitude of suspending judgment.¹¹ As FRIEDMAN 2013 convincingly argues, suspending judgment is a genuine attitude, not the mere absence of belief or disbelief. Rather, as Friedman puts it, suspending judgment about a content is an attitude that “expresses or represents or just is [your] neutrality or indecision” about that content (180). According to traditional models of full belief, you suspend judgment about a proposition just in case it contains some but not all of your doxastic possibilities. According to imprecise credence models, you suspend judgment about a probabilistic content just in case it contains some but not all probability measures in your representor. According to LEVI 1980, for instance, “[c]redal ignorance entails suspension of judgment between alternative systems of evaluations of hypotheses with respect to credal probability” (185). As KAPLAN 2010 puts it, imprecise credence models represent “a doxastic option, indecision, whose cogency orthodox Bayesian Probabilism wrongly refuses ever to countenance” (49). In short, imprecision *just is* the suspension of probabilistic judgment.

Having spelled out this interpretation of imprecise credence models, we can identify a second useful characterization of the notion of a pointwise constraint defined above. A pointwise constraint contains the representors of all and only those agents who *believe a certain probabilistic content*. By the definition given above, a constraint is pointwise just in case it is the power set of some set S of probability measures.

9. For an extended defense of the claim that probabilistic beliefs are attitudes toward probabilistic contents, see §1.2–3 and §3.6 of Moss 2017.

10. For simplicity, I talk about sets of probability *measures* rather than sets of probability *spaces* in the remainder of this paper. Although probabilistic contents are defined to be the latter rather than the former in Moss 2017, this difference does not matter for my arguments. Also, I set aside potential differences between what is represented by a probability measure and by its singleton set, assuming that the belief states of precise agents may be represented by either object.

11. According to imprecise credence fans, at least. This hypothesis about the structure of mental attitudes may well be rejected by opponents of imprecise credences.

Being a member of S is the test that each member of your representor must pass in order for your representor to satisfy the constraint. And when each member of your representor is indeed contained in the set S , that just amounts to saying that the set of measures S is a probabilistic content that you believe.

In addition to identifying certain constraints as global, we can also identify a special subset of global constraints, namely those that do not contain the representors of any precise agents.¹² These constraints are global in an especially strong sense:

C is *strongly global* if and only if: for all $R \in C$, $|R| > 1$

All other global constraints are merely *weakly global*:

C is *weakly global* if and only if: C is global, and for some $R \in C$, $|R| = 1$

Strongly global constraints are like the property of spreading out to fill a football field, and other properties that only groups can have. By contrast, weakly global constraints are like the property of lifting a piano. Although groups can lift pianos, so can very strong individuals. But the property of lifting a piano is still global in an interesting sense—namely, because a group does not have this property in virtue of each member of the group having it.

A word of caution: although it can be helpful to compare global constraints and collective readings of predicates, it is important not to overstate the analogy between them. For instance, one might at first be tempted to assume that any global constraint must contain at least one non-singleton set. But this assumption is false. For instance, consider the set of representors containing exactly one probability measure—that is, the constraint corresponding to the rational requirement to have precise credences. This set is a global constraint. Having exactly one measure in your representor does not involve each of your representor members having a certain property. The notion of a global constraint essentially depends on the richness of imprecise credence models, but not because every global constraint must contain the representor of some imprecise agent.

2 A puzzle about evidence gathering

In sections 4 and 5 of this paper, I use the foregoing notions of globalness to solve familiar problems for fans of imprecise credences. But first, I want to address a prob-

¹² Any non-empty constraint of this sort must be global, since every non-empty pointwise constraint contains the representor of each precise agent who believes the corresponding probabilistic content.

lem that has not yet been discussed in the literature. The problem is that without global constraints on rationality, imprecise agents will be forced to make questionable decisions about whether to gather evidence. Here is an example:

Phone a Friend: You are about to be offered a chance to guess whether p is true. If your guess is correct, you will win 100 dollars. If your guess is incorrect, you will lose 100 dollars. Also, before you face this offer, you have the option of paying 20 dollars right now to phone a friend and find out whether p is true.

Let *Later p* be the proposition that if you do not pay now to find out whether p is true, you will take up the later offer and guess that p is true. Let *Later not- p* be the proposition that if you do not pay now, you will take up the offer and guess that p is not true.¹³ Suppose that your representor contains just two probability measures, with the following features:

$$m_1(p) = .99, m_1(\textit{Later } p) = 1, m_1(\textit{Later not-}p) = 0$$

$$m_2(p) = .01, m_2(\textit{Later } p) = 0, m_2(\textit{Later not-}p) = 1$$

The members of your representor strongly disagree about the likelihood of p , and also about the likelihood that you will later guess that p is true.¹⁴ Their opinions about how you *will* act are not independent of their first-order credences that determine how you *should* act. This sort of dependence in your imprecise credences is described and defended by WILLIAMS 2014 in response to another diachronic decision puzzle. As Williams puts it, the idea is that “each credence assumes that the agent will do what is rational” by the lights of that credence function (26)—which in this case, means guessing that p is true just in case it is likely that p is true.

Unfortunately, *Phone a Friend* presents a problem for imprecise credence fans. According to each member of your representor, you should not pay 20 dollars to find out whether p is true. The expected utility of foregoing the phone call and simply guessing about p is 99 dollars, whereas the expected utility of making an informed guess is merely 80 dollars. As a result, practically any decision theory for imprecise agents will say that it is *impermissible* for you to phone your friend to find out whether p is true. As JOYCE 2010 explains, there is a “consensus among proponents of the imprecise model” that the rules for rational decision making “should never recommend one act over another when every member of your committee says that the utility of the latter exceeds that of the former” (311). Applied to the case of *Phone a Friend*, this conclusion entails that it is impermissible for you to phone your friend. This is a

13. These conditionals should be read as material conditionals.

14. Throughout, I assume that you are certain that if you phone a friend to find out whether p is true, you will then guess accordingly.

counterintuitive result. The members of your representor have wildly divergent opinions about the value of guessing that p is true. In light of this fact, rationality should at least *permit* you to gather evidence that would settle this dispute and tell you what to guess in order to win 100 dollars.

How should the fan of imprecise credences respond? There are a couple of options, both of which involve endorsing global requirements on imprecise agents, rules that forbid rational agents from having a representor containing only m_1 and m_2 . The first requirement is a rule proposed by LEVI 1980—namely, that your representor must be *convex*, in the following sense:

R is *convex* if and only if: for all $f, g \in R$ and $0 \leq \lambda \leq 1$, $\lambda f + (1 - \lambda)g \in R$

If your representor is convex, then it contains all linear averages of its members. As a result, the credences assigned by members of your representor will also form a convex set. Let us introduce some helpful notation. Where R is a representor, and p is a proposition, let us define $R[p] =_{df} \{m(p) : m \in R\}$. Speaking loosely, $R[p]$ is “the imprecise credence assigned by R to p .” If R is convex, then for any proposition p , $R[p]$ will be an interval of real numbers. The set of convex representors is a global constraint. A representor can be convex while a proper subset of it is not, and so the constraint of convex representors is not the power set of any set of probability measures. Among the many global requirements that might govern imprecise agents, convexity is certainly one of the first that comes to mind. Roughly speaking, the idea of this requirement is that your representor members must “fill in any gaps in the football field.” In other words, if your representor includes some probability measures, it must also include all those between them.

The requirement of convexity forestalls our counterintuitive verdict about *Phone a Friend*—and indeed, it provides a general solution to other problems of this same sort. As long as your representor is convex, any pair of confident representor members like m_1 and m_2 will be accompanied by a moderate probability measure that assigns .5 to the proposition p that you might be acting on later. According to this third probability measure, the later option to guess that p is worthless unless you first find out whether p is true, and so you should be willing to pay up to 50 dollars to gain that information. As long as your representor contains measures of this sort, it is no longer possible to derive the conclusion that you are rationally required to forego gathering evidence in cases like *Phone a Friend*.

Since the convexity requirement is endorsed by some imprecise credence fans, it is important to appreciate that it solves our present problem. At the same time, though,

the requirement itself is highly controversial. The main argument against the requirement is due to JEFFREY 1987. According to Jeffrey, you can “judge propositions A , B to be irrelevant to each other without having any particular judgmental probabilities for them or their conjunction” (586). For instance, you can be certain that whether there is water on Mars is independent of whether a certain coin landed heads, even if you have maximally imprecise credences about both propositions. But as Jeffrey points out, the convexity requirement precludes rational agents from having this particular combination of beliefs:

The bare judgment of irrelevancy would be represented by the set $I = \{P | P(AB) = P(A)P(B)\}$, but by no proper subset, and by no one member. Levi disallows such judgments: he requires the sets that represent indeterminate probability judgments to be convex. (586)

The set of probability measures representing the bare judgment of irrelevancy is not convex, and so it cannot constitute the representor of any rational agent.

As I see it, Jeffrey’s observation alone does not yet constitute a strong argument against the convexity requirement. Fans of convexity could reasonably complain that it is extremely unrealistic to assume that a rational agent could believe a bare judgment of irrelevancy about some propositions while lacking any other belief whatsoever. But on behalf of opponents of convexity, I want to propose a way to strengthen Jeffrey’s argument, challenging convexity without making this unrealistic assumption. Here is a much weaker assumption: a rational agent can believe that A and B are irrelevant to each other, while still having imprecise credences in A and B . For fans of imprecise credences, this certainly seems like an unobjectionable combination of belief states—yet the convexity requirement rules out any such state as rationally impermissible. The set I defined by Jeffrey has the following significant feature: for any $R \subseteq I$, R is convex only if its members either all assign the same precise credence to A , or all assign the same precise credence to B .¹⁵ Hence the convexity requirement entails that any rational agent with a representor in I must have a precise credence in at least one of A or B . To sum up where we stand: requiring representors to be convex would indeed solve our puzzle about evidence gathering. Levi and other proponents of convexity should welcome this motivation for their global constraint. But in light of our strengthened result about convexity and independence, one might also reasonably doubt whether imprecise agents are rationally required to have convex representors.

¹⁵. See claim 1 of the appendix for a proof of this result.

3 A global independence requirement

In light of the controversial nature of the convexity requirement, I am going to state an additional solution to our puzzle about evidence gathering. Rather than faulting your credences in p for not being convex, we could instead fault them for being intimately connected with your credences about how you will later act. Imagine filling out the details of the *Phone a Friend* case by saying that there is no fact of the matter that settles exactly which member of your representor will determine how you respond in any given decision situation. If that is right, then there is no evidence that could reasonably settle ahead of time whether you would guess that p in *Phone a Friend*. Even from the perspective of an individual member of your representor, there is no more reason to suppose that you will later guess correctly than that you will guess incorrectly. Accordingly, rationality should require you to have imprecise credences about how you will guess—and moreover, it should require these credences to be *independent* of your first-order beliefs about p .

This independence requirement does not amount to belief in the bare judgment of irrelevancy discussed by JEFFREY 1987. Jeffrey’s judgment of irrelevancy is a probabilistic content, a set containing all probability measures that share a certain property. Accordingly, you satisfy this independence requirement for p and *Later p* just in case your representor is contained in a certain pointwise constraint, namely:

$$I_p =_{df} \{R : R \subseteq \{m \mid m(p \wedge \text{Later } p) = m(p)m(\text{Later } p)\}\}$$

As you are deciding whether to phone your friend, each member of your representor assigns an extreme credence to *Later p*. Hence each member of your representor considers p and *Later p* to be trivially independent, and so your representor is indeed contained in the pointwise constraint I_p defined above.

By contrast, a global interpretation of the relevant independence requirement yields a successful solution to our puzzle. Consider the following global requirement: for any probability measures m_1 and m_2 in your representor, there must be a third representor member m_3 such that $m_3(p) = m_1(p)$, and such that m_3 is certain that you will later act as prescribed by your currently having m_2 as your credence function.¹⁶ When you satisfy this requirement, it is just as if your *representor members themselves have imprecise credences* about how you will act. Any precise credence function in your representor is accompanied by many counterpart functions, each of which is certain that you will act on the recommendations of a different representor

16. In fact, several global notions of independence could be used to solve our puzzle. For relevant discussion, see Couso et al. 2000. See also Moss 2015 for a precursor to the above independence requirement and for discussion of another puzzle that it could be used to solve.

member. The set of these counterpart functions is the representor of an agent who suspends judgment about how you will act. Rather than demanding that you believe that p and *Later p* are independent, our independence requirement demands that you *suspend judgment* about each proposition in light of the other. This fact explains why our independence requirement is not a pointwise requirement, but a global requirement of rationality—because it does not require probabilistic belief, but rather the suspension of probabilistic judgment.

By contrast with the pointwise independence constraint discussed above, our global independence constraint is not satisfied by your representor in *Phone a Friend*. In order to produce a representor that satisfies the constraint, we must add the following probability measures to your representor:

$$m_3(p) = .99, m_3(\textit{Later } p) = 0, m_3(\textit{Later not-}p) = 1$$

$$m_4(p) = .01, m_4(\textit{Later } p) = 1, m_4(\textit{Later not-}p) = 0$$

As soon as these probability measures are added, the act of declining to gather evidence about p will no longer have maximal expected value according to every member of your representor. According to m_3 and m_4 , the expected utility of guessing without gathering evidence is -99 dollars, whereas the expected utility of gathering evidence and then guessing is 80 dollars. Hence it no longer follows that you are rationally required to forego gathering evidence in this case.

The independence requirement introduced in this section serves as a useful template for global independence requirements on imprecise credences. *Phone a Friend* motivates objective rational requirements of this sort—namely, rules that say that your first-order credences must be independent of your credences about your future actions. Even if there is a fact of the matter about which members of your representor would determine how you would act in various situations, rationality might sometimes require your predictions about such facts to be independent of the first-order beliefs on which you are acting. Of course, your predictions may sometimes depend on your first-order beliefs—such as, for instance, when you are offered a bet at long odds on the proposition that you will at some point accept at least one bet at long odds. Hence it is a substantive project to identify the global independence requirements that govern our imprecise credences. For present purposes, what matters is that we have a way of spelling out what epistemic humility requires when imprecise agents lack a very specific sort of evidence, namely evidence about which member of their representor will govern their own later actions. This humility amounts to the satisfaction of a certain global requirement, one that prevents rational agents from being required to forego sensible acts of evidence gathering.

4 The problem of belief inertia

Fans of imprecise credences sometimes suggest that rational agents can have *radically imprecise* credences in a proposition, i.e. credences that span the range from 0 to 1. At first glance, this suggestion raises a problem: in certain circumstances, rational agents with radically imprecise credences could not ever come to have more precise credences, since rationally updating on any evidence proposition would leave their credences unchanged. In other words, radically imprecise credences can be *inert*.¹⁷ As WALLEY 1991 observes, “[i]f the vacuous previsions are used to model prior beliefs about a statistical parameter for instance, they give rise to vacuous posterior previsions” (93). RINARD 2013 gives an example:

[C]onsider an urn about which you know only the following: either all the marbles in the urn are green (H₁), or exactly one tenth of the marbles are green (H₂). . . if your initial credence in H₁ is (0, 1), it will remain there. It will be impossible for you to become confident in H₁, no matter how many marbles are sampled and found to be green. (160–1)

As long as your credence is entirely divided between H₁ and H₂, your imprecise credence in H₁ will be inert with respect to the proposition that all of the sampled marbles have been green, no matter how many marbles have been sampled. In this sense, your having radically imprecise credences precludes inductive learning. Following BRADLEY 2015, Vallinder identifies this conclusion as the “problem of belief inertia.”¹⁸

Faced with this problem, one natural response is to say that certain objective constraints prevent rational agents from ever having the inert credences mentioned above. Against this response, VALLINDER 2017 argues that belief inertia presents a serious problem even for fans of imprecise credences who endorse such objective constraints. Here is his statement of the problem that remains: “even if they can’t give us an exact characterization of which imprecise priors are permissible, they should at least be able to show that none of the permissible priors give rise to widespread belief inertia. Before that has been done, it seems premature to think that the problem has been solved” (24). According to Vallinder, fans of imprecise credences are under no obligation to identify a comprehensive list of objective constraints on rational credences. But in order to solve the problem of belief inertia, they must at least identify a set of constraints strong enough to guarantee that inductive learning is possible.

17. To give a precise definition: a representor R is *inert* with respect to a constraint C and a set of propositions E if and only if for all $p \in E$, $\{m|_p : m \in R\} \in C$. The set E is often implicitly determined by context to be the set of evidence propositions that an agent might learn.

18. For further discussion of this problem, see JOYCE 2010 and §4.6.3 of BRADLEY 2012.

JOYCE 2010 proposes such a set of constraints as a response to this very problem. Joyce argues that any rational agent will indeed be able to engage in inductive learning, in virtue of the fact that her representor will be contained in two other constraints:

Perhaps the right way to secure inductive learning is to sharpen your credal state by (a) throwing out all the pigheaded committee members. . . and (b) silencing “extremist” elements by insisting that each committee member assign a credence to [H₁] that falls within some sharpened interval. (291)

The exact details of this proposal do not matter for our purposes. The point is that Joyce is attempting to derive a global requirement of rationality from pointwise requirements. The idea is that there are certain rational constraints on precise credence functions, namely constraints against pigheaded and extremist credences. Fans of imprecise credences should endorse these requirements as constraints on the individual members of your representor. According to Joyce, any representor satisfying these pointwise constraints will also satisfy the constraint of not being inert with respect to assigning radically imprecise credences to a proposition.

Unfortunately, VALLINDER 2017 demonstrates that Joyce’s proposal does not solve the problem of belief inertia. Vallinder produces a representor that satisfies the pointwise constraints that Joyce proposes, although it is still incapable of inductive learning. In short, you can be stubborn in your credences without being maximally imprecise. Even in the absence of pigheaded and extremist representor members, your credences can be inert with respect to more moderate imprecise credence assignments. Vallinder concludes that the essential problem remains, since “even this weaker form of belief inertia means that no matter how much evidence the agent receives, she cannot converge on the correct answer with any greater precision than is already given in her prior credal state” (13).

How, then, should imprecise credence fans solve the problem of belief inertia? As I see it, the correct response does not involve deriving anti-inertia requirements from pointwise requirements of rationality. Anti-inertia requirements on imprecise credences are indeed intimately connected with rational requirements on precise agents—but not because the former consist in the pointwise application of the latter. Rather, many anti-inertia requirements on imprecise agents are genuinely global requirements. They are intimately connected with traditional rules governing precise agents because they are themselves the direct analogs of traditional rules against inert belief states.

To spell this out: among the rational requirements governing both propositional and probabilistic beliefs, one familiar sort of rule is that you should not be stubborn in your beliefs. In the full belief case, this rule entails that you ought not have full beliefs that are rationally unrevisable. The *locus classicus* for this rule is QUINE 1951:

“no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?” (40). This claim is generally interpreted as not merely descriptive but normative: you ought to be such that there is no proposition such that you believe it and could never stop believing it. Even the law of excluded middle should be open to rational revision.

When it comes to probabilistic beliefs such as credences, the most familiar rule against stubbornness is the rule of *Regularity*, which prohibits rational agents from having credence 1 in any proposition. As LEWIS 1980 puts it, Regularity is “required as a condition of reasonableness: one who started out with an irregular credence function (and who then learned from experience by conditionalizing) would stubbornly refuse to believe some propositions no matter what the evidence in their favor” (268). As with all rules of rationality governing precise agents, this version of Regularity can be easily extended to a pointwise rule governing imprecise agents, namely the requirement that each member of your representor satisfy the precise Regularity condition. This pointwise rule simply amounts to requiring belief in the probabilistic content that the likelihood of each proposition is in $(0, 1)$.

When it comes to injunctions against stubbornness, though, these familiar rules are just the beginning of the story. For instance, your probabilistic beliefs include much more than your assignments of extreme credences to propositions. A more general rule against stubbornness should forbid the existence of any probabilistic content such that you believe it and could never stop believing it. In addition, as explained in section 1, your attitudes toward contents include not only the attitude of belief, but also the attitude of suspending judgment. The central insight of “Two Dogmas” can be extended to the latter attitude. Just as any belief should be in principle revisable, there should be no content about which you would suspend judgment come what may. Finally, our rule against being stubborn in maintaining various attitudes can be extended to more general rules against being inert with respect to changes of attitude. For instance, as a rational agent, you must not only be able to stop believing contents; you must also be able to start believing contents. Just as any thread might eventually be disentangled from your web of belief, many external threads might eventually be woven into it.

These rules against being inert with respect to changes of attitude hold not just for propositional attitudes, but also for attitudes with probabilistic contents. For any input constraint, there is an output constraint containing the representors that are not inert with respect to that input. A wide range of these output constraints could

be objective constraints of rationality. And many such output constraints are global. In fact, even a pointwise input constraint can generate a global output constraint of representors that are not inert with respect to it. For instance, consider the constraint containing representors of agents that believe that it is more than .5 likely that a certain coin landed heads. This is a pointwise constraint. But the set of representors that are not inert with respect to it is a global constraint. Although believing that a coin probably landed heads just amounts to each member of your representor assigning at least .5 probability to the proposition that it landed heads, *being such that you could stop believing this content* does not correspond to any pointwise test.¹⁹

At this point, our discussion illuminates a compelling response to the problem of belief inertia. In his response to Joyce, Vallinder implicitly assumes that rules against inert credences must be grounded in other rational requirements, or at least that fans of imprecise credences bear the burden of deriving the former from the latter. But when it comes to relations of grounding and entailment, global rules against inert imprecise credences have just the same status as other rules against doxastic stubbornness. The rule of Regularity is often introduced as an objective constraint on rational credences, defended on the grounds that rational agents should be able to change their minds. A wide range of rules against inert imprecise credences can be defended on exactly the same grounds. Whether your credences are radically or moderately imprecise, your credences are forbidden from being inert for just the same reasons as other beliefs are forbidden from being inert. No belief state should be immune to revision, including the state of having more than .5 credence that all the marbles in a certain urn are green, as well as the state of not having more than .5 credence in this same proposition. We can solve the problem of belief inertia by accepting global rules against inertia as fundamental objective constraints.

5 Violations of Reflection principles

Like many traditional rules of rationality, the principle of Reflection has traditionally been interpreted as imposing a constraint on the credence functions of precise agents. As introduced by VAN FRAASSEN 1984, the principle states that your conditional credence in a proposition, conditional on your assigning it credence r at some later time, must equal this same real number r (244). In other words:

$$\text{Precise Reflection} \quad Cr_0(p|Cr_1(p) = r) = r$$

¹⁹. See claim 2 of the appendix for a proof of this result.

As many authors have noted, this principle stands in need of qualification.²⁰ *Precise Reflection* should not govern your credences when you believe that you might forget information, for instance, or when you fear that you might learn false information or fail to update rationally. BRIGGS 2009 makes the helpful observation that a suitably qualified version of *Precise Reflection* is simply a consequence of the Kolmogorov axioms, and so the former may be considered just as unobjectionable as the latter.

In the context of *Precise Reflection*, Cr_0 and Cr_1 are precise credence functions, mapping propositions to real numbers. How should this principle be extended to constrain representors, objects that are not even functions defined on propositions? At first glance, this question might appear to have an obvious answer—namely, that Reflection constrains your current imprecise conditional credences in a proposition, given hypotheses about your later imprecise credence in it.²¹ In other words:

$$\text{Value Reflection} \quad R_0[p|R_1[p] = S] = S$$

In support of the principle of *Value Reflection*, WHITE 2009 says that it is natural to suppose that “if you know that you will soon take doxastic attitude A to *heads* as a result of *rationally* responding to new information *without loss of information*, then you should *now* take attitude A to *heads*... This is a generalization of Bas van Fraassen’s (1984) Reflection principle” (178).²²

However, *Value Reflection* leads to a problem for fans of imprecise credences. According to *Value Reflection*, rational agents cannot anticipate having credences that *dilate*, becoming less precise over time. But according to fans of imprecise credences, dilation is sometimes the inevitable consequence of rational updating. Here is an example from WHITE 2009:

Coin game. You haven’t a clue as to whether p . But you know that I know whether p . I agree to write ‘ p ’ on one side of a fair coin, and ‘ $\neg p$ ’ on the other, *with whichever one is true going on the heads side* (I paint over the coin so that you can’t see which sides are *heads* and *tails*). We toss the coin and observe that it happens to land on ‘ p ’. (175)

Suppose that at the start of the coin game, you have credence $(0, 1)$ in p . When you observe the coin land on ‘ p ’, you should update your representor by conditionalizing each member of it on the following proposition: that p is true if and only if the coin landed heads. As a result, you will come to have credence $(0, 1)$ in *heads*. In short, some members of your representor take the result of the coin toss as confirming *heads*,

20. See CHRISTENSEN 1991, TALBOTT 1991, MAHER 1992, and BOVENS 1995.

21. An imprecise conditional credence is defined as follows: $R[p|q] =_{df} \{m(p|q) : m \in R\}$.

22. For similar remarks, see also SCHOENFIELD 2012 and TOPEY 2012.

while others take the result as disconfirming it. Hence your credences in *Coin Game* constitute a counterexample to *Value Reflection*:

$$R_0[\text{heads}|R_1[\text{heads}]] = (0, 1) = \{.5\} \neq (0, 1)$$

This counterexample has none of the usual trappings of traditional problems for Reflection. We can stipulate that you are certain that you will not forget information or update on false information, for instance, and that you are certain that you will update your representor by conditionalizing each member of it on the proposition that you learn. As a result, one might be tempted to conclude that dilation is irrational, and that the same goes for the imprecise credences that license this diachronic behavior.

How should fans of imprecise credences solve this problem? The correct diagnosis does not involve rejecting the permissibility of your imprecise priors, nor the updating rule that results in their dilation.²³ Rather, we should reject the principle of *Value Reflection* itself. At first glance, this principle may appear to be an uncontroversial extension of *Precise Reflection*. But as I shall argue, *Value Reflection* is actually much stronger than any principle supported by the normative facts that ground *Precise Reflection*. The appropriate extension of *Precise Reflection* is another weaker principle.

The idea of tempering *Value Reflection* in response to dilation examples is discussed briefly by SCHOENFIELD 2012 and TOPEY 2012. Both authors consider something like the following substitute for *Value Reflection*:

$$\textit{Identity Reflection} \quad R_0[p|R_1 = X] = X[p]$$

Unlike *Value Reflection*, the principle of *Identity Reflection* is not violated by your *Coin Game* credences. Before you look at the coin, you have an imprecise conditional credence in *heads*, conditional on the hypothesis that you will later be certain that the coin lands on '*p*' and adjust your other credences accordingly. The same goes for your conditional credences given other hypotheses about the identity of your later representor. Conditional on any such hypothesis, your current credence in *heads* is radically imprecise, matching the credence that you anticipate later assigning to *heads*.

Although *Identity Reflection* is consistent with the permissibility of dilation, one might worry that this version of Reflection is overly restricted in scope. Both Schoenfield and Topey raise something like this concern:

We don't want the principles that tell us how to defer to be applicable only in cases where we know what the expert's entire representor is, since we rarely have such

23. The phenomenon of rational dilation is generally accepted as a consequence of imprecise credence models; see SEIDENFELD & WASSERMAN 1993 for further discussion. BRADLEY & STEELE 2014 investigate alternative rules for updating imprecise credences and conclude that no reasonable updating rule forbids rational dilation.

information. (SCHOENFIELD 2012, 207)

It isn't the case that any psychological difference renders Reflection inapplicable. If it were, Reflection would be applicable only when a person had acquired perfect knowledge of her entire future credal state. And no one ever has such knowledge. So, if Reflection is to be at all useful as a principle, some psychological differences must be irrelevant to its applicability. (TOPEY 2012, 485)

In response to these concerns, it should be conceded that an imprecise agent hardly ever knows what representor she will have at a later time. But fortunately, this fact does not limit the scope of *Identity Reflection*. Just like *Precise Reflection*, the principle of *Identity Reflection* imposes significant rational constraints on your credences, even when you are not certain of your future credal states. *Identity Reflection* imposes constraints on your conditional credences, conditional on hypotheses about various states that you think that you might be in later. These constraints on your conditional credences indirectly constrain your current unconditional credences. For example, suppose that you do not know what your later representor will be. But say you have .5 credence that your representor will be Q and .5 credence that it will be R , where $Q[p] = (.1, .2)$ and $R[p] = (.2, .3)$. From *Identity Reflection*, it follows that you are rationally required to believe that p is more than .15 likely and less than .25 likely, which is indeed a substantive constraint on your current credences.²⁴

That being said, there is something right about the spirit of the complaints raised by Schoenfield and Topey. Fans of imprecise credences should value Reflection principles that are easy to operationalize. *Identity Reflection* constrains your credences in light of your opinions about extremely strong hypotheses. A more valuable Reflection principle would constrain your current credences in light of more local features of your opinions about your future credences—for instance, constraining your current imprecise credence in p in light of your estimates of your future imprecise credence in that same proposition. Can we find a Reflection principle of this sort?

As we search for such a principle, it is again useful to pay attention to the distinction between global and pointwise constraints. Throughout this paper so far, global constraints have proved useful in solving several problems that could not be solved by pointwise constraints. At this point, though, the tables have turned. In extant discussions of Reflection principles for imprecise agents, *Precise Reflection* has been too hastily extended to global requirements of rationality. For example, the principle of *Value Reflection* corresponds to a global constraint on your current credences, namely the set of representors that treat your future self as an expert about the likelihood of every proposition. (In order to treat your future self as an expert about a proposition,

24. See claim 3 of the appendix for a proof of this result.

your credences might be required to spread out to fill a certain interval, for instance, and this does not amount to the satisfaction of any pointwise constraint.)

Fans of imprecise credences should part with this trend in the literature, endorsing a pointwise Reflection principle rather than a global requirement. The traditional principle of *Precise Reflection* is intimately connected with the traditional rule for rational updating, requiring you to defer to your future credences when you are certain that you will update rationally. This traditional updating rule is generally extended to imprecise agents as a rule that targets the individual elements of a representor, saying that your later representor must contain just those functions that result from conditionalizing each member of your representor on the information you learn. An accompanying Reflection principle for imprecise agents should similarly target the individual elements of a representor. Here is the rough idea behind an attractive Reflection principle: each individual member of your representor should defer to her later credences, as long as she is certain that you will update rationally.

The obvious difficulty with this rough idea is that it is not as if each individual member of your representor has opinions about what her later credences will be. At any given time, your total belief state is represented by a set of credence functions; there are no further facts about the cross-temporal identity of members of this set. To continue the metaphor of the rough idea: at best, each member of your representor can only ever learn that her later credence in p will be contained in a certain set—namely, your later imprecise credence in p . However, this information is still enough to significantly constrain your credences. For instance, consider the fact that as a precise agent, you violate *Precise Reflection* if you have .8 conditional credence in p , given the proposition that your later credence in p is contained in $(.6, .7)$. Similarly, as an imprecise agent, you violate an important Reflection principle if some member of your representor has .8 conditional credence in p , given the proposition that your later imprecise credence in p is $(.6, .7)$. This idea is captured by the following general Reflection principle for imprecise agents:

$$\textit{Pointwise Reflection } R_0[p|R_1[p] = S] \subseteq S$$

In other words, every probability measure m in your current representor R_0 must be such that $m(p|R_1[p] = S) \in S$. By contrast with other imprecise Reflection principles considered so far, the principle of *Pointwise Reflection* is a pointwise constraint, imposing a universal condition on the probability measures in your representor.

Fortunately, just like *Identity Reflection*, the principle of *Pointwise Reflection* is consistent with the rational permissibility of dilation. In *Coin Game*, you have .5 credence in *heads* conditional on the proposition that you will later have radically imprecise

credences in *heads*, but this is consistent with *Pointwise Reflection*, since the former credence is among the latter. Meanwhile, unlike *Identity Reflection*, the principle of *Pointwise Reflection* is easy to operationalize. The principle does not constrain your credences conditional on extremely strong hypotheses about your future representor, but rather on hypotheses about your later credence in one particular proposition. And moreover, *Pointwise Reflection* is a strong rational requirement. Most notably, in ordinary cases of learning where your credences shrink rather than dilate, *Pointwise Reflection* ensures that evidence about your later credences significantly constrains your current credences. For instance, in the special case where S is a singleton and your representor is non-empty, *Pointwise Reflection* entails the intuitive rule of *Value Reflection* considered at the very start of this section: $R_0[p|R_1[p] = S] = S$.

Pointwise Reflection constitutes a natural generalization of *Precise Reflection*. As mentioned at the start of this section, BRIGGS 2009 derives a qualified version of *Precise Reflection* from the probability axioms, given some modest background assumptions. As long as you are certain that you will rationally update on veridical evidence, simply having a coherent credence function will guarantee that you satisfy *Precise Reflection*. We can derive a similarly qualified version of *Pointwise Reflection*, given the same modest background assumptions. As long as you are certain that you will rationally update on veridical evidence, merely having a representor of coherent credence functions will guarantee that you satisfy *Pointwise Reflection*.²⁵ This result justifies the principle of *Pointwise Reflection* as a legitimate Reflection principle for imprecise agents, by contrast with overly ambitious principles such as *Value Reflection*.

In conclusion, sometimes dilation is simply an effect of rationally updating imprecise credences. Accordingly, dilation is consistent with the correct extension of *Precise Reflection* to imprecise agents. Although it is initially tempting to extend *Precise Reflection* to an extremely strong global requirement, the spirit of this principle is best captured by a pointwise requirement. According to *Pointwise Reflection*, each member of your representor must take rational account of your later informed opinions. Distinguishing pointwise requirements from global requirements has thus proven useful in both directions. Articulating global independence and inertia constraints helps us solve some challenging problems for fans of imprecise credences. Articulating a pointwise Reflection constraint has helped us solve another. The notion of a global constraint is not only theoretically interesting, but also significant for the development and defense of the epistemology of imprecise credences.

25. See claim 4 of the appendix for a proof of this result.

Appendix

Claim 1. If $R \subseteq \{P | P(AB) = P(A)P(B)\}$, R is convex only if $|R[A]| = 1$ or $|R[B]| = 1$.

Proof: Assume R is convex. Let $P_1, P_2 \in R$. By convexity, $.5P_1 + .5P_2 \in R$. Hence:

$$\begin{aligned}
 .5(P_1(A) + P_2(A)).5(P_1(B) + P_2(B)) &= .5(P_1(AB) + P_2(AB)) \\
 P_1(A)P_1(B) + P_2(A)P_1(B) + P_1(A)P_2(B) + P_2(A)P_2(B) &= 2P_1(AB) + 2P_2(AB) \\
 P_1(AB) + P_2(A)P_1(B) + P_1(A)P_2(B) + P_2(AB) &= 2P_1(AB) + 2P_2(AB) \\
 P_2(A)P_1(B) + P_1(A)P_2(B) &= P_1(AB) + P_2(AB) \\
 P_2(A)P_1(B) + P_1(A)P_2(B) &= P_1(A)P_1(B) + P_2(A)P_2(B) \\
 P_2(A)[P_1(B) - P_2(B)] &= P_1(A)[P_1(B) - P_2(B)] \\
 P_2(A) &= P_1(A)
 \end{aligned}$$

Since P_1 and P_2 are arbitrary elements of R , it follows that $|R[A]| = 1$, as desired.

Claim 2. $C_{open} =_{df} \{R : R \text{ is not inert with respect to } \{Q : Q[H] \subseteq [.5, 1]\}\}$ is global.²⁶

Proof: It suffices to produce R_1, R_2 such that $R_1 \notin C_{open}$, $R_2 \in C_{open}$, and $R_1 \subseteq R_2$. As VALLINDER 2017 demonstrates, for any given interval (c_-, c_+) , there exists some representor that is inert with respect to assigning that imprecise credence to H . Let R_1 be a representor that is inert with respect to assigning $(.5, 1)$ to H , and let R_2 be the union of this representor with the credence function corresponding to a uniform density function over possible chance hypotheses.

Claim 3. If $R_0[R_1 = Q] = .5$, $R_0[R_1 = R] = .5$, $Q[p] = (.1, .2)$, and $R[p] = (.2, .3)$, then by *Identity Reflection*, $R_0[p] \subseteq (.15, .25)$.

Proof: Let m be an arbitrary element of R_0 . By the probability calculus, we have:

$$\begin{aligned}
 m(A) &= m(R_1 = Q)m(A|R_1 = Q) + m(R_1 = R)m(A|R_1 = R) \\
 m(A) &= .5a + .5b \text{ for some } a \in (.1, .2), b \in (.2, .3) \\
 m(A) &\in (.15, .25).
 \end{aligned}$$

26. I adopt the simplifying assumption that the elements of a representor are given by the expected value of a probability density function over possible chance hypotheses about the outcome of flipping a coin, where H is the proposition that the coin lands heads. As mentioned in footnote 17, the inertness relation is relative to a class of evidence propositions that is implicitly restricted by context. In this case, the evidence propositions are potential outcomes of a series of observed flips of the coin.

Claim 4. Assume that the evidence propositions that the agent might learn form a finite partition \mathcal{B} , and that the agent is certain that conditionalization is the right updating procedure.²⁷ Assume that the following qualifications hold for every $B \in \mathcal{B}$:

- i. $R_0[R_0[A|B] = R_1[A|B]] = 1$
- ii.²⁸ $R_0[B \equiv R_1[B] = 1] = 1$
- iii. $R_0[R_0[A|B] = S] = 1$ if and only if $R_0[A|B] = S$

Then $R_0[A|R_1[A] = S] \subseteq S$.

Proof: The argument closely follows the derivation of Qualified Reflection on p. 69 of BRIGGS 2009. Let m_0 be an arbitrary element of R_0 . Then we have:

$$\begin{aligned}
 m_0(A|R_1[A] = S) &= \frac{m_0(A \wedge R_1[A] = S)}{m_0(R_1[A] = S)} \\
 &= \frac{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(A \wedge R_1[B] = 1)}{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(R_1[B] = 1)} \\
 &= \frac{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(A|R_1[B] = 1)m_0(R_1[B] = 1)}{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(R_1[B] = 1)} \\
 &= \frac{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(A|B)m_0(B)}{\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(B)}
 \end{aligned}$$

By (i) and (iii), we have $R_0[A|B] = R_1[A|B]$ for every $B \in \mathcal{B}$. Hence we have:

$$\sum_{B \in \mathcal{B}: R_1[A|B] = S} m_0(A|B) = \sum_{B \in \mathcal{B}: R_0[A|B] = S} m_0(A|B)$$

By the definition of $R_0[A|B]$, we can conclude that $m_0(A|B) \in S$, and hence that $m_0(A|R_1[A] = S) \in S$. Since m_0 was chosen to be an arbitrary element of R_0 , it follows that $R_0[A|R_1[A] = S] \subseteq S$, as desired.

27. The latter assumption mimics the third idealizing assumption introduced on p. 69 of BRIGGS 2009. According to my best understanding of Briggs, this assumption is intended to be interpreted in a way that licenses the second step of the derivation of Qualified Reflection on p. 69. In order to preserve the close analogy between our arguments, I shall use the same language for the assumption that licenses the second step of my derivation of *Pointwise Reflection*.

28. This premise is inspired by premise (ii) on p. 69 of BRIGGS 2009, which Briggs informally glosses as the claim that the agent is “certain that she will update on veridical evidence” (70). Although Briggs formally defines premise (ii) as the claim that $Cr_0(B|Cr_1(B) = 1) = 1$, it seems to me that the derivation of Qualified Reflection on p. 69 requires a stronger claim, namely that $Cr_0(B \equiv Cr_1(B) = 1) = 1$.

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