Regulating Experience Goods When Consumers Cannot Identify Their Source[1]

Preliminary: Please Do Not Quote

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Abstract

We consider markets where consumers cannot discern a product's quality prior to purchase and can never identify the firm which produced the good. This characterizes many fruits and vegetables and an increasing number of imports from countries with unfamiliar alphabets. We assume producers of such goods choose the quantity to sell and a distribution of quality within the chosen quantity. When the constant per unit cost of production is strictly convex in quality, we show that firms would choose to produce only a single quality. If identical firms simultaneously choose quality and quantity to maximize profits, a Nash equilibrium is shown to exist in pure strategies and it is symmetric and interior. We demonstrate that under quite general conditions minimum quality standards can improve welfare. When the final product is assembled from components manufactured in-house or purchased on competitive markets by each Cournot competitor, minimum quality standards can be imposed on the final product or, alternatively, on one or more of the components of that product. We compare the positive and normative effects of each policy.

1 Introduction

In many markets, consumers cannot evaluate the quality of a good before buying it. How can the the pleasure of consuming a French brie or a California navel orange be judged without consuming them? How can the feel of a Bic ball-point pen be determined without writing with it? Consumers assess the quality of such goods by purchasing them repeatedly and eventually learn to anticipate their quality.

When the quality of a good cannot be discerned prior to purchase, we call it an "experience good." But there are really *two* distinct classes of experience goods. In the first, the consumer knows the identity of the producer. In the second, it is either impossible or too costly for the consumer to identify the producer. A Bic ballpoint pen belongs to the first class of experience goods while a French brie or a California navel orange belongs to the second class—which farm produced either product is not apparent.

Although little attention has been paid in the IO literature to this second class of goods, they are becoming the subject of countless news stories. At this moment, U.S. beef is the subject of massive protests in South Korea because of fears that it will expose Koreans to mad cow disease. Meanwhile, U.S. consumers—having experienced Chinese toothpaste, cold medicines, toys, and other products containing lead, antifreeze, and other poisons—are developing an aversion to the "Made in China" label.

When goods are imported from distant countries, possibly with different languages or alphabets, identifying the producer of a particular product may be too costly for a consumer. But not all experience goods in this second class are imports. Who knows which particular apple orchard produced a given Washington apple or what orange grove produced a particular California navel? In some circumstances, output from different firms are pooled together *before* their quality is assessed. Olmstead and Rhode (2003) describe the case of cotton grown in the South in the early twentieth century. Partly because of the high cost of testing quality, sometimes only samples of cotton pooled from many growers were tested. This prevented an individual farmer from increasing his own specific reputation. A similar situation recently confronted tomato growers. Although tomatoes are grown on separate farms, they are pooled together for washing and processing. In the recent salmonella outbreak, it was thought that the contamination originated after the tomatoes from the various individual growers were pooled. In such cases, an individual grower has no way to distinguish the quality of his produce from that of his competitors.

Some goods in this second class of experience goods have many ingredients or components. In such cases, the quality of the final good is a function of the quality of each underlying component. These components may be produced in-house or purchased on competitive markets. Producers of this second class of experience goods are in a difficult situation. They know the quality of the goods they produce and the components from which they are made. But they realize that there is no way to distinguish the quality of *their* product from the quality of the other products lumped together in the consumer's mind. They share a "collective reputation." Not surprisingly, a producer does not have as much incentive to make a product of high quality as he would if consumers distinguished his products from those of his competitors.

In markets exhibiting collective reputation, high-quality production can be difficult to maintain. Akerlof's famous lemons paper (1970) can be interpreted as a characterization of the dangers of collective reputation in the used-car market. To prevent this degradation of quality, minimum quality standards are often imposed as a form of quality (and consumer) protection. Concern for health and safety first prompted Congressional action in the 1880s and 1890s on issues ranging from "pickles dyed with copper salts" to "trichinosis in beef."[1] In 1891, worried by the declining reputation of American meat abroad, Congress sent inspectors to all slaughterhouses preparing meat for export. Throughout the early Twentieth Century, policymakers struggled to create meaningful minimum quality standards. In 1938 Congress passed the Food, Drug, and Cosmetics Act, legislative action which strengthened the regulatory power of the government. By the mid-Sixties, nearly half of all food products were regulated in some way. Today the USDA offers fee-based voluntary grading of nearly all fruits and vegetable. The USDA also enforces mandatory quality standards in 26 industries, regulating over four billion dollars in business annually.

Perhaps because there has been little discussion of this second class of experience goods, such attempts to impose minimum quality standards are often interpreted by the antitrust authorities (rightly or wrongly) as attempts to cartelize the market. For example, the Department of Justice made the following comment about the plan of the Federal agricultural marketing board governing Michigan tart cherries to impose minimum quality standards: "For the reasons set forth in these comments, DOJ also opposes those provisions in the proposed marketing order providing for minimum quality standards for tart cherries. We oppose those provisions because they also appear to establish volume controls; that is, they appear to give the administrative board the discretion to vary quality standards from year to year in response

[1] Gardner (2004). The following historical summary draws heavily from Gardner's work, a must for those interested in the USDA's employment of minimum quality standards.

to the size of the cherry crop." The response of the Department of Justice raises an interesting set of questions. In an oligopoly, must the imposition of a minimum quality standard result in smaller aggregate output and lower welfare?

Experience goods such as cell phones, computers, etc. are often composed of different ingredients or components. This leaves policy makers a choice. They can impose minimum quality standards on the final product or on one (or more) of the underlying components. Our analysis permits a comparison of the positive and normative effects of each policy. We find that imposing a quality standard on a few components of a multicomponent product (or a few processes of a multiprocess product) will typically induce the regulated firm to *reduce* the quality of the unregulated components or processes. We explore the generality of this conclusion.

Klein-Leffler (1981) and Shapiro (1983) considered markets for experience goods of the first type. In their formulations, the quality of products cannot be distinguished prior to purchase, but consumers can identify the producer of each product. In the steady state of Shapiro's model, identical firms exhibit heterogeneous behavior with some specialized to one quality and receiving one price and other firms specialized to different qualities and receiving different prices. In these formulations one firm's quality choice has no effect on another firm's future reputation for quality.

In contrast, the recent IO literature on "collective reputations" investigates the situation where consumers do not distinguish the products of different agents (be they firms or farms)—such as U.S. tomatoes, Washington apples, Chinese toys, and so forth. As a result, all firms sell at a common price and share a common reputation for quality. The first paper in this literature was Winfree and McCluskey (2005).[2] In their model, quantity is exogenous, entry is prohibited, and firms are constrained to offer a single quality. Under these assumptions, Winfree and McCluskey show that when firms share a collective reputation they have an incentive in the steady state of their dynamic model to free-ride and to produce low quality goods. In a recent working paper, Rouvière and Soubeyran (2008) retain the assumption of a fixed output and the constraint that firms choose a single quality but permit entry. They show that free-riding causes entry into the market to be suboptimal.

[2]Tirole (1996) coined the term "collective reputation" and was the first to analyze the phenomenon. However, his focus was not on the strategic interaction of firms but on the reputations of workers. In Tirole's formulation, an agent's own past behavior is imperfectly observed and he is assessed not only on the basis of his own past actions but also on the collective past actions of others. In contrast, past behavior of our firms are not observed and they have no individual reputations, only a collective one.

Our paper is closest to a working paper by Fleckinger (2007). Both our papers, although static, build on Winfree and McCluskey (2005) by endogenizing the quantity decision. But since the two studies were developed independently they are otherwise based on different assumptions. Fleckinger, for example, constrains each firm to a single quality whereas firms in our model are free to produce multiple qualities. Fleckinger assumes that inverse demand is multiplicatively separable and in addition that one of the multiplicative factors is linear. We work with a much wider class of inverse demand functions and consider the implications of this abstraction. Finally, unlike Fleckinger, we consider products composed of multiple components. The quality of the final product depends on the quality of each component. After demonstrating that the quality of such goods in the unregulated equilibrium is suboptimal, we compare the positive and normative effects of imposing minimum quality standards on the final good to the effects of imposing standards on one or more of its components.

We proceed as follows. In the next section, we present a generalized model of Cournot oligopoly in which each profit-maximizing firm chooses both quantity and quality. We characterize the equilibrium of the game and investigate the positive and normative effects of minimum quality and quantity standards. Section 3 extends the analysis to the case where the quality of each final good derives from the quality of the components from which it is made. We compare a minimum quality standard imposed on the quality of the final good to a standard imposed instead on one or more of the underlying components. Section 4 concludes the paper.

2 Model of Oligopoly

2.1 A Preliminary Simplification

Suppose n firms simultaneously choose how to distribute their production of a good over the quality set $K = [0, \infty)$. We define the strategy of a firm i to be the tuple $\xi_i = (\mu_i, Q_i)$, where μ_i is a probability measure on the Borel algebra of K and $Q_i \in [0, \bar{Q}]$ is the total output of firm i.[3] Note that mass points are allowed, i.e. a firm i can produce a single certain quality with positive probability. Let $Q(\mu_1, ..., \mu_n) = Q_1(\mu_1) + \cdots + Q_n(\mu_n)$ be the total output in the market. The per unit cost of producing a quality k is given by $c(k) : K \to [0, \infty)$, which is strictly increasing away from 0 and strictly

[3] We define the Borel algebra of K to be the minimal σ-algebra containing the closed interval subsets of K.

convex. Given a probability measure μ_i, the total cost bill of firm i is:

$$C(\mu_i, Q_i) = Q_i \int_K c(k) d\mu_i \tag{1}$$

where here we use the Lebesgue integral.

We finally define the collective reputation in the market by:

$$R(\mu_1, ..., \mu_n) = \sum_{i=1}^{n} \frac{Q_i}{Q} \int_K k d\mu_i. \tag{2}$$

We assume consumers are unable to discern the individual quality of any firm's good prior to consumption and can never determine which firm produced the good. Rather, consumers only observe the collective reputation of the experience good. Inverse demand is given by the function $P(Q, R)$: $[0, \bar{Q}] \times K \to [0, \infty)$. Given a strategy profile $\xi = (\xi_1, ..., \xi_n)$ of the n firms, the profit of a firm i is given by:

$$\Pi_i(\xi_i, \xi_{-i}) = Q_i[P(Q, R) - \int_K c(k) d\mu_i], \tag{3}$$

where Q and R are as given above.

This formulation is a substantial departure from the previous literature on collective reputation. Winfree and McCluskey (2005), Fleckinger (2007), and others arbitrarily restrict each firm to a *single* quality level. The possibility therefore remains that Nash equilibria where firms produce *several* qualities at once have been ruled out by assumption. In addition, the Nash equilibria which have been identified in the literature may be artifacts of the restriction that no firm can offer multiple qualities. That is, once that restriction is relaxed, a firm may have a profitable deviation from the strategy profile identified in the literature as a Nash equilibrium.

The prior literature assumes strict convexity of the cost function and this permits us to establish the following surprising result.

Theorem 2.1 *Given convex cost functions, there can be no Nash equilibrium of the game where some active firm i ($Q_i > 0$) chooses a probability measure that assigns mass to more than one point.*

Thus, the restriction invoked in these papers does not limit the generality of their results.

Proof Suppose the contrary—that we have a Nash equilibrium strategy profile $\xi = (\xi_1, ..., \xi_n)$ in which a firm i chooses an output $Q_i > 0$ and a probability measure μ_i which does not consist of a single mass point. Figure 1 will be helpful in understanding the result. Let $\bar{k}_i = \int_K c(k) d\mu_i$. Let us consider the the strategy $\xi_i^\star = (\mu_i^\star, Q_i)$ of firm i, where $\mu_i^\star(\{\bar{k}_i\}) = 1$ and vanishes otherwise. Suppose the strategy profile of the other firms, denoted by ξ_{-i}, is unchanged. It is clear that the total output and the collective reputation in the market does not change. Thus, the price firm i receives does not change as well. However, since the per unit cost function is strictly convex in quality, by Jensen's inequality we have that $c(\bar{k}_i) < \int_K c(k) d\mu_i$. Thus firm i's cost bill is lower, which implies profits are higher. Therefore, firm i has a profitable unilateral deviation and ξ fails to constitute a Nash equilibrium. ∎

By a similar argument, no Nash equilibrium in which firms are constrained to choose a single quality would be eliminated if they were allowed instead to produce multiple qualities. This follows since, by Jensen's inequality, a unilateral deviation to multiple qualities is always strictly less profitable than a deviation to a single-quality profile with the same mean and, by hypothesis, no such single-quality deviation is profitable.

2.2 Existence, Characterization, and Uniqueness of Nash Equilibrium

Given the previous discussion, we now assume without loss of generality that each firm chooses to produce a single quality. We now also assume that inverse demand is twice-differentiable, $P_1(Q, R) < 0$, $P_2(Q, R) > 0$, and $P_{11}(Q, R), P_{22}(Q, R) < 0$ for all $R \geq 0$ and $0 \leq Q < \bar{Q}$, where $\bar{Q}$ is the horizontal intercept of the family of inverse demand curves: $P(\bar{Q}, R) = 0$ for all R. Moreover, we suppose that $P(Q, 0) > 0$ and $P_2(Q, R) \to 0$ as $R \to \infty$ for all $Q < \bar{Q}$. Inverse demand depends now on the aggregate quantity of goods sold and on their quality averaged across the firms, which we again refer to as their collective reputation (denoted R). More precisely, collective reputation is now the quantity-weighted average of the qualities the n firms sell:

$$R = \frac{q_i k_i}{q_i + Q_{-i}} + \frac{\sum_{j \neq i} q_j k_j}{q_i + Q_{-i}}. \tag{4}$$

The variables q and k in (4) denote quantity and quality respectively; the subscripts i and $-i$ refer respectively to firm i and to every other firm; $Q_{-i} = \sum_{j \neq i} q_j$ and $k_{-i} = (k_j)_{j \neq i}$. The right-hand side of (4) is undefined when $q_i = Q_{-i} = 0$. As for the per unit cost function, we continue to assume that $c(k)$ is strictly increasing away from 0 and strictly convex. We moreover assume that it is twice differentiable and $c'(0) = c''(0) = 0$.

Consider the game where each firm simultaneously chooses its output and quality to maximize the following payoff function:

$$q_i[P(q_i + Q_{-i}, R(k_i, q_i, k_{-i}, Q_{-i})) - c(k_i)]. \tag{5}$$

We define firm i's profit whenever it is inactive ($q_i = 0$) as zero. This assignment never conflicts with (5) since that equation, evaluated at $q_i = 0$, gives the same result when $Q_{-i} > 0$ and is undefined when $Q_{-i} = 0$.

Since firm i maximizes profits, its decisions must satisfy the following pair of complementary slackness (denoted c.s.) conditions for $Q_{-i} > 0$:

$$q_i \geq 0,\ P(Q, R) - c(k_i) + q_i P_1(Q, R) + q_i P_2(Q, R)\frac{\partial R}{\partial q_i} \leq 0,\ \text{c.s.} \tag{6}$$

$$k_i \geq 0,\ q_i[P_2(Q, R)\frac{\partial R}{\partial k_i} - c'(k_i)] \leq 0,\ \text{c.s.} \tag{7}$$

where, from (4), $\frac{\partial R}{\partial k_i} = q_i/Q$ and $\frac{\partial R}{\partial q_i} = (k_i - R)/Q$.

The first three terms of equation (6) are standard. They reflect the excess of the marginal gain from selling another unit over the marginal cost of producing it. The last term, $q_i P_2(Q, R)\frac{\partial R}{\partial q_i}$, is novel and captures an additional consequence (positive or negative) of expanding output marginally. If firm i's quality is greater than the reputation in the market, then increasing output will increase the collective reputation of the good, which will raise price. On the other hand, if firm i's quality is below average, then increasing output will decrease price. The intuition for equation (7) is straightforward. If the firm produces no output, any quality is optimal. If the firm is active ($q_i > 0$) and quality is set optimally, then a marginal increase in quality would raise the revenue from sales of the goods by as much as it raises their cost of production.

We suppose that the profit function is pseudoconcave so that when the first-order conditions are satisfied, a global maximum has been achieved. We seek to establish the following:

Theorem 2.2 *There exists one or more pure-strategy equilibria and they are interior and symmetric.*

We proceed as follows. We first demonstrate that there exists at least one symmetric Nash equilibrium of the game. We then show that any Nash equilibrium (symmetric or otherwise) of the game must be interior. Finally, we show that there can be no asymmetric Nash equilibria.

Lemma 2.3 *There exists a symmetric equilibrium.*

Proof Assume each firm chooses the same pair (q, k). Given this symmetry, we can write the Kuhn-Tucker conditions as:

$$q \geq 0,\ \ P(Q,k) + qP_1(Q,k) - c(k) \leq 0,\ \text{ c.s.} \tag{8}$$

$$q = Q/n, \tag{9}$$

and

$$k \geq 0,\ \ q[P_2(Q,k) - nc'(k)] \leq 0,\ \text{ c.s.} \tag{10}$$

where $R = k$.

Now to begin, let $\hat{k}(Q)$ be the unique solution to the equation $P(Q,k) = c(k)$ given Q. We know that there exists a unique solution for all $Q < \bar{Q}$ since then $P(Q,0) > 0$, $c(0) = 0$, inverse demand is weakly concave in quality, and the cost function is strictly convex in quality. Clearly, $\hat{k}(\bar{Q}) = 0$ Moreover, we see that:

$$\frac{d\hat{k}}{dQ} = \frac{P_1(Q,\hat{k})}{-[P_2(Q,\hat{k}) - c'(\hat{k})]}. \tag{11}$$

But for all $Q < \bar{Q}$, $P(Q,\hat{k}) - c(\hat{k})$ is strictly decreasing at $(Q,\hat{k})$, which implies that $P_2(Q,\hat{k}) - c'(\hat{k}) < 0$. Since inverse demand is strictly decreasing in Q, we conclude that $\frac{d\hat{k}}{dQ} < 0$.

Next, we consider equations (8) and (9). Given a particular quality k, we would like to study the solution to these complementary slackness conditions, which we denote by $(\breve{q}(k), \breve{Q}(k))$. Note that if $k \geq \hat{k}(0)$, then $P(Q,k) - c(k) \leq 0$ for all Q and the solution to (8) and (9) is $\breve{Q}(k) = \breve{q}(k) = 0$. Now suppose that $k < \hat{k}(0)$. Then $P(0,k) - c(k) > 0$. Moreover, by totally differentiating equation (8) with respect to Q, we find that $\frac{dq}{dQ} = -\frac{P_1(Q,k) + qP_{11}(Q,k)}{P_1(Q,k)} < 0$. Finally, we note that $P(\bar{Q},k) = 0$ for all k and so when $Q = \bar{Q}$, the solution to (8) is $q = 0$. So consider the function $f(Q) = nq(Q)$ where $q(Q)$ is the solution to (8) given Q. We are looking for a fixed point of this function. We know that this function is strictly decreasing in Q, $f(0) > 0$, and $f(\bar{Q}) = 0$. This implies that the curve will cross the 45^o line at some $Q < \bar{Q}$. Then $(Q/n, Q)$ defines the desired solution. We conclude that we have solution curves $\breve{Q}(k)$ and $\breve{q}(k)$ which are defined for $0 \leq k \leq \hat{k}(0)$. Since $P_1(Q,k)(1 + \frac{1}{n}) + \frac{Q}{n}P_{11}(Q,k) < 0$ for all (Q,k), it follows from the implicit function theorem that these curves are smooth over this interval.

Now given $0 \leq Q < \bar{Q}$, let us consider the solution curve $\breve{k}(Q)$ to the equation (10). If the firm is inactive $(q = 0)$, then any quality choice solves equation (10). However, we will assume (without, as we will show, loss of generality) that $\breve{k}(0) = \lim_{Q \to 0} \breve{k}(Q)$. Thus, $\breve{k}(Q)$ is continuous at $Q = 0$. Note that $P_{22}(Q,k) - nc''(k) < 0$, so by the implicit function theorem in any neighborhood of a solution to the equation, the solution curve is smooth. Next, note that for any $Q \in (0, \bar{Q})$, the left hand side of (10) is (1) strictly positive at $k = 0$ since $P_2(Q, 0) > 0$ and $c(0) = 0$; (2) strictly negative for $k \to \infty$; and (3) continuous. Hence, there is at least one root. Moreover, since the left-hand side is strictly decreasing in k, there is exactly one root for any $Q \in (0, \bar{Q})$. Therefore, $\breve{k}(Q)$ is a well-defined positive function for all $Q \in [0, \bar{Q})$. Finally, note that at the point $(Q, \hat{k}(Q)$, the cost curve must be steeper than the price curve since the cost function is strictly increasing and strictly convex away from 0, while inverse demand is strictly increasing and concave in quality. In particular, this means that $\breve{k}(Q) < \hat{k}(Q) < \hat{k}(0)$ for all $0 \leq Q < \bar{Q}$.

Finally, let $T(k) = \breve{k}(\breve{Q}(k))$. Since $\breve{Q}(k)$ takes values in $[0, \bar{Q})$, this is a continuous mapping from the nonempty, convex, compact set $[0, \hat{k}(0)]$ to $[0, \hat{k}(0)]$. Therefore, by Brouwer's theorem there exists at least one fixed point $k^\star$ to this mapping and an associated image $Q^\star = \breve{Q}(k^\star)$. Pseudoconcavity of the profit functions implies that each firm's strategy $(Q^\star/n, k^\star)$ is profit-maximizing and hence the profile of these strategies forms a Nash equilibrium. ∎

The following two lemmas demonstrate that any Nash equilibrium must be interior.

Lemma 2.4 *There can be no equilibrium (symmetric or otherwise) in which a firm produces a positive quantity of the minimum quality.*

Proof If it is optimal to produce at minimum quality ($k_i = 0$), then since $c'(0) = 0$ condition (7) requires that $P_2(Q, R)\frac{q_i}{Q} \leq 0$. But each of the factors to the left of this inequality is strictly positive since, by hypothesis, $q_i > 0$. So the inequality can never hold. Therefore, every firm with $q_i > 0$ must have $k_i > 0$. ∎

Intuitively, since the cost function is flat at the origin but inverse demand is strictly increasing in quality, an active firm producing a minimal quality can always increase his profit by marginally increasing his quality choice. At the margin, costs will remain the same but the price will increase.

Lemma 2.5 *There can be no equilibrium (symmetric or otherwise) in which some firm is inactive.*

Proof First, suppose that all the other firms produce nothing ($Q_{-i} = 0$). If firm i also produces nothing, it would earn a payoff of zero. But this cannot be optimal. For, by unilaterally setting $k_i = 0$ and $q_i > 0$, its payoff would increase to $q_i P(q_i, 0) > 0$. So there can be no Nash equilibrium with $q_i = 0$ when $Q_{-i} = 0$. [4]

Nor can there be a Nash equilibrium with $q_i = 0$ when $Q_{-i} > 0$. In that case, one or more of the rival firms is producing a strictly positive amount. Label as firm j the active firm with the smallest quality. Hence, $k_j - R \leq 0$. Since firm j produces a strictly positive amount, its first-order condition in (6) must hold with equality. Since the terms $q_j P_1(Q, R)$ and $q_j P_2(Q, R)(k_j - R)/Q$ are respectively strictly and weakly negative, (6) implies $P(Q, R) - c(k_j) > 0$. But since the cost function is strictly increasing, $P(Q, R) - c(0) > 0$ and this same complementary slackness condition (6), which must hold for firm i as well, implies that $q_i > 0$, contradicting the hypothesis that $q_i = 0$. ∎

From the two lemmas above, we conclude that the symmetric equilibria found previously must be interior. We conclude the proof of the theorem by demonstrating that in no equilibrium do any two firms choose different qualities or different quantities.

Lemma 2.6 *There exist no pure strategy asymmetric Nash equilibria.*

Proof Since any equilibrium must be interior, the pair of first-order conditions for each firm must hold with equality. These imply that the following equation must hold in any equilibrium:

$$[k_i - (R - \frac{QP_1(Q, R)}{P_2(Q, R)})]c'(k_i) + \{P(Q, R) - c(k_i)\} = 0. \tag{12}$$

Define the left-hand side as $\Gamma(k_i; Q, R)$. For each equilibrium, there may be a different (Q, R). But given those aggregate variables, every firm $(i = 1, \ldots, n)$ will have $\Gamma(k_i; Q, R) = 0$. However, this equation cannot have more than one root. We see $\frac{\partial \Gamma}{\partial k_i}(k_i; Q, R) = [k_i - (R - \frac{QP_1(Q,R)}{P_2(Q,R)})]c''(k_i)$ and (12) requires that at any root, the first factor in $\frac{\partial \Gamma}{\partial k_i}(k_i; Q, R)$ must be strictly negative.[5]

Hence, there can be no more than one root. So, for any given equilibrium with its (Q, R) a unique k_i satisfies equation (12). But since equation (12) must hold for *every* firm, each firm must choose the same quality in this equilibrium. Denote it $k(Q, R)$. Moreover, since every firm will be active

[4] Our discarding of all but one value for $\breve{k}(0)$ when a firm is inactive could have resulted in the elimination of additional fixed points. However, since every fixed point is a Nash equilibrium and we have just established from first principles that there are no Nash equilibria with any firm inactive, in fact we could not have eliminated any equilibria.

[5] The term in braces in (12) must be strictly positive since $q_i > 0$ (from Lemma 2.5), condition (8) therefore holds with equality, and $P_1 < 0$ by assumption.

and reputed quality will equal $R = k(Q, R)$, equation (6) implies that $q_i = \frac{P(Q,R)-c(k(Q,R)}{-P_1(Q,R)}$. Since the right-hand side of this equation is independent of i, every firm will produce the same quantity in this equilibrium. Denote it $q(Q, R)$. $\blacksquare$

One may also wonder about uniqueness of Cournot-Nash equilibrium. If one assumes that inverse demand is additively separable ($P_{12} = 0$), then $\breve{k}(Q)$ becomes a horizontal line. Since $\breve{Q}(k)$ is a well-defined function, there then can only be one point of intersection, i.e. a unique equilibrium. We conjecture that if P_{12} is small (but nonzero) then one can also prove uniqueness. More generally, if we were to place certain restrictions on the signs of cross partial third derivatives we would also be able to get uniqueness. However, it is important to recognize that the comparative statics and welfare considerations of the following sections hold at any equilibrium of the game, even if there is more than one.

2.3 Regulation and Welfare

We now consider the economic welfare that is achieved under such equilibria and, more importantly, the role regulations can play in increasing welfare. We define economic welfare by:

$$W(Q, k) = \int_0^Q P(u, k)du - Qc(k). \tag{13}$$

A social planner seeking to maximize economic welfare would choose Q and k to solve:

$$P(Q, k) = c(k) \tag{14}$$

$$\int_0^Q P_2(u, k)du = Qc'(k). \tag{15}$$

Note that if additive separability is assumed ($P_{12} = 0$), equation (15) reduces to $P_2(Q, k) = c'(k)$. Since the left-hand side is independent of aggregate sales (Q) and strictly decreases in k while the right-hand side is strictly increasing in k, this equation uniquely defines the socially optimal quality.

Recall that in the market setting, every firm sets its quality in the Nash equilibrium to solve $P_2(Q, k) = nc'(k)$. This implies that under additive separability, the equilibrium quality level emerging in the market setting will be socially optimal under monopoly and suboptimal under oligopoly. It is, therefore, likely that the discrete change from monopoly to duopoly will be welfare improving: the induced deterioration in quality is apt to have a small adverse effect on welfare (negligible if n could be increased marginally), while the expansion in total output will likely have a more substantial, positive effect on welfare. However, as the market becomes more and more concentrated, the continued deterioration in quality may in fact induce each firm to cut back on output to such an extent that total market output falls. If this occurs, then increased firm entry may in fact lower welfare. This has important policy implications. If the market is highly concentrated, then it may be optimal for the government to undertake measures to increase competition. However, after a point increased competition is likely to decrease total economic welfare.

2.3.1 Positive and Normative Effects of Minimum Quality Standards

There are a variety of instruments available to regulators seeking to improve economic welfare. We begin by considering the effect of an exogenous binding minimum quality standard which we denote by $\bar{k}$. Note first that when a standard marginally raises quality, the marginal impact on welfare can be calculated as:

$$\frac{dW}{d\bar{k}} = \frac{dQ}{d\bar{k}}\{P(Q,\bar{k}) - c(\bar{k})\} + [\int_0^Q P_2(u,\bar{k})du - Qc'(\bar{k})]. \tag{16}$$

Welfare changes because (1) industry output will change (the first term) and (2), even if industry output didn't change, welfare would change because of the resulting changes in the gross surplus and costs of producing the old output at a marginally enhanced quality. To calculate the marginal impact on quantity we totally differentiate equation (8) to find:

$$\frac{dQ}{d\bar{k}} = \frac{P_2(Q,\bar{k}) - c'(\bar{k}) + \frac{Q}{n}P_{12}(Q,\bar{k})}{-[P_1(Q,\bar{k})(1+\frac{1}{n}) + \frac{Q}{n}P_{11}(Q,\bar{k})]}. \tag{17}$$

Since inverse demand is strictly decreasing and weakly concave in output, the denominator is strictly positive. The sign of the numerator is ambiguous however. If output is marginally increased, the total marginal effect of the standard on price is given by $P_2(Q,\bar{k}) + \frac{Q}{n}P_{12}(Q,\bar{k})$, while the increase in cost is simply $c'(\bar{k})$. The sign of the numerator depends on the signs and magnitudes of these respective terms. Given these two equations we have the following results:

Theorem 2.7 *If inverse demand is additively separable, for any binding minimum quality standard below the social optimum, a marginal increase will raise aggregate output and welfare. For any binding minimum quality standard above the social optimum, a marginal increase will decrease aggregate output and welfare.*

Given additive separability, we recall that the equilibrium quality equals the social optimum under monopoly and is strictly less than the social optimum for less concentrated market structures. This implies that minimum quality standards are only beneficial when there is more than one firm in the market.

Proof Since inverse demand is additively separable, we have that $P_{12} = 0$. Let $\tilde{k}$ denote the socially optimal quality. That is, $\tilde{k}$ solves the equation $P_2(Q, k) = c'(k)$. So for $\bar{k} < \tilde{k}$ we have $P_2(Q, \bar{k}) > c'(\bar{k})$ and the opposite for $\bar{k} > \tilde{k}$. Looking at the numerator of equation (17), we therefore find if the standard is below the social optimum, a marginal increase will raise output, while for a standard above the social optimum, a marginal increase will lower output.

Turning now to equation (16), the term in braces, $P(Q, \bar{k}) - c(\bar{k})$, is always strictly positive since $P(Q, \bar{k}) - c(\bar{k}) = -\frac{Q}{n}P_1(Q, \bar{k}) > 0$ for $\bar{k}$ either below or above the social optimum. Finally, by additive separability the last two terms can be rewritten as $Q[P_2(Q, \bar{k}) - c'(\bar{k})]$. Again, this expression is strictly positive for a standard below the social optimum and strictly negative for a standard above the social optimum. We conclude that welfare is strictly increasing in binding standards below the social optimum and strictly decreasing in binding standards above the social optimum. ∎

We have shown that given the assumption of additive separability, minimum quality standards can improve welfare under oligopoly market settings. It turns out that minimum quality standards can improve welfare when there is more than one firm under considerably more general inverse demand functions. To begin, we need some notation. Given unregulated equilibrium values $(Q^\star, k^\star)$, let $\zeta(u) = \frac{P_2(Q^\star, k^\star)}{Q^\star}u$. That is, $\zeta(u)$ is the line emanating from the origin and passing through the point $(Q^\star, P_2(Q^\star, k^\star))$.

Theorem 2.8 *Suppose that $n \geq 2$, $P_{12} \geq 0$, and the unregulated equilibrium admits values $(Q^\star, k^\star)$. If $\int_0^{Q^\star}[P_2(u, k^\star) - \zeta(u)]du > 0$, then a minimum quality standard which just binds will marginally increase welfare.*

Proof Since $P_2(Q, k) = nc'(k)$ in equilibrium and the MQS just binds, i.e. $\bar{k} = k^\star$, we can reduce equation (17) to:

$$\frac{dQ}{d\bar{k}} = \frac{P_2(Q^\star, k^\star)(1 - \frac{1}{n}) + \frac{Q^\star}{n}P_{12}(Q^\star, k^\star)}{-[P_1(Q^\star, k^\star)(1 + \frac{1}{n}) + \frac{Q^\star}{n}P_{11}(Q^\star, k^\star)]}. \tag{18}$$

Since $P_{12} \geq 0$, it follows that $\frac{dQ}{dk} \geq 0$ for all $n \geq 1$. To complete the proof, we simply note that:

$$\int_0^{Q^\star} P_2(u, k^\star)du > \int_0^{Q^\star} \zeta(u)du = \frac{Q^\star}{2}P_2(Q^\star, k^\star) \geq \frac{Q^\star}{n}P_2(Q^\star, k^\star) = Q^\star c'(k^\star). \tag{19}$$

Thus, by equation (16), welfare marginally increases. ∎

At first glance, the bound on the integral $\int_0^{Q^\star} P_2(u,k^\star)du$ may appear quite restrictive since it incorporates the output and quality level of the unregulated equilibrium. However, there is a large class of inverse demand functions which will satisfy this assumption automatically. Indeed, if P_2 is constant, linearly increasing, or increasing and strictly concave in Q, the assumption will be satisfied. The function could also be strictly convex in certain regions as well, as long as it spends a sufficient amount of "time" above the ray defined by $\zeta(u)$.

Rather than regulating quality, the government may instead seek to regulate quantity, especially under monopoly which yields the socially optimal quality but less than the socially optimal output. If inverse demand is additively separable, firms make their quality decisions independent of their quantity decisions. Thus, regulators may create incentives to change a firm's production without altering the quality choices of the firm. Therefore, given additively separable inverse demand, optimal minimum quantity regulations or production subsidies can maximize social welfare under monopoly and increase welfare under oligopoly.

2.3.2 Department of Justice Criticism of MQS

Recall the objection of the Department of Justice to quality standards on agricultural produce such as Michigan cherries. Presumably, the Department disregarded the suboptimal quality which arises when consumers cannot trace a basket of cherries to the farm which produced it. In our model of collective reputation, however, quality in the absence of regulation is below the social optimum because no firm can distinguish itself from the others. We have shown that under general assumptions, a minimum quality standard imposed on an oligopoly can raise both output and welfare. Intuitively, quality is below the social optimum and the regulation increases it. In addition, consumers want more of the product at any given price because of its enhanced quality and firms are motivated to expand output. Hence, the quality standard raises both quality and aggregate output. Under additive separability and other more general conditions, it also increases welfare.

If consumers do not care about quality, however, it is possible to explain the Department of Justice's reasoning. Suppose that $P_2(Q,R) = 0$ for all (Q,R) and maintain all other assumptions.[6] We first claim that there now exists a unique Nash equilibrium which is symmetric and in which each firm produces a positive amount of the minimum quality. To see this, first note

[6] Since $P_2 = 0$ it follows that $P_{12} = 0$ as well.

that there can be no equilibrium in which any active firm sets its quality above the minimum ($k = 0$). This follows since such a firm could strictly reduce its costs while preserving its gross revenue by dropping its quality to the minimum. Hence, the only candidates for equilibria are strategy profiles where every active firm produces at the lowest quality level. Standard treatments in the Cournot oligopoly literature establish that there exists a unique symmetric equilibrium when every firm has the identical constant marginal cost ($c(0)$ in our case). In equilibrium, every firm produces $q_i = Q^*/n$, where Q^* solves: $P(Q, 0) + \frac{Q}{n}P_1(Q, 0) - c(0) = 0$. To verify that the strategy profile $\{k_i = 0, q_i = Q^*/n\}$ for $i = 1, \ldots, n$ forms a Nash equilibrium when firms choose quality as well as quantity, we must check that no firm could profitably deviate by raising quality above the minimum and altering quantity at the same time. Since consumers do not value quality, such a deviation affects price only because it alters total output. Since increases in quality raise costs, the firm can do better by simply changing output and keeping quality at its minimum level; but, as the standard proofs establish, no such deviation is ever profitable. Therefore, the strategy profile in which each firm produces an output $Q^\star/n$ of minimum quality forms a Nash equilibrium.

In terms of welfare, by studying equations (16) and (17), it is clear that since $P_2 = P_{12} = 0$, we have $\frac{dQ}{d\bar{k}} \leq 0$ and $\frac{dW}{d\bar{k}} \leq 0$ with equality holding only at $\bar{k} = 0$. Thus, minimum quality standards only serve to lower both output and economic welfare, which is exactly what the Department of Justice forewarns.

What about the effects of the minimum quality standards on profits? Minimum quality standards raise per unit costs of every firm. Seade (1985) identified a necessary and sufficient condition for a cost increase to raise the profit of every firm in an industry of Cournot competitors. Kotchen and Salant (2008) demonstrate that this condition is equivalent to local convexity of the total revenue function. If this condition holds and an equilibrium exists, firms would have an incentive to press for minimum quality standards even if consumers cared nothing about quality. If consumers do value quality, firms benefit from minimum quality standards not merely because of the price increase induced by the reduction in aggregate output but because of the increase in price induced by the improvement in quality. Assuming that a Nash equilibrium exists in our model under the Seade condition, that condition would be sufficient (but no longer necessary) for firms to profit from minimum quality standards.

3 Multiple Component Products

In many instances, the good consumers buy is composed of a number of different parts (or, alternatively, is produced using a number of different processes). The quality of the final good is then typically a function of the quality of each component. Suppose a good sold by n firms is composed of m components. Assume some of these components are produced in-house while others are purchased on competitive markets at prices we assume are exogenous. Alternatively, assume some processes are performed in-house while others are contracted out at competitive prices. We assume that any quality is available for purchase. Each firm is assumed to know the quality of the components it is producing or purchasing. The per unit cost of producing a component j, identical across all firms, is given by $\sigma_j(\kappa_j) : [0, \infty) \to [0, \infty)$, a twice differentiable, strictly convex function which is strictly increasing away from 0. Assume that $\sigma_j(0) = \sigma'_j(0) = 0$ for all j. The j^{th} function also gives the prices at which the continuum of qualities of component j can be purchased on competitive markets. Thus, $\sigma_j(\kappa_j)$ generally gives the per unit cost a component j at quality κ_j facing producers of the final good and this cost is identical across all producers.[7]

The quality vector $(\kappa_1, ..., \kappa_m)$ of the components determines the quality of the final good. Below we establish that given relatively mild assumptions, we can recover the cost functions of the previous section. Therefore, all of the results of the previous section regarding the final good will continue to hold true. For instance, under oligopoly market conditions and additively separable inverse demand, the quality of the final good will be socially suboptimal. We then compare the imposition of minimum quality standards on the final good to their imposition on a proper subset of the underlying components. We find that imposing a minimum quality standard on a proper subset of the processes or components will typically induce firms to degrade the quality of the other processes or components. In the next subsection we explore the generality of this conclusion. We provide an extreme example where firms do not respond to the regulations in this way; however, we illustrate that given a sufficiently stringent regulation, firms will still seek to undercut the regulator's efforts, although in a different fashion.

[7]Note these costs are independent of how any particular firm chooses to vertically integrate its production.

3.1 How Firms Respond to Minimum Quality Standards on a Subset of Components: the Standard Case

To begin, assume that the quality of the final good is given by $f(\kappa_1, ..., \kappa_m)$, where $f : [0, \infty) \to [0, \infty)$ which is additively separable.[8] Given the assumption of additive separability, we can write $f(\kappa_1, ..., \kappa_m) = f_1(\kappa_1) + \cdots + f_m(\kappa_m)$. We assume for all j that $f_j(\kappa_j)$ is strictly increasing and weakly concave; therefore, $f(\kappa_1, ..., \kappa_m)$ inherits these properties. We finally assume that $f_j(0) = 0$ for all j. Given the cost of enhancing the quality of each component, firms set their qualities so that the final good has a given quality (or better) and is produced at least cost. Since the cost of achieving any given quality for any of the m components is the same across firms, each firm achieves a given minimum quality k of the final good at the same cost. Hence, we omit the firm subscript on the minimum cost function and denote it simply as $c(k)$. To derive $c(k)$, we formulate the following constrained optimization problem. For any quality k of the final good, a firm solves:

$$c(k) = \min_{(\kappa_1,...,\kappa_m)\geq 0} \sum_{j=1}^{m} \sigma_j(\kappa_j) \text{ subject to } k \leq f(\kappa_1, ..., \kappa_m). \tag{20}$$

We have the following result:

Theorem 3.1 *Suppose that f is twice continuously differentiable. For any k, the optimization problem above has a unique solution $(\kappa_1^\star(k), ..., \kappa_m^\star(k))$. The function $c(k) = \sigma_1(\kappa_1^\star(k)) + \cdots + \sigma_m(\kappa_m^\star(k))$ is then twice differentiable, strictly increasing away from 0, and strictly convex. It satisfies $c(0) = c'(0) = 0$.*

Proof See the appendix.

Since the minimized cost function $c(k)$ satisfies all of the assumptions in Section 2, the firms described in that section could have been producing goods comprised of multiple components. In particular, there is a symmetric equilibrium in which the quality k of the final product solves $P_2(Q, k) = nc'(k)$. To express $c'(k)$ in terms of the underlying costs of the components, we must investigate the solution to the cost minimization problem.

The firm will set the quality of every component above the minimum level since increasing marginally the quality of any component in the neighborhood

[8]Additive separability is not used in Theorem 3.1 but the subsequent analysis relies on this assumption.

of its minimum quality raises its price but does not increase its per unit cost. The optimum will, therefore, be interior. When component j has quality κ_j, the increase in its quality necessary to increase the quality of the final good by one unit is $\frac{1}{f_j'(\kappa_j)}$. Hence, the marginal cost of increasing the final good quality by one unit by altering any single component j is $\frac{1}{f_j'(\kappa_j)}\sigma_j'(\kappa_j)$. Suppose $\frac{\sigma_j'(\kappa_j)}{f_j'(\kappa_j)} < \frac{\sigma_l'(\kappa_l)}{f_l'(\kappa_l)}$ for any pair of components. Then marginally increasing the quality of component j by enough to raise the final good quality by one unit and marginally decreasing the quality of component l by enough to lower the final good quality by one unit would yield the same final good quality at a lower cost. Evidently, when the cost of achieving a given quality of the final good is minimized, the m marginal costs must be equated. That is, for any two components $j \neq l$ we must have $\frac{\sigma_j'(\kappa_j)}{f_j'(\kappa_j)} = \frac{\sigma_l'(\kappa_l)}{f_l'(\kappa_l)}$.

Figure 2 depicts the solution of this cost-minimization problem graphically. The firm sets the quality of each component so that every marginal cost equals the height (λ) of the horizontal line. The horizontal line in turn is adjusted so that the final good quality is k. If the exogenous target quality (k) is increased to *any* level, the cost-minimizing way to achieve the new target involves enhancing the quality of every component in such a way that marginal costs remain equal but at a higher level. If the exogenous quality target is marginally increased by one unit, then the cost per unit increases at the rate $\frac{\sigma_j'(\kappa_j)}{f_j'(\kappa_j)}$. That is, $c'(k) = \frac{\sigma_j'(\kappa_j)}{f_j'(\kappa_j)}$ for all j.[9]

Let us now assume additively separable inverse demand. We will maintain this assumption henceforth. Since in a Nash equilibrium $P_2(Q,k) = nc'(k)$, we have for each firm,

$$P_2(Q,k)f_j'(\kappa_j) = n\sigma_j'(\kappa_j), \text{ for } j = 1, \ldots, m \tag{21}$$

where $k = \sum_{j=1}^{m} f_j(\kappa_j)$. These $m+1$ equations define the equilibrium quality levels of the m components and the overall quality chosen by each firm. Equation (21) can be interpreted as follows. The marginal cost of increasing the quality of component j by one unit is $\sigma_j'(\kappa_j)$. Because enhancing by one unit the quality of one of its m components raises the quality of its final good by $f_j'(\kappa_j)$ units and because this firm's output is $1/n^{th}$ of industry output, the firm's enhancement of the quality of its one component raises the collective reputation by $f_j'(\kappa_j)/n$ units. For each unit increase in the collective reputation of the quality of the industry's goods, the price per unit received by every seller rises by $P_2(Q,k)$. So by raising by one unit the quality of one component, the firm raises its per unit gross revenue by

[9] As discussed in the Appendix, this is a consequence of the envelope theorem.

$P_2(Q,k)f_j'(\kappa_j)/n$. Equation (21), therefore, simply says that every firm sets the quality of every component so that the marginal benefit of enhancing quality matches the marginal cost of enhancing quality.

3.1.1 Minimum Quality Standards on Components

If a minimum quality standard on the final good exceeds the equilibrium quality of the final good in the absence of regulation, then firms will respond by enhancing the quality of every component. If the standard is set at the level a social planner would select, every firm would choose component qualities to minimize the cost of achieving that standard and in the process would duplicate the planner's quality choices for each component.

Suppose instead that the regulator imposed the minimum quality standard on a *single* component. This might *seem* superficially attractive in cases where the same component is used in several products for which the regulator is responsible. We will see, however, that such a policy creates difficulties. Moreover, our analysis suggests that these difficulties will persist if the standard is imposed on any proper subset of the components.

Let a binding minimum quality standard on a *single* component (denoted $\bar{\kappa}_l$) compel each firm to raise its quality above the unregulated equilibrium level κ_l. Assume provisionally that firms do not adjust the quality of any of the other $m-1$ components. Then the quality of the final good produced by every firm would improve and this would depress $P_2(Q,k)$ without shifting $\sigma_j'(\kappa_j)$ for $j \neq l$. Let $\tilde{k}$ denote the new provisional quality of the good. As a result, $P_2(Q,\tilde{k})f_j'(\kappa_j) < n\sigma_j'(\kappa_j)$ for $j \neq l$. But this inequality implies that each firm can profitably alter its behavior in response. By reducing the quality of any component (never to its lower bound),[10] each firm could reduce its costs per unit by more than it would reduce its revenue per unit; consequently it would increase its profits. As a result, the unit increase in the quality standard on component l would induce the firm to degrade the quality of every unregulated component. As a result, the quality of the final good would increase by less than $f_l(\bar{\kappa}_l) - f_l(\kappa_l)$.

The regulator can impose a quality standard on a single component to achieve the quality level the planner would select for the final good. But

[10] The reduction in the quality of each of the $m-1$ components is never so severe that a firm would set one of them at its lower bound. For, at the lower bound, the marginal cost of quality enhancement is zero while, by assumption ($P_2(Q,R) > 0$ for all Q, R), the marginal benefit of quality enhancement is always strictly positive.

firms will not achieve this quality at least cost.[11]

This tendency of the firms to offset partially the efforts of the regulator to improve the suboptimal quality of the final good is a robust result. It continues to hold as long as a proper *subset* of the m components is regulated by a minimum quality standard. Each firm will reduce the quality of every unregulated component in response to the forced quality improvements of the other components. Hence, whatever overall quality is achieved will not be achieved at least cost.

3.2 An Example without Partial Offsets

Suppose instead that the quality of the final good is determined by the quality of its shoddiest component:

$$k = \min(\kappa_1, ..., \kappa_m). \tag{22}$$

Each firm i now solves the constrained maximization problem:

$$c(k) = \min_{(\kappa_1,...,\kappa_m)\geq 0} \sum_{j=1}^{m} \sigma_j(\kappa_j) \text{ subject to } k \leq \min(\kappa_1, ..., \kappa_m). \tag{23}$$

[11]These conclusions can be verified more formally. Consider an MQS ($\bar{\kappa}_l$) binding on the unregulated equilibrium quality of component l. From equation (21) and the equilibrium relation $k = f_l(\bar{\kappa}_l) + \sum_{j\neq l} f_j(\kappa_j)$, we obtain the following system of equations:

$$\frac{dk}{d\bar{\kappa}_l} = \sum_{j=1}^{m} f_j'(\kappa_j)\frac{d\kappa_j}{d\bar{\kappa}_l} = f_l'(\bar{\kappa}_l) + \sum_{j\neq l} f_j'(\kappa_j)\frac{d\kappa_j}{d\bar{\kappa}_l}$$

$$f_j''(\kappa_j)P_2(Q,k)\frac{d\kappa_j}{d\bar{\kappa}_l} + f_j'(\kappa_j)P_{22}(Q,k)\frac{dk}{d\bar{\kappa}_l} = n\sigma_j''(\kappa_j)\frac{d\kappa_j}{d\bar{\kappa}_l} \text{ for } j \neq l,$$

where we recognize that $\frac{d\bar{\kappa}_l}{d\bar{\kappa}_l} = 1$. We then solve:

$$\begin{aligned} \frac{dk}{d\bar{\kappa}_l} &= [f_l'(\bar{\kappa}_l) + \sum_{j\neq l} \frac{(f_j'(\kappa_j))^2 P_{22}(Q,k)\frac{dk}{d\bar{\kappa}_l}}{n\sigma_j''(\kappa_j) - f_j''(\kappa_j)P_2(Q,k)}] \\ \Rightarrow \frac{dk}{d\bar{\kappa}_l} &= \frac{f_l'(\bar{\kappa}_l)}{1 - \sum_{j\neq l} \frac{(f_j'(\kappa_j))^2 P_{22}(Q,k)}{n\sigma_j''(\kappa_j) - f_j''(\kappa_j)P_2(Q,k)}}. \end{aligned}$$

Since inverse demand is strictly increasing and strictly concave in quality, while each component's quality aggregation function is strictly increasing and strictly concave and its cost function is strictly increasing and strictly convex, we conclude that $0 < \frac{dk}{d\bar{\kappa}_l} < f_l'(\bar{\kappa}_l)$. This then implies that $\frac{d\kappa_j}{d\bar{\kappa}_l} < 0$. This implies that when an MQS is imposed on a single component, the firms will respond by adjusting the qualities of the *other* components downward. However, the overall quality of the final product will increase.

Theorem 3.2 *For any k, the optimization problem above has a unique solution The function $c(k)$ is twice differentiable, strictly increasing away from 0, and strictly convex. It satisfies $c(0) = c'(0) = 0$.*

Proof It is clear that the firm must choose $\kappa_j \geq k$ for all $j = 1, ..., m$. Since the cost function of each component is strictly increasing in the quality of the component, it follows that in fact each of these conditions must simultaneously hold as equalities. Thus the cost function of the final good is simply given by:

$$c(k) = \sum_{j=1}^{m} \sigma_j(k). \tag{24}$$

That $c(k)$ is twice-differentiable, strictly increasing away from 0, strictly convex, with $c(0) = c'(0) = 0$ follows since it inherits each of these properties from the underlying component cost functions $\sigma_j(\cdot)$. ∎

Theorem 3.2 parallels Theorem 3.1 and shows that our analysis in Section 2 remains applicable even if each oligopolist produced a good composed of m components the shoddiest of which determines the quality of the final good. If such firms compete, there is again a symmetric equilibrium admitting a quality k of the final product which solves the equation $P_2(Q, k) = nc'(k)$. Differentiating (24), we can express $c'(k)$ in terms of the marginal costs of the components: $c'(k) = \sum_{j=1}^{m} \sigma_j'(k)$. The equilibrium quality of the final good solves:

$$P_2(Q, k) = n \sum_{j=1}^{m} \sigma_j'(k). \tag{25}$$

The equilibrium quality of each component satisfies $\kappa_j = k$ for all j. The intuition underlying (25) is straightforward: in equilibrium, the marginal cost each firm would incur if it raised the quality of the final good ($\sum_{j=1}^{m} \sigma_j'(k)$) equals the marginal benefit of raising that quality ($P_2(Q, k)/n$).

3.2.1 Minimum Quality Standards on Components

While firms will typically partially undermine the effort of the regulator to improve product quality, this need not always occur. To illustrate the limits of the effect we have been investigating, consider the case where the quality of a product is determined by the quality of its shoddiest component: $k = \min(\kappa_1, \ldots, \kappa_m)$. Let $k^\star$ be the quality of the final product in the unregulated equilibrium. When a binding minimum quality standard is placed on the final product, firms will respond by raising the quality level of each component to the level specified by the standard. For, that is the most profitable way to comply with the regulation. Hence, once again the regulator can achieve the quality a planner would choose if he can impose a standard on the quality of the final good.

Suppose instead the government imposes a binding minimum quality standard on a *single* component, which we denote $\bar{\kappa}_l > k^\star$ for $1 \leq l \leq m$. Note that given this MQS, the cost function facing a firm is given by:

$$\bar{c}(k, \bar{\kappa}_l) = \begin{cases} \sum_{j \neq l} \sigma_j(k) + \sigma_l(\bar{\kappa}_l) \text{ if } k \leq \bar{\kappa}_l \\ \sum_{j=1}^{m} \sigma_j(k) \text{ if } k > \bar{\kappa}_l. \end{cases} \tag{26}$$

If a cost minimizing firm wants to produce the final good at a quality level less than or equal to that specified by the standard, it will set the quality of each of the $m-1$ unregulated components to this level and set the quality of the m^{th} component to the level required by the MQS. However, if the firm wishes to produce the final good at a quality level greater than that specified by the standard, it must set the quality of each of the m components to that level. Thus, there is a kink in the cost function at $\bar{\kappa}_l$.

$$\frac{\partial \bar{c}}{\partial k}(k, \bar{\kappa}_l) = \begin{cases} \sum_{j \neq l} \sigma_j'(k) \text{ if } k < \bar{\kappa}_l \\ \sum_{j=1}^{m} \sigma_j'(k) \text{ if } k > \bar{\kappa}_l, \end{cases} \tag{27}$$

In Figure 3, we depict the increasing but discontinuous marginal cost curve as a function of the quality of the final good (k). The marginal cost jumps from the lower branch to the upper branch at $k = \bar{\kappa}_l$. Both branches are relevant because, intuitively, raising the quality of the good above the standard imposed on the l^{th} component requires increasing the quality of all m components while reducing the quality of the good below $\bar{\kappa}_l$ involves reducing the quality of the $m-1$ unregulated components.

In Figure 3, we therefore derived the lower portion of the marginal cost curve by summing the marginal costs of the $m-1$ unregulated components when each is set at quality k and the upward portion by adding to that sum the marginal cost of the m^{th} component when its quality is also set at k. In this way, the upward portion coincides exactly with the marginal cost function in the absence of any regulation, while the lower portion lies strictly below it. That is, for $k < \bar{\kappa}_l$, we have $\frac{\partial \bar{c}}{\partial k}(k, \bar{\kappa}_l) < c'(k)$, where $c'(k)$ is the cost function in the absence of an MQS. Similarly, for $k > \bar{\kappa}_l$, we have $\frac{\partial \bar{c}}{\partial k}(k, \bar{\kappa}_l) = c'(k)$.

The downward-sloping curve in Figure 3 is the marginal benefit curve, $P_2(Q, k)/n$. It intersects the upper branch of the marginal cost curve (or its extension) once at $k = k^*$ and its lower branch (or its extension) once at $k = k^0$. That is, k^0 is the solution to the equation $\frac{P_2(Q,k)}{n} = \sum_{j \neq l} \sigma_j'(k)$ while k^* solves $\frac{P_2(Q,k)}{n} = \sum_{j=1}^{m} \sigma_j'(k)$.

It is clear from Figure 3 that $k^\star < k^0$. Now suppose that $\bar{\kappa}_l \in (k^\star, k^0)$. Then $P_2(Q,k)/n$ will pass through the discontinuity in $\frac{\partial \bar{c}}{\partial k}(k, \bar{\kappa}_l)$.[12] If the standard is marginally increased, the discontinuity occurs at a higher k. Therefore, the firm will raise the quality of the $m-1$ unregulated components up to the level of the regulated component. For any quality lower than this level, it is clear that the marginal benefit of raising quality (P_2/n) is strictly greater than the marginal cost. However, for any quality above the level set by the standard, the marginal benefit of lowering quality is strictly greater than the marginal cost of lowering quality. See Figure 3. If the quality standard on component l is sufficiently stringent, however, the marginal cost of enhancing the quality of the $m-1$ unregulated components further would no longer be smaller than the marginal benefit of further increases (P_2/n) and each firm would *refuse* to enhance further the quality of unregulated components in response to increases by the regulator in $\bar{\kappa}_l$. This occurs if $\bar{\kappa}_l > k^0$. When such a stringent quality standard is imposed on component l, the firm no longer provides the quality of the final good at least cost.

To summarize, there is a limit to the effectiveness of minimum quality standards imposed on a single component or, by extension, on a proper subset of the components. Namely, using such devices regulators can increase the quality of a good up to k^0 but no further.

3.3 Welfare Consequences of Regulating Component Quality Instead of Final Good Quality

We would finally like to understand the impact minimum quality standards on components have on welfare in both the standard case and the case with no partial offsets. We first recognize that in the standard case, at least at the margin, such regulations can be welfare improving. We have the following result:

Theorem 3.3 *In the neighborhood of the unregulated equilibrium, a minimum quality standard on a single component will be welfare improving if* $n > 1$ *and welfare neutral if* $n = 1$.

Proof The proof follows the same lines of Theorem 2.8 and exploits the fact that $\frac{dk}{d\bar{\kappa}_l} > 0$. See Appendix B.

Furthermore, in the extreme case without partial offsets, within its range of effectiveness a minimum quality standard on a single component can achieve

[12] A slight modification on the proof of existence of Cournot-Nash equilibrium demonstrates that the quality set by each firm in equilibrium will be $\bar{\kappa}_l$.

the same effects as a minimum quality standard on the final product. Firms will set the same quality level, produce the same output, incur the same costs and achieve the same welfare regardless of what type of standard regulators choose. Accordingly, we can use Theorem 2.7 to understand the conditions under which a minimum quality standard on a single component will be welfare improving. We conclude that in both cases, regulating the quality of a minimum quality standard can be welfare improving.

However, in both the standard case and after k^0 for the extreme case with no partial offsets, each firm resists the efforts of the regulator and, as a result, the quality of the final good is not achieved at least cost. The consequences for welfare may be dire. Under additive separability ($P_{12} = 0$), aggregate output does not influence the quality choice of any firm; nevertheless, as equation (8) reflects, the cost of a firm's quality choice directly influences its chosen output. If $P(0)$ is sufficiently small, the unnecessarily high cost per unit induced by imposing a standard on a single component may cause each firm to produce little output or to cease producing altogether.[13] Hence, continued increases in the minimum quality standard, even those below the level specified by the social optimum, may in fact lower welfare.

4 Conclusion

This paper has developed a general model of collective reputation in which firms operating in an oligopolistic market choose quantity and quality levels to maximize profit. We have reached a number of conclusions:

(1) Given the assumptions of convex cost curves, we have proven that each firm will produce only a single quality.

(2) There exists a Nash equilibrium to the game and it is interior and symmetric.

(3) Given additively separable inverse demand, the socially optimal quality level is achieved under monopoly. Firm entry will increase welfare in the neighborhood of monopoly. However, as the market becomes less concentrated, the continued deterioration in quality caused by free-riding may actually lower total market output, thus decreasing economic welfare.

[13] The existence of such an equilibrium does not contradict Lemma 2.5 since that result holds if and only if firms are unregulated. If a firm is unregulated, it can always drop its per unit cost to zero by producing goods of minimum quality. If a firm is regulated, however, it must incur the cost per unit of meeting the quality standard on each item it produces. If it is too costly to meet the minimum quality standard on the single component, therefore, each firm may find zero production to be its best option.

(4) Under additive separability, minimum quality standards improve economic welfare under oligopoly market settings and can achieve the social optimum. Such standards can also increase welfare given oligopoly under significantly more general inverse demand settings.

We also considered the case where each firm's product is composed of components produced in-house or purchased on competitive markets. In this case, the quality of the final good is an increasing function of the quality of the underlying components. We concluded:

(5) Regulating the quality of one or more components, although welfare-improving up to a point, may reduce or eliminate social surplus before the quality of the final good reaches the social optimum.

A Proof of Theorem 3.1

The objective function in (20) is continuous since it is the sum of continuous functions. Since the objective function is strictly convex and the constraint is convex (by the quasiconcavity of f), we know that the optimization problem has at most one solution. The constraint set can be rewritten as an equality since, as should be evident, the optimum can never occur when the inequality is strict. If the constraint holds as an equality, the constraint set is compact. Therefore, the Weierstrass Extreme Value Theorem guarantees that the constrained optimization problem has at least one solution. These two observations jointly imply that the optimization problem has exactly one solution, which we now investigate. Since the linear independence constraint qualification is satisfied, the Kuhn-Tucker conditions must be satisfied at the optimum. Given that both the constraint and the objective function are differentiable, the following Kuhn-Tucker conditions must hold:

$$\kappa_j \geq 0,\ \sigma_j'(\kappa_j) - \lambda f_j'(\kappa_j) \geq 0, \text{ with c.s. for all } j = 1, ..., m \tag{28}$$

$$\lambda \geq 0,\ k - f_j(\kappa_1, ..., \kappa_m) \leq 0 \text{ with c.s.} \tag{29}$$

For $k = 0$, it is clear that the constraint is satisfied and the total cost minimized by choosing $\kappa_j = 0$ for all j since $\sigma_j(0) = 0$ for all j. Now suppose $k > 0$. We will show that $\lambda > 0$ and consequently the quality of every component exceeds its minimum and is set so that marginal costs of enhancing quality are equalized. Suppose instead that $\lambda = 0$. Since $k > 0$, $f(\kappa_1, ..., \kappa_m) \geq k > 0$, it follows at least one $\kappa_j > 0$ since $f_j(0) = 0$, $f'(\kappa_j) > 0$ for all κ_j for all j. The marginal cost associated with the quality of that component is strictly positive ($\sigma_j'(\kappa_j) > 0$) since the function is strictly increasing and passes through the origin. But this contradicts the Kuhn-Tucker conditions since if $\kappa_j > 0$ then (28) requires that $\sigma_j'(\kappa_j) = 0$. We conclude that $\lambda > 0$. Condition (28) then implies that $\kappa_j > 0$ for $j = 1, \ldots, m$.

Now for any k, let $(\kappa_1^\star(k), ..., \kappa_m^\star(k))$ be the solution to the optimization problem characterized above. We define $c(k) = \sigma_1(\kappa_1^\star(k)) + \cdots + \sigma_m(\kappa_m^\star(k))$. It is clear by the discussion in the previous paragraph that $c(0) = 0$. Additionally, by the envelope theorem (Mas-Colell et. al. Theorem M.L.1) we have $c'(k) = \lambda^\star$. In particular, $c'(0) = 0$ and $c'(k) > 0$ for $k > 0$. By the implicit function theorem, which holds in neighborhood of the optimum induced by k, we have that $c''(k) = d\lambda/dk$ exists for all k.

It remains to be shown that $c(k)$ is strictly convex. Assume that when the firm wants at least quality k the m-tuple of quality minimizing costs is κ; and when the firm wants at least quality $\hat{k} \neq k$ the minimizing vector is $\hat{\kappa}$. Hence, $k \leq f(\kappa)$ and $\hat{k} \leq f(\hat{\kappa})$. Suppose the firm wants at least quality $tk + (1-t)\hat{k}$ for $t \in (0,1)$. We wish to show that the minimum cost, $c(tk+(1-t)\hat{k}) < tc(k)+(1-t)c(\hat{k})$. Consider the quality vector $t\kappa+(1-t)\hat{\kappa}$. We note that $f(t\kappa + (1-t)\hat{\kappa}) \geq tf(\kappa) + (1-t)f(\hat{\kappa}) \geq k$. So while it may not be optimal, the m-tuple $t\kappa + (1-t)\hat{\kappa}$ is feasible when $tk + (1-t)\hat{k}$ is to be achieved. Now

$$\begin{aligned} tc(k) + (1-t)c(\hat{k}) &= t\sum_{j=1}^{m} \sigma_j(\kappa_j) + (1-t)\sum_{j=1}^{m} \sigma_j(\hat{\kappa}_j) \\ &= \sum_{j=1}^{m} [t\sigma_j(\kappa_j) + (1-t)\sigma_j(\hat{\kappa}_j)] \\ &> \sum_{j=1}^{m} \sigma_j(t\kappa_j + (1-t)\hat{\kappa}_j) \\ &\geq c(tk + (1-t)\hat{k}). \end{aligned}$$

The strict inequality follows from the strict convexity of the cost function of each component and the weak inequality follows because the cost-minimizing quality vector may differ from feasible quality vector we have been considering. This completes the proof. ∎

B Proof of Theorem 3.3

Recall that the unregulated equilibrium is specified by the following equations:

$$P(Q^\star, k^\star) - c(k^\star) + \frac{Q^\star}{n} P_1(Q^\star, k^\star) = 0 \tag{30}$$

$$P_2(Q^\star, k^\star) f_j'(\kappa_j^\star) = n\sigma_j'(\kappa_j^\star), \text{ for } j = 1, \ldots, m \tag{31}$$

$$k^\star = f_1(\kappa_1^\star) + \cdots + f_m(\kappa_m^\star). \tag{32}$$

Now let us suppose that we set an MQS $\bar{\kappa}_l$ on component l. Let $\bar{c}(k, \bar{\kappa}_l)$ denote the new cost function. We know then that $\bar{c}(k^\star, \kappa_l^\star) = c(k^\star)$. Moreover, it is clear that:

$$\frac{\partial \bar{c}}{\partial \bar{\kappa}_l}(k^\star, \kappa_l^\star) = \sum_{j=1}^{m} \sigma_j'(\kappa_j^\star) \frac{d\kappa_j}{d\bar{\kappa}_l} = \sigma_l'(\kappa_l^\star) + \sum_{j \neq l} \sigma_j'(\kappa_j^\star) \frac{d\kappa_j}{d\bar{\kappa}_l}. \tag{33}$$

Welfare under the regulation is given by:

$$W = \int_0^Q P(Q, k) - Q\bar{c}(k, \bar{\kappa}_l). \tag{34}$$

In particular, when the MQS just binds we find that:

$$\frac{dW}{d\bar{\kappa}_l} = \frac{dQ}{d\bar{\kappa}_l}\{P(Q^\star, k^\star) - c(k^\star)\} + [\int_0^{Q^\star} P_2(u, k^\star)\frac{dk}{d\bar{\kappa}_l}du - Q^\star \sum_{j=1}^{m} \sigma_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l}]. \tag{35}$$

But we can then calculate:

$$\begin{aligned}
\frac{dQ}{d\bar{\kappa}_l} &= \frac{P_2(Q^\star, k^\star)\frac{dk}{d\bar{\kappa}_l} - \sum_{j=1}^{m} \sigma_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l} + \frac{Q^\star}{n}P_{12}(Q^\star, k^\star)\frac{dk}{d\bar{\kappa}_l}}{-[P_1(Q^\star, k^\star)(1+\frac{1}{n}) + \frac{Q^\star}{n}P_{11}(Q^\star, k^\star]} && (36)\\
&= \frac{P_2(Q^\star, k^\star)\frac{dk}{d\bar{\kappa}_l} - \frac{1}{n}P_2(Q^\star, k^\star)\sum_{j=1}^{m} f_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l}}{-[P_1(Q^\star, k^\star)(1+\frac{1}{n}) + \frac{Q^\star}{n}P_{11}(Q^\star, k^\star]} && (37)\\
&= \frac{dk}{d\bar{\kappa}_l}\frac{P_2(Q^\star, k^\star)(1-\frac{1}{n})}{-[P_1(Q^\star, k^\star)(1+\frac{1}{n}) + \frac{Q^\star}{n}P_{11}(Q^\star, k^\star]}. && (38)
\end{aligned}$$

Here we have used the fact that $\frac{dk}{d\bar{\kappa}_l} = \sum_{j=1}^{m} f_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l}$. Since $\frac{dk}{d\bar{\kappa}_l} > 0$, we conclude that $\frac{dQ}{d\bar{\kappa}_l} \geq 0$ with equality holding iff $n = 1$. Finally note that by additive separability, we have:

$$\begin{aligned}
& \int_0^{Q^\star} P_2(u, k^\star)\frac{dk}{d\bar{\kappa}_l}du - Q^\star \sum_{j=1}^{m} \sigma_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l} && (39)\\
= \; & Q^\star[P_2(Q^\star, k^\star)\frac{dk}{d\bar{\kappa}_l} - \sum_{j=1}^{m} \sigma_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l}] && (40)\\
= \; & Q^\star[P_2(Q^\star, k^\star)\frac{dk}{d\bar{\kappa}_l} - \frac{1}{n}P_2(Q^\star, k^\star)\sum_{j=1}^{m} f_j'(\kappa_j^\star)\frac{d\kappa_j}{d\bar{\kappa}_l}] && (41)\\
= \; & Q^\star\frac{dk}{d\bar{\kappa}_l}P_2(Q^\star, k^\star)(1-\frac{1}{n}) && (42)\\
\geq \; & 0, && (43)
\end{aligned}$$

with equality holding iff $n = 1$. Since the unit margin in the unregulated equilibrium must be strictly positive, we conclude that if there is more than one firm, then in the neighborhood of the unregulated equilibrium, a minimum quality standard on a single component will strictly improve welfare. ∎

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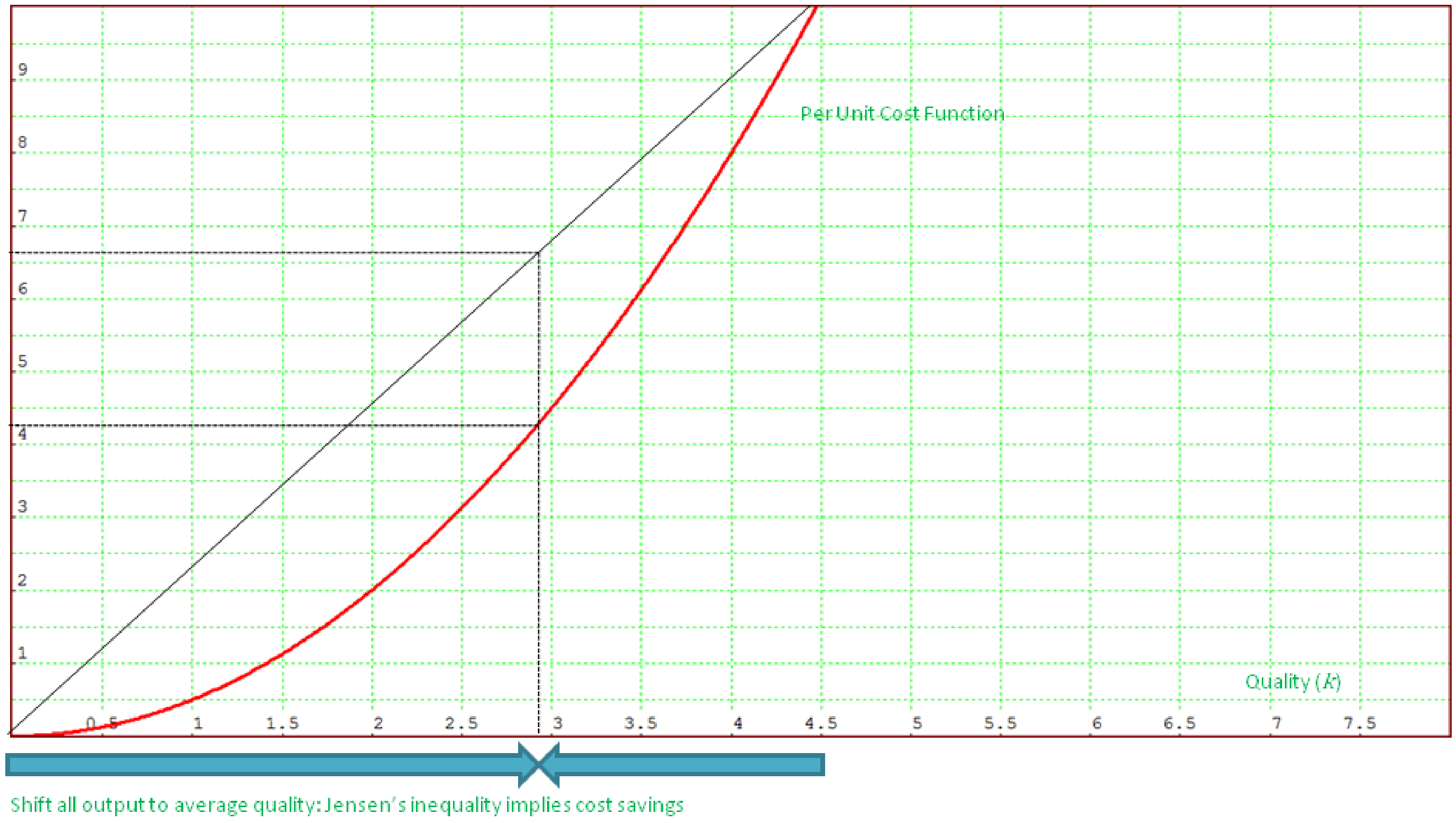

Figure 1: Strictly Convex Cost Function Insures Cost Savings When Product Line Has Diverse Qualities

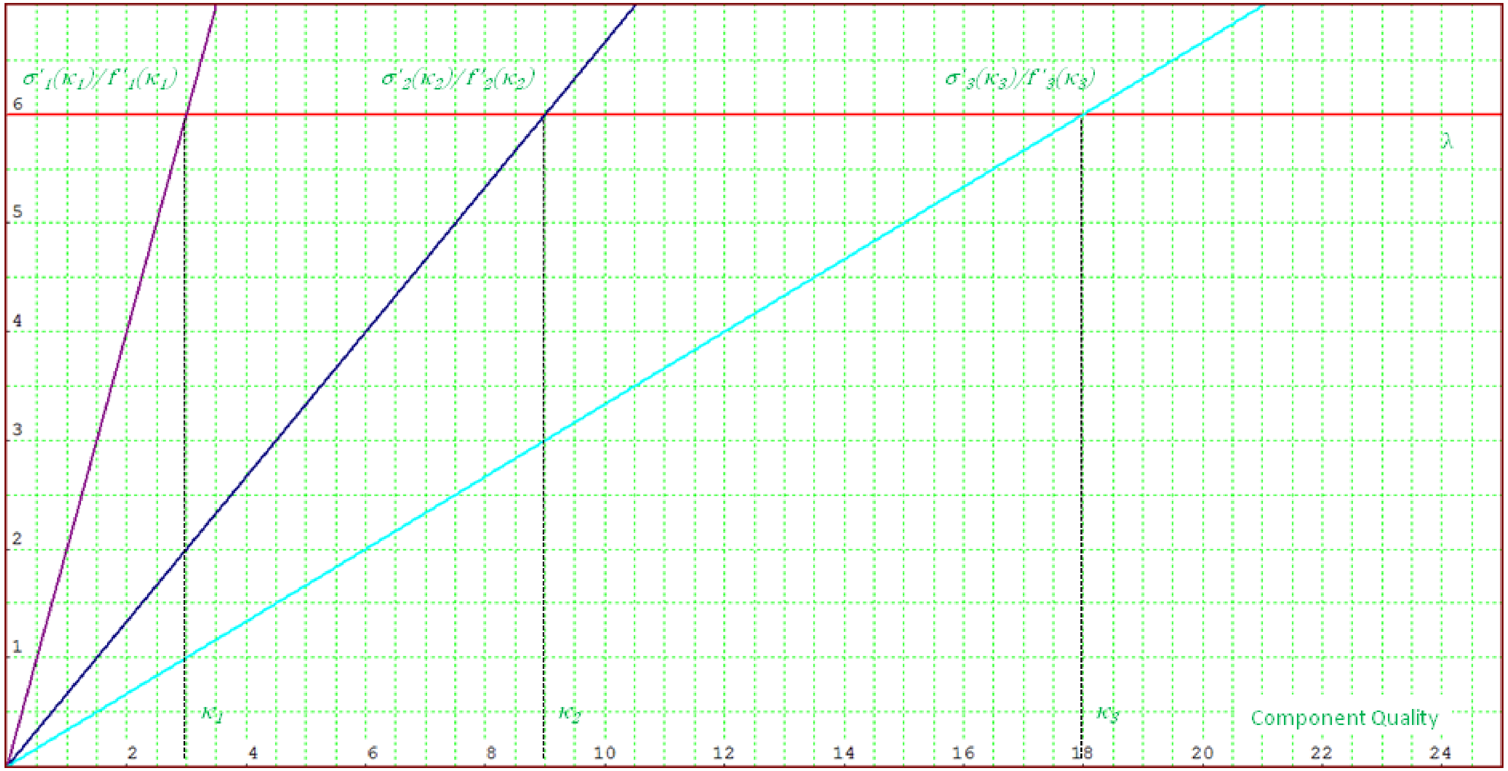

Figure 2: Solution to Cost Minimization Problem in the Standard Case

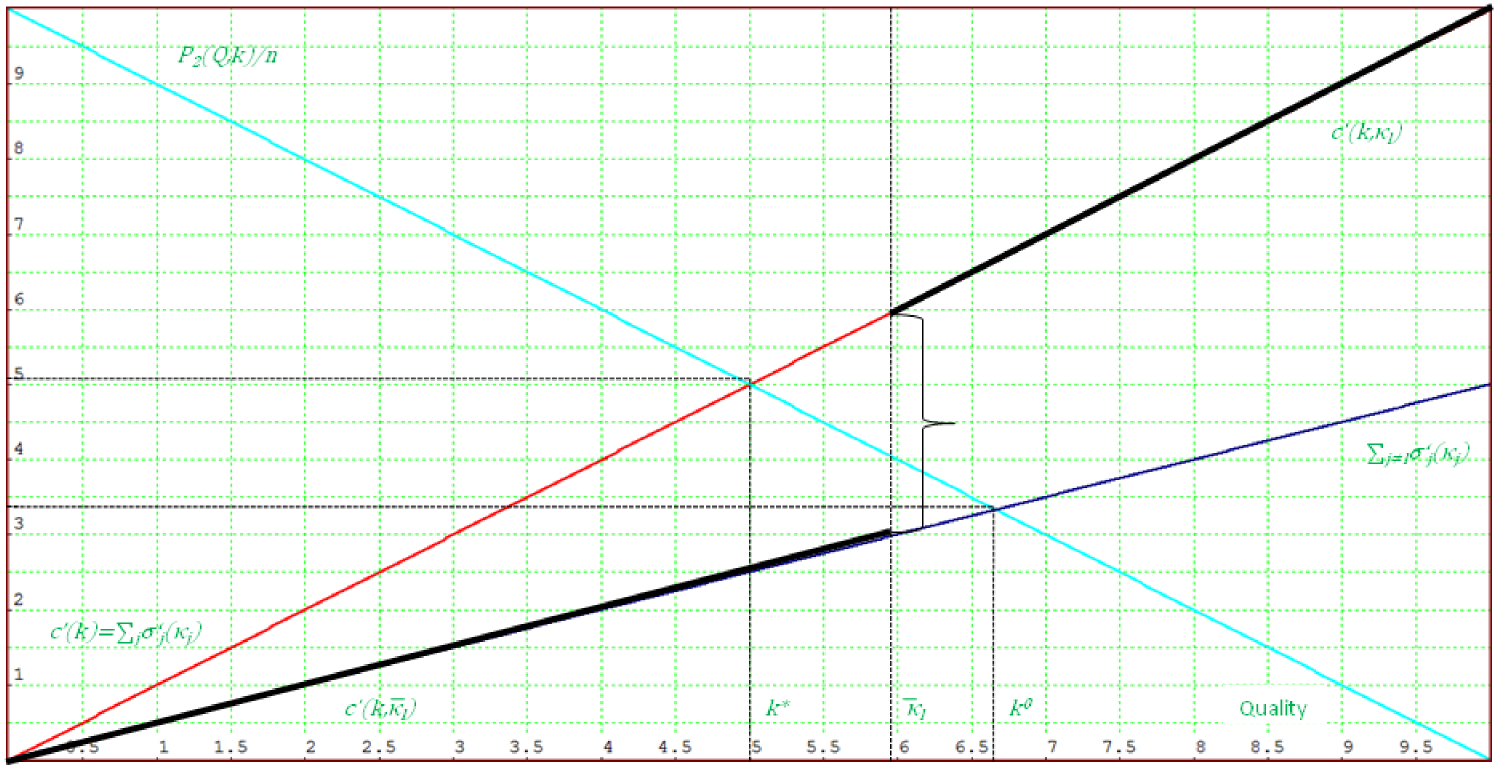

Figure 3: Regulation in Case without Partial Offsets