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Source: *The Journal of Political Economy*, Vol. 86, No. 4 (Aug., 1978), pp. 627-648
Published by: The University of Chicago Press
Stable URL: http://www.jstor.org/stable/1840382

Market Anticipations of Government Policies and the Price of Gold

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This paper is an analysis of the effects of anticipations of government sales policies on the real price of gold. Although the risk of a future government gold auction depresses the price, it also causes the price to rise in percentage terms faster than the real rate of interest and at an increasing rate. Even risk-neutral investors require this rate of return as inducement to hold gold in the face of the asymmetric risk of a price collapse. Announcements making a government auction more probable cause a sudden drop in the price. Government attempts to peg the price or to defend a price ceiling with sales from its stockpile must result eventually in a sudden attack by speculators.

Between March 1968, when central banks stopped their pegging operations, and December 1974, when the U.S. government announced its first gold auction, the path of the real gold price (shown in fig. 1) exhibited two striking features. First, the price rose at a rate much greater than the real rate of interest for intervals as long as 8 months. Second, each upward surge was interrupted by a sharp setback. During that 6-year period, little or no gold was actually decumulated from the massive stockpiles controlled by world governments. But since the role of gold in the international monetary system had been reduced, the possibility persisted that significant sales to the private market might some day occur. Because of its profound

Our colleague, Jeffrey Shafer, showed us that, if there were some reason why price might fall to a constant floor, observed prices would accelerate away from that floor. For his initial idea and subsequent insights, we wish to acknowledge our debt. Helpful comments on Salant and Henderson (1974) and several memoranda by us which formed the basis for this paper were received from Ralph Bryant, George Henry, Walter Salant, Steven Salop, Charles Siegman, Edwin Truman, and Henry Wallich. No one but the authors is responsible for the paper's remaining shortcomings. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Federal Reserve System or other members of its staff.

[*Journal of Political Economy*, 1978, vol. 86, no. 4]

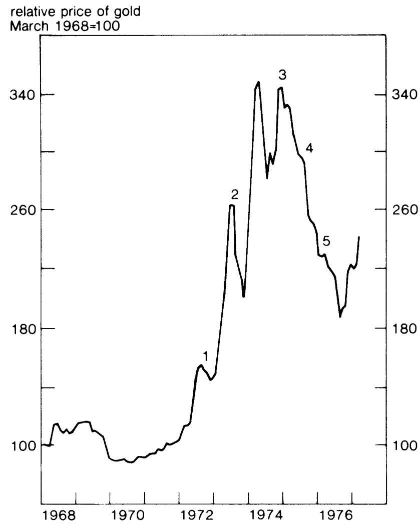

FIG. 1.—Monthly index of the dollar price of gold relative to the U.S. Consumer Price Index. *1*, First explicit reports that the United States and the IMF were considering gold sales. *2*, Reports of impending agreement to permit central bank sales. *3*, Announcement of first U.S. auction. *4*, Announcement of second U.S. auction. *5*, Agreement reached in Jamaica on IMF gold sales.

consequences, this possibility could not be disregarded by market participants owning gold. Government stocks are about 25 times as large as global annual production at its peak and are roughly equal to estimates of the entire stock remaining underground on the planet.[1] In our view, market anticipations that governments would sell a portion of these stocks at an unpredictable time exerted a significant influence on the gold price during this period. Since then, the United States and the International Monetary Fund have publicly sold, or announced schedules to sell, about 4 percent of the total stock overhang. It seems reasonable to expect further sales in the future.

The purpose of this paper is to explain the effects of anticipations of government gold policies on the path of the gold price. Since gold, like any other ore, can be depleted (through use in dentistry and industry) but not created, the conventional theory of depletable resources should be useful in understanding the behavior of the gold price.[2] However, it is argued

[1] Data on annual world gold extraction appear in Samuel Montagu & Co. (1977), and data on gold reserves held by world governments and international organizations are included in International Monetary Fund (1977).

[2] The basic theory of exhaustible resources was developed by Hotelling (1931). Solow (1974) provides a lucid exposition of this theory. Analyses of gold as an exhaustible resource can be traced to Paish (1938). Thomas Wolfe, the former director of the Office of Domestic Gold and Silver Operations of the Treasury, has emphasized that "[g]old is among the few minerals that could reach a critical supply situation within this century, taking account of available reserves above and below ground" (1977, p. 35).

in Section I that without some modification the standard theory cannot be used to explain how the price of a resource traded by competitive speculators could persistently rise by more than the rate of interest, nor can it be employed to account persuasively for the timing of the observed breaks in the price. In Section II, the standard Hotelling model is extended to the case where risk-neutral agents anticipate an auction of an additional stock at an unknown time. Properties of the anticipations model are established in Section III. The consequences of anticipations of other government policies involving gold sales are considered in Section IV. Section V contains concluding remarks.

I. The Inability of the Standard Model of Exhaustible Resources to Explain the Observed Movements in the Gold Price

To review the simplest model of an extraction industry, suppose a collection of competitive firms with zero extraction costs and fixed stocks of known size sell to consumers whose demand at any time depends only on the price then prevailing in the market.[3] Under such circumstances, only one sequence of prices will make the optimal decisions of extractors and consumers consistent at all times. The equilibrium price path must begin at a specified level and rise at the rate of interest during all periods of positive sales. If the price did not rise at that rate, some prices would have lower discounted values than others. Mine owners maximizing the present value of their profits would sell nothing in those periods, and excess demand would then result. Similarly, if the initial price were set too low (high), the cumulative amount demanded by consumers would exceed (fall short of) the cumulative supply, an indication that some of the intertemporal sequence of markets would fail to clear.

When speculators with neither costs of storage nor initial inventories are added to this model, the equilibrium price path does not change. Extraction in a period where speculators buy (sell) exceeds (falls short of) gold consumption, the difference going into (coming from) speculative inventories. While competitive extractors make profits because of the rents on the scarce resource they own, competitive speculators make no profits.

In this simple form, the exhaustible-resource model fails to capture the most salient characteristics of the price path of gold since 1968. It does not predict persistent increases in the price at greater than the rate of interest, nor can it explain either the existence or timing of most drops in the price.[4]

[3] Consumers, who purchase goods and services produced using gold, ultimately determine how much gold is bought by jewelry fabricators, dentists, and electronics firms. It is these derived demands which add up to the flow demand curve. Gold used for these purposes is assumed not to be used again, either because of the taboos of society or because of prohibitive costs of recovery.

[4] It does suggest that gold extraction might fall as price rises, a phenomenon which has occurred, to the puzzlement of some analysts.

However, since the assumptions of this naive model do not reflect several prominent features of the gold market, the omission of some important characteristic may account for the poor predictive performance of the model.

Three characteristics of gold distinguish it from the exhaustible resource of the simple model. First, extraction costs in the gold industry are not negligible and have risen. Second, the gold market is not competitive but is dominated by one seller, the South African Reserve Bank.[5] Third, gold holders cannot know with certainty the size of the stock which will be available for private use since the possibility always exists that governments may sell to the private market some of their enormous holdings.

Of these three distinguishing features of the gold market,[6] only the third could account for the basic characteristics of the observed gold price path. For, while the presence of soaring extraction costs or of monopolistic behavior could conceivably give rise to the observed pattern of prices in

[5] The Reserve Bank acquires and markets all South African production, an amount equal to three-quarters of the world total. This arrangement creates a presumption that the Reserve Bank takes account of its influence on market price as Machlup (1969) speculated. Since, in 1970 and 1971, the bank sold part of its supply to the IMF and some central banks in order to avoid reducing the higher price on the free market, the inference that it considers its market power in its sales decisions seems inescapable. The prices the Reserve Bank received from the two different sets of buyers may be used to infer the bank's estimate of the elasticity of the demand for gold at the time. In both 1970 and 1971, South Africa sold gold to the IMF and some central banks at $35 an ounce and to the free market at the higher market price of $40. Such behavior implies that South Africa estimated the marginal revenue of selling another unit on the private market to be $35 or, equivalently, estimated the elasticity of the (excess) demand facing it to be about 8. To infer the elasticity of *aggregate* demand from this information requires an additional assumption about how South Africa perceived its dominant position in the gold market. If it did not take into consideration how other sellers would react to changes in its sales (the "Cournot" assumption), the elasticity of aggregate demand would simply equal the elasticity of excess demand, estimated above, multiplied by South Africa's market share at the time. If it considered that other extractors would sell more when it sold less (the "Stackelberg" assumption), the implied elasticity of aggregate demand would be lower. For models of a dominant resource extractor which utilize the "Cournot" and the "Stackelberg" solution concepts, see Salant 1976 and 1978, respectively.

[6] There may be a fourth characteristic of gold which distinguishes it from the exhaustible resource of the simple model. In the simple model speculators and zero-cost extractors are willing to hold the existing stock of gold only if its price is increasing at the rate of interest. However, it has been suggested that some people may derive services from the mere presence of bullion. Such individuals would be willing to rent gold from its owners in order to obtain these services. As long as gold owners receive a positive rental income, the rate of increase in the gold price sufficient to induce them to hold title to the gold stock would be even *smaller* than the rate of interest. (In fact, industrial depletion may eventually be choked off altogether, and owners of the remaining stock would then earn only a rental income.) Hence, although the introduction of a demand for services derived from the presence of gold would increase the realism and complexity of the model, it would contribute nothing to an explanation of the observed, rapid surges in the price. An analysis of the effects of introducing a demand for services derived from the stock of an exhaustible resource into the simple model is available on request.

the absence of speculators,[7] *neither factor could explain the observed pattern in a world where competitive speculators can enter and do operate.* Since competitive gold speculators have no extraction costs and negligible storage costs, they prevent the price from persistently rising in percentage terms faster than the rate of interest. For, if speculators came to foresee a more rapid increase, they would attempt to make unlimited profits by borrowing, buying gold in one period, and selling it in the next—thereby carrying cheaper ore into a period in which it would have been more expensive. Such actions would cause the price path to change until all foreseeable opportunities for profitable arbitrage were eliminated.

II. An Alternative Model to Explain Gold Price Movements

Since competitive speculators place an upper bound on the rate of increase of the gold price under the circumstances considered so far, a model which purports to explain movements in these prices must explain why speculators did not flood the gold market when persistently faced with apparently unexploited profit opportunities. We believe the key ingredient in such an explanation is the persistent anticipation of gold holders that a portion of the large stocks held by the world's governments would be sold on the private market at an unpredictable time. This possibility had been mentioned repeatedly in trade journals, official statements, and newspapers long before the U.S. government finally announced its first gold auction.[8] In the words of the *New York Times* (May 24, 1974), "The 'sword of Damocles' over gold's high price is the huge dormant supply in the central banks." It is argued here that the threat that this sword would drop would have been sufficient to cause rapid increases in the gold price even in the absence of rising extraction costs and South African market power.

To clarify how the risk of government selling affects the path of the gold

[7] Hotelling (1931) and Solow (1974) study competitive extractors with positive marginal extraction costs. The behavior of a single monopolistic extractor is analyzed in Hotelling (1931), Weinstein and Zeckhauser (1975), Lewis (1976), and Stiglitz (1976). Although both Weinstein and Zeckhauser (1975) and Lewis (1976) indicate that an unconstrained monopolistic extractor might set a price path rising faster than the rate of interest, Salant (1976) and Stiglitz (1976) show that such paths cannot characterize an equilibrium in the presence of speculators. The constraints imposed on a monopolist by competitive speculators are completely analogous to those considered by Smithies (1941). He considered the single producer of a commodity who wishes to sell it in spatially separated markets at different prices but who knows he cannot (because of arbitrageurs) let prices diverge between markets by more than transportation costs. In an intertemporal model the markets are separated in time, and the cost of transporting through time is the rate of interest.

[8] For example, Samuel Montagu & Co., Ltd. cautioned, "Serious consideration has therefore also been given to sales of gold by monetary authorities, and these cannot be excluded in the future" (1973, p. 12). A year later, the warning was renewed that "the very large gold holdings of investors and central banks cannot be ignored" (1974, p. 13).

price, we retain most of the assumptions of the simple model discussed in the previous section. It is assumed that mine owners have an initial gold stock of known size ($\bar{I}$) which they extract costlessly and sell competitively. Speculators without storage costs or initial inventories are free to purchase gold and subsequently resell it. The consumers' demand curve for gold [$D(\cdot)$] is assumed to be stationary and downward sloping with a "choke" price (P_c) above which demand is zero.

What makes the modified model different is the possibility of government gold sales. To treat the simplest case, we assume a known amount ($\bar{G}$) may be sold in a single auction at an unknown date.[9] Mine owners and speculators are assumed to assess the odds of a sale in the next period,[10] given that none has so far been announced, as some constant α.[11] Let P_t be the price which will emerge at time t in the absence of an auction and f_t be the real price which will result if the auction occurs. Since agents are assumed to be risk-neutral, they act to maximize discounted expected profits. The stock of gold in private hands at the beginning of period t in the absence of an auction is denoted S_t.

Under these assumptions, five conditions determine the equilibrium paths of the variables in the model. In the initial period the private sector has what gold is in the mines:

$$S_0 = \bar{I} \qquad \text{(initial condition).} \tag{1}$$

[9] The extension to the case of anticipations of multiple auctions announced at random times is available on request.

[10] The simplifying assumption that market participants regard the timing of government gold sales as an entirely exogenous random process implies that agents cannot use market variables (like price) to improve their forecasts of government behavior. Although this assumption eliminates interesting considerations of game theory—which may well be relevant in the future—we think that market participants have had no reasonable basis for believing that government sales decisions to date were related to market behavior. For a not dissimilar view, see Wolfe (1977, p. 35).

[11] Alternatively, agents might have reasons to believe that the probability of an auction at t, given none has occurred before then, will vary over time. This complication can be incorporated merely by adding a time subscript to α in eq. (4). If this alternate specification implies that an auction will occur "soon enough" with probability one, the market would fully discount the event and the stock overhang would not have the effects described in the text. This point has occasionally been adduced as an objection to our analysis. We believe it to be valid as a logical possibility but demonstrably false as an empirical proposition. That is, as long as the market anticipates with probability one that the government will sell its stock some time before the date when $\bar{I}$ units would be depleted along a path rising at the rate of interest and generating cumulative demand of $\bar{G} + \bar{I}$, then the auction would occur soon enough, and the uncertainty about the exact timing of the auction would not matter. In such a situation, however, the announcement of an auction would not affect the price since the market would already have discounted the event that it would receive the stock in sufficient time. In reality, however, the announcement of each auction has triggered precipitous price declines. *This indicates that the market did not fully discount such announcements before they occurred.* Therefore, the overhang *should* have the effects described in the text.

Private gold holders sell all their gold before the endogenously determined terminal period (T) when price chokes off demand:

$$\begin{aligned} P_T &= P_c, \\ S_T &= 0, \end{aligned} \qquad \text{(terminal conditions).} \tag{2}$$

If an auction occurs at time t, the private sector then owns $S_t + \bar{G}$ units of gold and, by assumption, no longer operates under the threat of a future auction. Hence, price grows over time at the real rate of interest (r) from the auction price (f_t) to the choke price. The potential auction price is determined by the condition that cumulative consumption along the price path exactly equals the stock in private hands:

$$\sum_{x=0}^{\infty} D[f_t(1 + r)^x] = \bar{G} + S_t \qquad \text{(equation determining potential auction price).} \tag{3}$$

Equation (3) determines the potential auction price as an implicit function of the stock remaining in private hands at the time of the auction.

Since consumers demand gold in every period before T, mine owners and speculators must be willing both to sell gold and to carry some inventory of the metal. Risk-neutral private gold holders will be indifferent between these two activities only when they yield the same discounted expected profit. Opportunities for profitable arbitrage exist unless the price in period t is equal to the discounted value of the price then expected to prevail in period $t + 1$:[12]

$$P_t = \frac{\alpha f_{t+1} + (1 - \alpha)P_{t+1}}{(1 + r)} \qquad \text{(arbitrage equation).} \tag{4}$$

The private stock is depleted between period t and period $t + 1$ by the amount of nonrecoverable consumption in period t:

$$S_{t+1} = S_t - D(P_t) \qquad \text{(depletion equation).} \tag{5}$$

[12] Eq. (4) indicates that the discounted price expected in period t to prevail one period in the future is equal to the price in period t. It can be verified that a similar relation holds for the price expected in period t to prevail k periods in the future. That is,

$$\frac{E(X_{t+k} \mid P_t)}{(1 + r)^k} = P_t,$$

where X_{t+k} is the random price in period $t + k$. The verification is accomplished by showing that each side of the equation above is equal to the same intermediate expression; X_{t+k} may take on the following $k + 1$ values: $f_{t+i}(1 + r)^{k-i}$ with probability $\alpha(1 - \alpha)^{i-1}$, $i = 1, k$; P_{t+k} with (complementary) probability $(1 - \alpha)^k$. Thus,

$$\frac{E(X_{t+k} \mid P_t)}{(1 + r)^k} = \sum_{i=1}^{k} f_{t+i}(1 + r)^{-i}\alpha(1 - \alpha)^{i-1} + P_{t+k}\left[\frac{(1 - \alpha)}{(1 + r)}\right]^k.$$

That P_t is equal to the expression on the right-hand side of this equation can be verified by starting with the arbitrage eq. (4) and iterating $k - 1$ times to obtain P_t as a function of f_{t+i} ($i = 1, k$) and P_{t+k}.

The model of equations (1) through (5) generates time paths for the price of gold in anticipation of an auction (P_t), the potential auction price (f_t), demand [$D(P_t)$], and the private stock of gold (S_t). The number of periods (T) required to consume the initial private stock ($\bar{I}$) if the anticipated auction does not occur prior to its exhaustion is also determined.

To show that equations (1) through (5) determine these variables a backward solution is utilized. At the end, the private stock (S_T) and the price (P_T) in the absence of an anticipated auction are indicated by the terminal conditions. Using the private stock and equation (3), f_T—the price that would occur in the final period if there were an auction—can be calculated. Substituting P_T and f_T into equation (4) yields P_{T-1}, the price that must prevail at the beginning of the previous period in the absence of an auction. The stock in private hands in the previous period can then be determined from equation (5).

It has been shown that starting with the final values of the private stock (S_T) and price (P_T) in the absence of an auction, values for those same two variables one period earlier can be obtained. The process can be repeated to construct the time paths of the endogenous variables. Eventually, the private stock reaches the level specified by the initial condition,[13] and the backward iteration is terminated; T is calculated by counting the number of backward steps required for termination.[14]

[13] If the procedure described in the text is followed, the private stock may never build up exactly to the initial inventory since the analysis of the text is in discrete rather than continuous time. Formally, the arbitrage eq. (4) linking the price at $T - 1$ to the expected price at T holds with an inequality rather than equality because zero stock is carried into period T. Thus, $P_{T-1} \epsilon \{[\alpha f_T + (1 - \alpha)P_c]/[1 + r], P_c\}$. To insure that cumulative demand builds up exactly to the initial inventory, set P_{T-1} at the lower end of this interval and work backward in the manner described in the text, stopping in the first period in which cumulative demand exceeds $\bar{I}$. Then increase P_{T-1} until cumulative demand over the same number of periods is reduced exactly to $\bar{I}$. That this can be accomplished for P_{T-1} less than the upper limit of the interval is a consequence of the following observation: cumulative demand along a path of a given length beginning with P_{T-1} at the upper end of the interval is identical with cumulative demand along a path one period shorter beginning with P_{T-1} at the lower end of the interval. Since, by assumption, demand along the latter path was found to be smaller than I, raising P_{T-1} to the upper end of the interval and working back one period more will reduce cumulative demand too much. Since cumulative demand over a given number of periods is a continuous function of P_{T-1}, there exists some P_{T-1} in the closed interval above for which cumulative demand exactly matches the initial stock.

[14] It might be assumed that gold holders are uncertain about the amount of gold the government might sell but that the distribution of possible auction sizes is independent of the time of the auction and the size of the private stock. Under these assumptions the solution technique is only slightly different. For every auction size ($\tilde{g}$), the initial price of the auction [$f_t(g)$] can be obtained from the following equation:

$$\sum_{x=0}^{\infty} D[f_t(\tilde{g})\cdot(1 + r)^x] = \tilde{g} + S_t.$$

Using the probability distribution on $\tilde{g}$, the expected auction price (m_t) can be computed. This number replaces f_t in the backward solution described in the text.

III. Properties of the Model with Anticipations of an Auction

The most striking property of the anticipations model is that, prior to the auction, the price must rise in percentage terms by more than the rate of interest and at an increasing rate. Risk-neutral extractors and speculators require this rate of return to induce them to hold gold in the face of the asymmetric risk of a price collapse which would result if the government announced a sale. According to the arbitrage equation, the *expected* price in $t + 1$ must have the same discounted value as the price in period t. Given the intuitive proposition that a government auction in any period would reduce the price ($f_t < P_t$, for all t),[15] the price which would occur in period $t + 1$ in the absence of the government sale must have a larger discounted value than the price prevailing in the previous period:

$$P_t(1 + r) = \alpha f_{t+1} + (1 - \alpha)P_{t+1} < \alpha P_{t+1} + (1 - \alpha)P_{t+1} = P_{t+1}. \tag{6}$$

If the price were ever to rise at a slower rate prior to the auction, the expected discounted capital gain from owning gold would be negative, and no one would be willing to hold it as an asset.

The percentage increase in the price in anticipation of an auction must itself increase as time passes. To establish this property, we use the result proved below that the potential auction price rises by less than the rate of interest $[f_{t+1} < f_t(1 + r)]$. The arbitrage equation can be rearranged to yield an expression for R_t—the percentage increase in the price between period t and period $t + 1$:

$$R_t \equiv \frac{P_{t+1} - P_t}{P_t} = r + \frac{\alpha}{1 - \alpha}\left[(1 + r) - \frac{f_{t+1}}{P_t}\right]. \tag{7}$$

Since it has been shown that the actual price prior to the auction rises by more than the rate of interest and since it is being supposed for the moment that the potential auction price rises by less than the rate of interest, the ratio (f_{t+1}/P_t) in the expression above must decline over time. Hence, R_t increases over time. From equation (7), it also is evident that there is an

[15] To prove that $P_t > f_t$ for all t, we assume the contrary ($P_t \leq f_t$) for some t. It can then be shown that if $P_t \leq f_t$ for some t, $P_{t+i} \leq f_{t+i}$ for $i = 1, T - t$. But since $P_T > f_T$ (assuming $\bar{G} > 0$, this follows from [2] and [3]), the hypothesis generating this implication must be false, and $P_t > f_t$ for all t. Specifically, if $P_t \leq f_t$ for some t, (A) $f_{t+1} \geq f_t(1 + r)$. *Proof:* This result is established by changing the premise in the argument in the text used to establish that $f_{t+1} < f_t(1 + r)$ from $P_t > f_t$ to $P_t \leq f_t$. However, A implies that (B) $P_{t+1} \leq P_t(1 + r)$. *Proof:* $(1 + r)P_t = \alpha f_{t+1} + (1 - \alpha)P_{t+1} \geq \alpha P_t(1 + r) + (1 - \alpha)P_{t+1}$. Hence, the potential auction price would have to be greater than or equal to the price in anticipation of an auction in the next period: (C) $P_{t+1} \leq f_{t+1}$. *Proof:* $f_{t+1} \geq f_t(1 + r) \geq P_t(1 + r) \geq P_{t+1}$. By induction, the potential auction price would have to be greater than or equal to the price in anticipation of an auction for all subsequent t; but this generates a contradiction since $f_T < P_c = P_T$.

upper bound to the percentage increase in the price in anticipation of the auction:

$$R_t \leq \frac{\alpha + r}{1 - \alpha}. \tag{8}$$

It has been shown that market participants who are risk-neutral require the gold price to rise by more than the interest rate in order for them to hold gold in the face of the possible capital loss. If holders of gold were *averse* to risk and therefore preferred a safe return to a risky one with the same expected value, the return to gold holders would have to include a risk premium. In that case, the rise of the gold price would have to be even steeper, and R_t could exceed the upper bound given by (8).

To show that the potential auction price rises by less than the rate of interest $[f_{t+1} < f_t(1 + r)]$, we utilize equations (3) and (5). If the government auction occurs at t, the price drops to f_t; depletion equals $D(f_t)$; and the price consequently rises in the following period to $f_t(1 + r)$. A price path rising at the rate of interest from this initial level generates a cumulative demand equal to the stock then in private hands $[S_t + \bar{G} - D(f_t)]$. In the absence of an auction at t, however, the price is higher $(P_t > f_t)$, the depletion is consequently lower $[D(P_t) < D(f_t)]$, and the stock remaining at $t + 1$, $[S_t - D(P_t)]$, is therefore larger. If the government then auctions $\bar{G}$ units, the price (f_{t+1}) must fall below $f_t(1 + r)$ for the market to absorb the larger total stock. Since this argument applies to any period, the potential auction price must always grow by less than the rate of interest $[f_{t+1} < f_t(1 + r)$ for *all* $t]$.

An important consequence of this result is that the longer the government waits to auction its gold, the smaller the real discounted value of its proceeds. The exact magnitude of the loss cannot be determined without knowledge of the demand curve and various exogenous magnitudes. However, a lower bound to the loss can be deduced without such information under fairly general assumptions as shown in the Appendix.

The three paths in figure 2 illustrate what has been learned so far about the effects of the anticipation of government sales when gold holders are risk-neutral. Path *AA* is the path of prices which would emerge with no possibility of government selling. Along this path, price rises at the rate of interest. Since the natural logarithm of price is plotted against time, this path is represented by a straight line. Path *BB* is the path that emerges in the continued anticipation of sales *which have not yet occurred*. The slope of *BB* is always greater than the slope of *AA*, and rises as time passes. Path *CC* shows the path of the natural logarithm of f_t, the price which would potentially emerge each period if the government auctioned its stock in that period. It rises at a rate which is always below the rate of interest. Prior to the auction, the gold price moves along *BB*. If the auction occurs, the price jumps to *CC* and rises from it along a line parallel to *AA*.

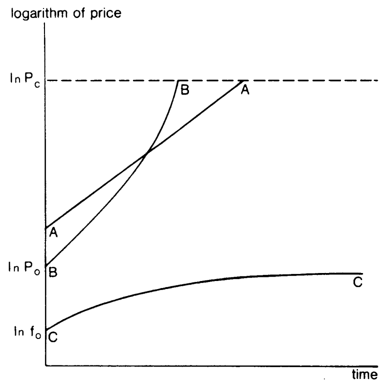

FIG. 2.—Price paths when a single auction is anticipated

In figure 2, the price path with risk (BB) begins lower, cuts the no-risk path (AA), and chokes demand sooner. To understand why this is so, recall that since the same stock ($\bar{I}$) must be depleted along both paths, cumulative demand along each path must be the same. Suppose the two paths began together. Since prices on the path with risk rise at a faster rate, this path would remain above the other and less would be consumed along it in each period. For the cumulative demand along each path to match, the price path with risk must begin lower. This implies that private gold stocks will be exhausted more quickly than they would be without the risk of government selling which fails to materialize.

That the price path with a risk of government sales should start lower and rise more steeply is intuitively appealing. Suppose society had a fixed stock of some exhaustible resource and had some chance, each period, of discovering a single additional deposit of known size. In that case, depleting more of the fixed stock in the early period than if there were no prospect of finding the additional amount would be sensible. However, if the additional stock were not "found," what remained after the early phase of rapid depletion should be rationed tightly. A group of competitive mine owners and speculators, acting in self-interest, would solve this planning problem by selling at low prices in early periods and very high prices later. Of course, in the case of government sales of gold, the "problem" solved by the market is artificial and man made; however, that makes the risk that additional supplies will topple the price no less real to private holders of gold.

It is helpful to consider what the model predicts will happen to the path of prices when unanticipated information becomes available. Suppose an official statement is made about the sale of monetary gold. If this statement causes speculators and mine owners to revise upward their estimate of the probability of government sales, the price of gold must fall and begin a new ascent;[16] if the intervention does not occur, the new path eventually rises above the old one and chokes off demand sooner. Figure 2 provides an illustration of this. When the odds of government selling increase (from zero to a positive fraction), the price immediately drops and follows the lower, steeper path. The reason for the drop, of course, is that holders of gold sell more quickly when faced with a higher probability of capital loss.

In fact, the sudden price declines of August 1972 and July 1973 (see fig. 1) appear to have resulted from exactly such revisions in the market's estimate of the odds of government sales. In the former episode, the revision was prompted by reports (*New York Times* August 2 and August 7, 1972) during the first week of August that the U.S. Treasury might consider selling gold to domestic users and by reports (*London Financial Times* August 4, 1972; *New York Times* August 7, 1972) in the second week of August that the Fund would recommend selling gold from its stocks (or from those of central banks) to the private market. In the latter episode, it was reported (*Washington Star News* July 14, 1973; *London Financial Times* July 21, 1973) during the second week of July 1973 that the Committee of 20 Deputies was in favor of official sales of gold in the free market as part of the reformed international monetary system.[17] In both episodes, the unanticipated reports triggered price declines although no official sales in fact took place; eventually, as the theory predicts, the price resumed its upward surge and surpassed its previous high.

The current discounted revenue which a mine owner or speculator can expect from the sale of a given amount of gold at any time in the future is equal to its current market value. When the odds of government sales increase, the market value of the existing stock falls. This serves to remind us that while the better prospect of additional supplies is socially beneficial the increased likelihood does injure owners of the existing stock. A country like South Africa, for example, would be injured by actions or announcements of world governments which substantially depress the gold price—just as it has benefitted substantially from previous international payments arrangements among nations which, in effect, constituted a commodity support program for gold.

[16] Suppose instead that the price path were initially *higher* after α increased. Then, as will be argued, eqq. (3) through (5) imply this path would have to *remain* higher until the choke price is reached. But then cumulative demand along the path with increased risk would be smaller than the initial stock, and excess supply would result. Hence, the path with increased risk must begin *strictly below* the other path, as is asserted in the text. Specifically, eqq. (3) through (5) imply that the path with increased risk must *remain* above the other path *if* it begins higher. From (7), the percentage increase in the price (R_t) is larger the larger is α and P_t and the smaller is f_{t+1}. If the path with increased risk ever approached the other path from above, it would have a larger α and P_t and a smaller f_{t+1}. Hence its percentage rate of increase in the next period would be higher, and the paths would never cross.

[17] As a noted market analyst observed, these reports "were instrumental in changing investors' attitudes to the gold market" (Samuel Montagu & Co., Ltd. 1974, p. 6).

IV. The Consequences of Market Anticipations of Alternative Government Policies

Up to now, the analysis has focused on the implications of market anticipations of a particular type of government gold policy, a single auction. In this section, the effects of market anticipations of three alternative gold policies are considered: a sequence of auctions, a price ceiling, and a pegged price.

It is worth considering the consequences of market anticipations of these alternative government policies for two reasons. First, since analysis of these cases involves the same methodology as that used in analyzing the case of anticipations of the announcement of a single auction, the general applicability of the approach is illustrated.[18] Second, it is conceivable that at some point in the future market participants might be given some reason to anticipate a government gold policy other than an auction. In addition, any analysis of the effects of anticipations of a policy requires a description of the effects of that policy when it is actually announced. Such a description provides a basis for understanding what happened in the gold market when the same policy or a similar policy was pursued in the past even if it had not been anticipated.

The approach used to analyze the effects of anticipations of each of the three alternative policies is essentially the same as that utilized in Sections II and III. In each case the first step in the analysis is to determine the initial price on the path which emerges if an announcement is actually made in any period that the government will undertake a particular action. It is shown for each policy how this price depends on the size of the private stock at the time of the announcement. In each case the time path of all variables could then be computed by replacing equation (3) with this new relationship, retaining the other equations of the model, and utilizing a backward solution as in Section II. For the three policies considered, the properties of the price path in anticipation of the policy announcement are described as they were in Section III for the case of a single auction.

To begin, consider the consequences of market anticipations of an announcement by the government that it will conduct two auctions at which $\bar{G}_0$ and $\bar{G}_1$ will be sold, respectively, where $\bar{G}_0 + \bar{G}_1 = \bar{G}$. Suppose it is known that the first auction will take place at the time of the announcement and that the second will take place $k + 1$ periods later. For any given private stock, the post-announcement price path will not depend on how the fixed total stock ($\bar{G}$) is allocated between the two separate auctions provided one condition is met. Assume first that the private stock is zero and let DD in figure 3, top, represent the post-announcement price path

[18] Two interesting examples of this approach have recently become available (see Long 1975 and Dasgupta and Stiglitz 1976).

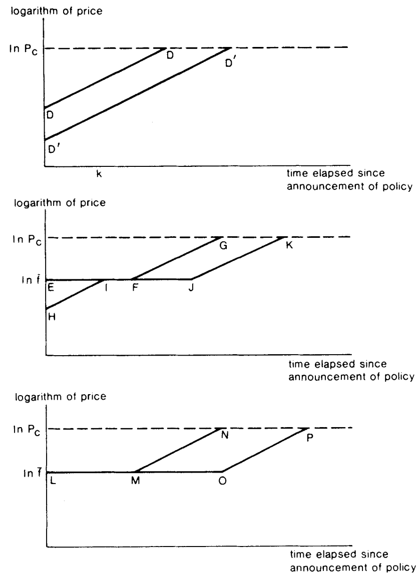

FIG. 3.—Post-announcement price paths associated with alternative government policies. Top, sequence of auctions. Middle, price ceiling. Bottom, price peg.

which would result if the entire government stock were sold in the first auction ($\bar{G}_0 = \bar{G}$). Now, suppose that $\bar{G}_0 < \bar{G}$. Provided the amount sold at the first auction, $\bar{G}_0$, equals or exceeds the cumulative consumption along DD for the $k + 1$ periods before the second auction occurs,[19] the post-announcement price path is still DD. Furthermore, as long as $\bar{G}_0$ meets this condition, that is, as long as a sufficient amount of gold is sold at the first auction, for any strictly positive private stock the post-announcement price path is the same no matter whether the government announces one auction or two.[20] In such cases, the first price on the post-announcement price path, f_t, is determined by equation (3), and the set of equations determining all the variables in each period is the same as in the case when a single auction is anticipated.

[19] If $\bar{G}_0$ is smaller than the cumulative consumption along DD for the $k + 1$ periods before the second auction occurs, the description of how to determine the post-announcement price path is considerably more involved. A complete analysis of the implications of market anticipations of the announcement of a pair of auctions when $\bar{G}_0$ takes on small values (including zero) is available from the authors on request. Assuming that $\bar{G}_0 = 0$ is equivalent to assuming that market participants anticipate that a single auction might be announced but that there will be a delay between the announcement of the auction and its execution.

[20] That is, as long as $\bar{G}_0$ meets the condition of the text, the post-announcement price path will have only a single segment with price growing at the rate of interest, for all positive private stocks. The alternative possibility, that such a path has a downward jump, can easily be ruled out.

Under such circumstances, therefore, the price path in anticipation of an announcement of a pair of auctions is *identical* with the case in which a single auction is anticipated. An analogous proposition can be established when a sequence of more than two auctions is anticipated.

Suppose instead that the market anticipates an announcement by the government that it will defend a real price ceiling, $\bar{f}$, by standing ready to sell an amount of previously acquired gold, $\bar{G}$. Two possible post-announcement price paths are shown in figure 3, middle. If the private stock at the time of the announcement is zero, the post-announcement price path is a path like *EFG*.[21] The initial price on the post-announcement price path, f_t, is equal to $\bar{f}$, and the government satisfies consumption demand at $\bar{f}$ for several periods as shown by *EF*. Eventually, private market participants realize that the remaining government stock can be exhausted along *FG*, a path which begins at $\bar{f}$ and rises at the rate of interest to the choke price.[22] Then the remaining government gold is purchased by market participants in a swift speculative "attack."[23] Consumption demand is satisfied from the newly acquired private stock.

If the private stock at the time of the announcement is strictly positive, the post-announcement price path is a path like *HIJK*. The first segment of the price path, *HI*, is a path which rises at the rate of interest and along which the entire private stock is just depleted by the time the price reaches $\bar{f}$. If a ceiling is announced, private market participants realize that when $\bar{f}$ is reached the government will satisfy demand for some time at that price and that they will receive no capital gains during this time. Since private market participants have no incentive to hold gold in the absence of capital gains, they attempt to sell gold, and the price drops to the first price on *HI*. The rest of the price path is *IJK* which is a lateral translation of *EFG*. For a while the government satisfies demand from its stock; then there is a speculative attack in which private market participants buy up the rest

[21] It is assumed in the text that the stock the government is willing to sell to defend the ceiling exceeds the cumulative demand along a price path beginning at the specified ceiling price and rising to the choke price at the rate of interest. Suppose, instead, that the government reserves were smaller:

$$\bar{G} < \sum_{x=0}^{\infty} D[\bar{f}(1 + r)^x].$$

In that case, the post-announcement price path would have no flat segment, and the initial price on that path would be determined by eq. (3). The price path in anticipation of an announcement that the government will defend a ceiling would then be indistinguishable from the price path in anticipation of an auction.

[22] Actually, the market price which prevails in the period in which the attack occurs may be anywhere in the closed interval between $\bar{f}$ and $\bar{f}(1 + r)$. Of course, if the market price lies above $\bar{f}$, those lucky enough to be able to purchase gold from the government make an immediate capital gain.

[23] The analysis implies that selling gold at a fixed real price results in a speculative attack. Of course, selling at a declining real price (or, equivalently, at a fixed nominal price in an inflationary world) makes the inevitable occur more quickly.

of the government stock. The larger the initial private stock, the lower the first price on the post-announcement price path, f_t, and the longer the time between the announcement and the time $\bar{f}$ is reached.[24] After $\bar{f}$ is reached the price path is the same no matter what the size of the private stock at the time of the announcement since only the government stock remains. Hence, by announcing that it is willing to defend a ceiling the government determines how much it will receive for its gold, but the timing of the required government sales and, therefore, the present value of the proceeds, is determined endogenously.

The properties of the price path which emerges in anticipation of an announced ceiling can now be summarized. At termination, when the private stock is zero, $P_T > f_T$ since $P_c > \bar{f}$. Using this condition, it can be demonstrated by repeating the proofs of Section III that in each period the price which emerges in the absence of an announcement is greater than the price which emerges if an announcement is made; that the first price on the post-announcement price path rises at less than the rate of interest; that the path of prices which emerges in the absence of an announcement rises at a rate greater than the rate of interest; and that this rate of increase itself increases over time. In short, the price path which emerges in anticipation of an announcement that a ceiling will be defended has properties similar to the corresponding price path when the event anticipated is an announcement of an auction.

Finally, consider the consequences of an announcement by the government that it will attempt to peg the gold price at a fixed real value, $\bar{f}$, by buying whatever gold is offered to it or by selling at that price whatever it has acquired plus its initial inventory of $\bar{G}$. Two possible post-announcement price paths are shown in figure 3, bottom. If the private stock at the time of the announcement is zero, the post-announcement price path is a path like LMN.[25] The government satisfies consumption demand for

[24] The first price on the post-announcement price path can be determined as follows:

$$A.\ \text{If } S_t = 0, \qquad f_t = \bar{f};$$

$$B.\ \text{If } S_t > 0, \qquad f_t \text{ solves } \sum_{x=0}^{\infty} E[f_t(1+r)^x] = S_t,$$

where $E[f_t(1+r)^x]$ is equal to $D[f_t(1+r)^x]$ when $f_t(1+r)^x < \bar{f}$ and is equal to zero when $f_t(1+r)^x \geq \bar{f}$; A and B determine the initial price if a ceiling is announced as an implicit function of the private stock and replace eq. (3) in the model of Secs. II and III.

[25] It is assumed in the text that the stock the government has on hand initially to peg the gold price exceeds the cumulative demand along a price path beginning at the specified peg and rising to the choke price at the rate of interest. Suppose, instead, that government reserves were smaller:

$$\bar{G} \leq \sum_{x=0}^{\infty} D[\bar{f}(1+r)^x].$$

Two situations might arise. First, it is possible that

$$\bar{G} + \bar{I} \leq \sum_{x=0}^{\infty} D[\bar{f}(1+r)^x].$$

several periods, and the price remains at $\bar{f}$ as shown by LM. After a time, private market participants realize that the remaining government stock can be exhausted along MN, a path which begins at $\bar{f}$ and rises at the rate of interest to the choke price. Then the remaining government gold is purchased by market participants in a speculative attack. Consumption demand is satisfied from the newly acquired private stock.

If the private stock at the time of the announcement is positive, the post-announcement price path is a path like LOP. Private market participants realize that the government can sustain $\bar{f}$ for a time and that they will receive no capital gains during this time. Since both speculators and zero-cost extractors are unwilling to hold gold when they earn no capital gains, they immediately sell their entire stocks to the government at $\bar{f}$.[26] The government alone then supplies gold to consumers at $\bar{f}$ for some time as shown by LO. Eventually, however, the augmented government stock is depleted to the point that it can be exhausted along OP, which is a lateral translation of MN, and the rest of the government gold is purchased in a

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In this situation, if a pegging policy is announced the speculators attack immediately. The initial price on the post-announcement price path in each period is the same as if the government had announced an auction of its stock. The price path in anticipation of a pegging announcement is therefore *identical* with the path in anticipation of an auction. Second, it is possible that

$$\bar{G} + \bar{I} > \sum_{x=0}^{\infty} D[\bar{f}(1 + r)^x].$$

In this situation, the price path in anticipation of a pegging announcement will have two phases. In the last phase, depletion will have reduced the private stock below a critical level, $\bar{I}^*$, where

$$\bar{I}^* = \sum_{x=0}^{\infty} D[\bar{f}(1 + r)^x] - \bar{G}.$$

During this last phase, the pegging model is indistinguishable from the auction model of Sec. II for the reason given in the preceding paragraph. Now, consider the first phase, during which the private stock is strictly greater than $\bar{I}^*$. At the end of this phase (T') the private stock is $\bar{I}^*$; the price if a pegging policy is announced is $f_{T'} = \bar{f}$; and the price in the absence of such an announcement is $P_{T'}$. Since the end of this phase is also the beginning of the phase known to be identical with the model of Sec. II, $P_{T'} > \bar{f}$. The price path in anticipation of a pegging announcement prior to T' can be computed by working backward from these conditions and noting that $f_t = \bar{f}$, $t \leq T'$. For private stocks larger than $\bar{I}^*$ the path in anticipation of a pegging announcement differs from the path in anticipation of an auction. It has each of the properties discussed in Sec. IV between the designation for n. 27 and the end of the section. To verify this claim it is sufficient merely to replace the terminal condition used there ($P_T > \bar{f}$) with the condition mentioned above ($P_{T'} > \bar{f}$ when $S_{T'} = \bar{I}^*$) and then to repeat the relevant arguments.

[26] With increasing marginal extraction costs, each mine extracts more gradually and sells to the government over time. Initially, extraction proceeds at a declining rate so that marginal profit (the fixed government buying price less marginal cost) rises at the rate of interest. During this phase, consumers buy gold either directly from mines or else from the government. Eventually, however, the remaining government gold stock begins to be worth more than the price charged by the government. At that moment, the entire government stock is purchased in a speculative attack. It can be shown that speculators and extractors then coexist in the market for some period of time with both supplying bullion to consumers. Finally, however, the speculators exhaust their supplies, and the mines alone supply consumers until demand is choked off.

speculative attack. The larger the private stock, the longer the post-announcement price path remains at $\bar{f}$ and the longer the time between the announcement and the attack. The first price on the post-announcement price path is always $\bar{f}$ no matter what the size of the private stock, that is, $f_t = \bar{f}$ for all t.[27] The final segment of the post-announcement price path always begins at $\bar{f}$ and rises to P_c at the rate of interest.

Since the first price on the post-announcement price path is the same, regardless of the private stock, it is not difficult to establish the properties of the price path which emerges in anticipation of the announcement. The arbitrage condition (eq. [4]) can be rearranged to yield

$$P_{t+1} = \frac{1 + r}{1 - \alpha} P_t - \frac{\alpha}{1 - \alpha} \bar{f}. \tag{9}$$

From this difference equation it follows that unless the initial price, P_0, strictly exceeds $[\alpha/(r + \alpha)]\bar{f}$ (which is less than $\bar{f}$), price will subsequently remain at this value or fall below it. However, at termination, the price must reach the choke which is strictly greater than $\bar{f}$. Hence, the initial price must strictly exceed $[\alpha/(r + \alpha)]\bar{f}$, and the price path in anticipation of a pegging announcement must rise monotonically. Equation (9) can be rearranged to yield

$$R_t = \frac{P_{t+1} - P_t}{P_t} = r + \frac{\alpha}{1 - \alpha}\left[(1 + r) - \frac{\bar{f}}{P_t}\right]. \tag{10}$$

The price path may have an initial phase in which the rate of increase in the price is smaller than the rate of interest since $[\alpha/(r + \alpha)]\bar{f} < P_t < [\bar{f}/(1 + r)]$ implies $0 < R_t < r$. The potential peg may be so high that if the policy were announced before private stocks were substantially depleted gold holders would experience a capital gain in anticipation of which they would willingly hold gold earning less than the rate of interest. But since the price must reach the choke level, it must eventually exceed $[\bar{f}/(1 + r)]$. Hence, there will always be a final phase in which the percentage rate of increase in price exceeds the rate of interest. Equation (10) also implies that the percentage rate of increase in price always increases over time when an announcement of a pegging policy is anticipated.

V. Concluding Remarks

This paper has focused on the gold market. But, in concluding, we would like to note its pertinence to other problems of current concern. The analysis of the previous section, for example, indicates that attempts to restrict the price of any exhaustible resource by means of a buffer stock will in-

[27] This is the equation determining the potential price if it is announced that the gold price will be pegged at $\bar{f}$ and replaces eq. (3) in the model of Secs. II and III.

evitably result in a speculative attack. In this sense, recent proposals for the international stabilization of the prices of certain metals seem destined to fail. The auction model of Sections II and III may be viewed as the market solution to the social problem of how best to deplete existing reserves when additional supplies are anticipated to arrive at an unknown time. From this vantage point, the model applies equally to the optimal utilization of a reservoir when rain is anticipated to relieve a drought and to the utilization of proved reserves when exploration is anticipated to result in new discoveries.

Finally, to put our analysis in perspective, it should be noted that anticipations can affect capital formation as well as resource depletion. As we have seen, anticipations of government sales at an unknown time do not merely depress the gold price but also—because that reduced price stimulates more rapid depletion—have other effects which our paper has examined. Symmetrically, anticipations that at some unknown time the government will sharply increase its demand for, say, housing services or for the skills of nuclear physicists do not merely raise the current worth of owning such assets but also—because that increased value stimulates more rapid *expansion* of their stocks—have additional consequences, among which are depressed rents or wages, prior to the anticipated increase in demand.

Appendix

Anticipations of a Government Auction: A Continuous Time Treatment

The anticipations model of Sections II and III may be recast in continuous time by retaining equations (1) and (2) and modifying (3)–(5):

$$\int_{x=0}^{\infty} D[f(t)e^{\hat{r}x}]dx = \bar{G} + S(t) \tag{3'}$$

$$\frac{\dot{P}(t)}{P(t)} = r + \hat{\alpha}\left[1 - \frac{f(t)}{P(t)}\right] \tag{4'}$$

$$\dot{S}(t) = -D[P(t)] \tag{5'}$$

where $\hat{\alpha}$ and $\hat{r}$ are the continuous time analogues of α and r and

$$\hat{\alpha} = -\ln(1-\alpha),$$
$$\hat{r} = \ln(1+r).$$

Equation (4′) may be derived in the following way. Holders of gold will force the current price to equal the discounted price now expected to prevail t moments in the future. This expected price is equal to the probability weighted average of the prices which would prevail at t if the auction occurred at different moments before t or if no auction occurred through t:

$$P(0)e^{\hat{r}t} = P(t)e^{-\hat{\alpha}t} + \int_{x=0}^{t} f(x)e^{\hat{r}(t-x)}\cdot\hat{\alpha}e^{-\hat{\alpha}x}\,dx.$$

Equation (4′) is obtained by differentiating this equation and simplifying the result.

The model can be solved most easily by utilizing the terminal conditions for P and S, working backward using the equations of motion (4′) and (5′), and stopping when S builds up to the initial inventory ($\bar{I}$). Alternatively, the initial price (P) can be determined by trial and error. For each initial price, the paths implied for P, f, and S can be deduced from equations (1) and (3′)-(5). Only one initial assignment of P will permit the terminal conditions (Z) to be satisfied.[28]

The properties of the model may be easily established. To prove that $f < P$ at every instant, assume the contrary ($f \geq P$, for some instant). From (4′), $\dot{P}/P \leq \hat{r}$. As will be shown below, (3′) implies that $\dot{f}/f = \{[\hat{r}D(P)]/[D(f)]\}$. Under the hypothesis that $f \geq P$, this would imply that $\dot{f}/f \geq \hat{r} \geq \dot{P}/P$. Hence, $f \geq P$ for all subsequent t. But since $f(T) < P(T)$, the consequence that $f \geq P$ for all subsequent t is inconsistent with the terminal conditions of the model. Hence the hypothesis ($f \geq P$ for some t), which generates that consequence, must be false.

Having established that $f < P$ for all t, we can infer immediately from (4′) that $\dot{P}/P > \hat{r}$. We will establish below that $\dot{f}/f < \hat{r}$. These two results and equation (4′) imply that $d/dt\{\dot{P}/P\} > 0$.

To establish that $\dot{f}/f = \hat{r}[D(P)/D(f)] < \hat{r}$, differentiate (3′) to obtain

$$\frac{df}{dS} = \frac{1}{\int_{x=0}^{\infty} e^{\hat{r}x} D'(fe^{\hat{r}x})\, dx} = -\frac{\hat{r}f}{D(f)},$$

use the chain rule to obtain $\dot{f}/f = 1/f \cdot (df/dS) \cdot \dot{S}$, and substitute the first result into the second equation.

To show conditions sufficient for the percentage increase in the potential auction price to decline over time, note that

$$\operatorname{sgn} \frac{d}{dt}\{\dot{f}/f\} = \operatorname{sgn} \frac{d}{dt}\{\ln \dot{f}/f\} = \operatorname{sgn} \frac{d}{dt}\{\ln D(P) - \ln D(f)\}$$
$$= \operatorname{sgn}\left\{\frac{\dot{f}}{f}\eta(f) - \frac{\dot{P}}{P}\eta(P)\right\},$$

where $\eta\,(\cdot)$ is absolute value of the point elasticity of demand. For all linear or concave demand curves and some convex demand curves, the magnitude of the demand elasticity rises with price. This is sufficient for the percentage increase in the auction price to decline over time as is illustrated in figures 2 and 4.

In this circumstance, a lower bound to the loss which the government would incur by postponing the auction can be derived. In figure 4, CC indicates the actual revenue per unit which the government would receive if it held the auction in x periods. On the other hand, CA indicates the revenue the government would receive if it sold its stock immediately and then let the proceeds earn interest for x periods. The difference represents the true loss from postponement. It is convenient to approximate the average revenue the government would receive from

[28] This latter approach can be used to verify that our results do not depend on the assumption that the demand curve has a choke price. If the demand curve were assumed to have no choke price, the terminal conditions (Z) would be replaced by $\lim_{t \to \infty} S_t = 0$. In this case, the trial and error procedure must be used to solve the model. Using this approach, it can again be shown (by contradiction) that $P_{(t)} > f_{(t)}$, for all t. Since all of the qualitative properties follow once this inequality is established, they continue to hold even if the demand curve is assumed to have no choke price.

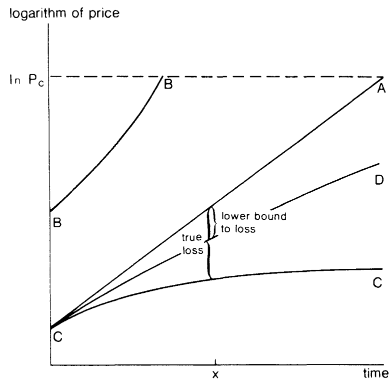

FIG. 4.—Lower bound on loss in government revenue from postponement of auction x periods.

an auction in x periods by assuming the potential auction price continues to grow in percentage terms at a constant rather than a declining rate. The approximation, which is represented by CD—the tangent to CC at C—overestimates the proceeds of the government and, therefore, underestimates its loss from postponing the auction for x periods.

A formula for this underestimate of the true loss can be computed as follows. If an auction occurred at t, the government would receive $f(t)$ per unit sold. If the auction occurred x years later, its discounted average revenue would be only $f(t + x)e^{-rx}$. Since $\dot{f}/f = D(P)/D(f) \cdot \hat{r}$, while, under the circumstances described above,

$$\frac{d}{dt}(\dot{f}/f) < 0, f(t + x)e^{-\hat{r}x} < f(t)e\{D[P(t)]/D[f(t)] - 1\}\hat{r}x,$$

the real discounted loss for a delay of length x is, therefore, equal to

$$\frac{f(t) - f(t + x)e^{-\hat{r}x}}{f(t)} > 1 - e\{D[P(t)]/D[f(t)] - 1\}\hat{r}x.$$

If, for example, demand at the current high price were 60 percent of the rate which would prevail if the auction were announced today and if the real rate of interest were 5 percent per year, then the postponement of the auction of a given amount for 10 years would cause a loss in real discounted revenue in excess of 18 percent.

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