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Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market

Stephen W. Salant

Board of Governors of the Federal Reserve System

The theory of exhaustible resources is modified to take account of the industrial organization of the world oil market. The cartel is viewed as a unified enterprise which dominates other extractors because of its larger reserves. Equilibrium price and sales paths are derived giving neither the dominant extractor nor the competitive fringe any incentive to change its intertemporal behavior. Under standard but simplified cost assumptions, it is shown that a disproportionate share of the increased profits resulting from the formation of the cartel goes to non-members and that the cartel's restriction on sales eventually leaves it the sole supplier of oil.

The current structure of the world's oil industry bears little resemblance to the extremes assumed in the theoretical literature on exhaustible resources. There is neither a single cartel (or firm) which owns all the world's oil and thus has unchecked power to set prices over time, nor is there an abundance of measureless, "Mom-and-Pop" oil extractors dotting the globe. Instead, the industry contains one cartel with more power than any other extractor; but other extractors do exist and have enough importance, collectively and perhaps individually, to restrain the full exercise of monopoly power.

In this paper, the conventional theory of exhaustible resources is modified to take account of this structure. Since many extractive industries besides oil have market structures "intermediate between monopoly and

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perfect competition”—as Hotelling (1931, p. 171) remarked long ago¹—the modified theory may also be applied to other exhaustible resources.

The world oil industry is regarded as a collection of independent firms from which consumers purchase oil at the same price. Each firm may be viewed as a participant in a Cournot noncooperative game—maximizing its own discounted profits while taking as given the sales path of the remaining firms. Assumptions about the extraction costs and oil reserves of each firm determine whether the Cournot equilibrium which results resembles more closely monopoly, competition, oligopoly, or price leadership.

As a simplification, every small extraction unit or “plant” is assumed to have the same oil stock and cost function. In the absence of a cartel, each plant is owned by a different firm. A cartel is said to form when more than one plant comes under control of the same firm. The cartel is then in the distinctive position of owning larger reserves than any other firm. Moreover, if the marginal cost function of each plant is upward sloping, the cartel can extract at the same rate as another firm for a smaller cost.

These two distinguishing characteristics give the cartel market power over every other firm. If it is assumed that the rest of the world’s oil stock is divided equally among a sufficiently large number of other extractors, a dominant firm model results. In such a case, each small firm acts like a competitor which takes as given the price path set by the cartel and chooses a sales path to maximize the sum of discounted profits. The cartel takes the sales path of the “competitive fringe” as given and chooses a price path—supported by cartel sales—to maximize its discounted profits. Any situation where each sector takes as given the optimal choice of the other and where neither can, under that assumption, increase its profits by altering its own strategy is called a Nash-Cournot equilibrium.²

This paper analyzes equilibrium properties of the dominant extractor model.³ In the next two sections, it is assumed that costs at each plant are a linear function of the rate of extraction (constant marginal costs). As a

¹ Although Hotelling recognized the importance of oligopoly in extractive industries, he did not analyze the case in detail. Since his pioneering article, theoretical research on exhaustible resources has focused exclusively on the rare extremes of monopoly and perfect competition.

² The equilibrium is named after Cournot and Nash because the first originated and the second generalized the assumption that each player ignores the effects of his actions on the strategies of the others. In contrast, Stackelberg assumed that one particular player can manipulate the reactions of the other. The unexplained asymmetry of this alternative approach is itself a drawback; moreover, it makes an extension of the analysis from duopoly to oligopoly impossible.

³ The versatility of the Cournot approach is further illustrated in Appendix B, where asymmetric oligopoly is shown to result when different assumptions are made about stocks owned by the various extractors. The dominant extractor model of the text is then shown to be a limiting case of asymmetric oligopoly.

result, the acquisition of several plants by the cartel does not enable it to extract more cheaply than another firm. Nonetheless, the cartel still has one important advantage over the other extractors: it owns more oil. To focus on this advantage, which has no counterpart in the theory of cartels selling an ordinary producible, marginal extraction costs are assumed to be zero.⁴ Section I considers the optimization problems of both the cartel and the competitive fringe under this cost assumption, and then shows how the equilibrium price and sales paths can be constructed. Qualitative properties of the resulting equilibrium are discussed in section II and comparisons are made to the more familiar extremes of monopoly and perfect competition. In section III, the dominant extractor model is reconsidered under the assumption that the marginal cost of extracting at each plant is upward sloping.

Three results emerge under either cost assumption: (1) the formation of the cartel raises the discounted sum of profits of the competitive fringe by an even greater percentage than those of the cartel; (2) after the initial jump in prices, the cartel continues to raise the real price of oil monotonically over time; and (3) no matter how small its supplies compared to those of the competitive industry as a whole, the cartel restricts its sales so as to take over the market after the competition has exhausted its reserves.

I. The Oil Cartel as a Dominant Extractor: The Case of Zero Marginal Costs

To begin, consider the problem facing the cartel. Suppose the cartel knows the stationary consumer demand curve, the sales path of the competitive sector, and the stock of its own resources. The sales of the competitive sector may include sales of competitive speculators who begin with nothing but are free to enter, purchase in any period, and sell a smaller or equal amount later. The cartel's problem is to pick a path of prices, supported by its sales, to maximize its discounted revenues. Alternatively, it can be thought of as picking its sales path over time.

Since the cartel takes the competitive sales path as given, it can deduct the given sales on each period from the consumer demand curve to obtain a sequence of excess demand curves. Each curve will indicate the amount of oil demanded of the cartel at a given price in excess of what the competitive fringe sells at that instant. Faced with these excess demand curves, the cartel must choose a sales path which maximizes the sum of discounted revenues, but which does not exceed its initial inventory.

⁴ To extend the results of the next two sections (and the appendixes) to the case of positive, constant marginal costs, reinterpret the words "price," "marginal revenue," and "choke level" as "net of the constant marginal cost."

To accomplish this, the cartel (like a monopolistic extractor) must compare the marginal revenue associated with selling another unit today to the discounted marginal revenue of selling it in the future. The marginal revenue associated with a sale by the cartel of Q^m units is simply the price it would receive for selling another unit less the loss it would incur, because of the slight reduction in price, on the inframarginal units (Q^m) it sells:

$$M = P(Q^m + \bar{Q}^c) - Q^m \cdot a(P),$$

where

Q^m is the sales of the cartel,

$\bar{Q}^c$ is the (given) sales of the competitive sector,

P is the price at which consumers would purchase $Q^m + \bar{Q}^c$ units,

M is the marginal revenue associated with the excess demand curve, and

$a(P)$ is the absolute value of the slope of the consumer demand curve (with respect to sales), expressed as a function of price.

Notice that this marginal revenue is derived from the *excess* demand curve faced by the cartel.

If the strategy of the cartel is optimal, it must generate marginal revenues of the same discounted value in all periods in which the cartel sells; once the cartel stops selling, the discounted marginal revenue must never exceed the marginal revenue on any period of positive sales. Violation of either condition would provide opportunity for additional profits. Hence, along the optimal path, the marginal revenue grows at the rate of interest until sales cease; then it grows at a smaller (or equal) rate. Cumulative sales along the optimal path must, in addition, equal the cartel's initial inventory (I^m); leaving some oil in the ground is never optimal.

Given the competitive sales path, the cartel picks its sales path to maximize discounted profits. Alternatively, it can be viewed as selecting the optimal price path, supported by its sales. The price path it selects may have any shape, since the excess demand curves it faces in successive instants depend on the arbitrarily given competitive sales path. The price path may have regions where the price rises by more than the rate of interest or even falls absolutely. However, we must ask if all other agents in the model would react to the chosen price path in the way assumed by the cartel. If not, the price path does not result in noncooperative equilibrium. To understand how the other agents will react, the competitor's problem must be considered.

The competitive sector takes the price path as given and knows its total stock. It chooses a sales strategy to maximize discounted profits without exceeding its initial inventory (I^c). Since the competitive sector contains

speculators who may *buy* for re-sale—as well as extractors—its “sales” may be negative (i.e., purchases) in some periods.

The competitor's optimization problem, like the cartel's, is entirely conventional. The competitors will always exhaust their stock. As long as the price rises by the rate of interest, different ways of selling a given stock have the same discounted value. If the price rises by a greater amount between two periods, the competitive speculators would enter and attempt to make *unlimited* profits by buying in the first period and selling in the next. If the price begins to rise by less than the rate of interest, the competitive sector will find selling its inventory before this region is reached to be optimal.

These two maximum problems provide enough information to characterize the equilibrium when it exists. Price rises at the rate of interest as long as competitors hold stocks. Afterward, it can rise at a smaller or equal rate. Price can never rise by more than the rate of interest in equilibrium. The marginal revenue derived from the excess demand curve must rise at the rate of interest, as long as the cartel makes positive sales; when they are completed, the marginal revenue can rise at a smaller or equal rate. Each sector ultimately sells its entire stock.

These conditions imply that, as long as the cartel begins with positive stocks, it will conserve enough to continue selling after the competitors drop out.⁵ For, suppose the cartel completed its sales before the competitors. Then the price path would rise at the rate of interest while the two sectors coexisted and would continue to rise at that rate after the cartel stopped selling. Now, compare some early moment when the cartel is selling to some later moment when its sales are zero. When its sales are positive, its marginal revenue will be less than the price; when its sales are zero, its marginal revenue will be equal to the price (since it has no inframarginal units on which to take losses). Since price is growing at the rate of interest, the marginal revenue would have to grow by more; but this would give the cartel an incentive to alter its strategy. Hence, in equilibrium, the competitors cannot continue selling after the cartel drops out.

If both sectors begin with positive stocks, they must both operate simultaneously in the market for a period of time. When the price reaches some level, the competitive fringe completes its sales and abandons the market to the cartel. Denote this “termination” price as P^* . Prior to the time P^* is reached, the two sectors coexist in the market; afterward, the cartel operates alone. This suggests a solution technique. We begin at P^* and the marginal revenue at that price associated with the consumer demand curve, $MR(P^*)$. Before P^* is reached, the price and marginal

⁵ In Section III, this result is shown to hold in a dominant extractor model with increasing marginal costs at each plant. In Appendix B, the result is shown for an oligopolistic market where the cartel owns more oil than any other single firm.

revenues grow at the interest rate; hence, u moments before termination of the first phase at P^* , the price $P(u, P^*)$ and marginal revenue $M(u, P^*)$ are:

$$P(u, P^*) = P^*e^{-ru}, \quad \text{and} \quad M(u, P^*) = MR(P^*)e^{-ru}.$$

The price and marginal revenue, u moments before termination at P^* , each depend on two variables: the sales of each sector at that time:

$$P(u, P^*) = P[Q^m(u, P^*) + Q^c(u, P^*)].$$

$$M(u, P^*) = P[Q^m(u, P^*) + Q^c(u, P^*)] - Q^m(u, P^*) \cdot a[P(u, P^*)].$$

Substituting, we obtain:

$$P^*e^{-ru} = P[Q^m(u, P^*) + Q^c(u, P^*)], \quad (1)$$

$$MR(P^*)e^{-ru} = P^*e^{-ru} - Q^m(u, P^*) \cdot a(P^*e^{-ru}). \quad (2)$$

These two "marginal" equations can be solved simultaneously to obtain the sales of each sector u moments before termination at P^* : $Q^c(u, P^*)$ and $Q^m(u, P^*)$. Only these sales paths will permit price and marginal revenue to grow at the rate of interest until they reach, respectively, P^* and $MR(P^*)$.

When the price reaches P^* , the competitors drop out and the cartel takes over the entire market. In this second phase, its marginal revenue must continue to grow at the rate of interest until demand is eliminated. Define the choke price (F) as the vertical intercept of the stationary demand curve. The marginal revenue begins at $MR(P^*)$ and grows at the rate of interest until it reaches F . We denote the cumulative sales of the cartel in the second phase as $\Delta(P^*)$.

The duration of the first phase (S) and the price (P^*) at which the competitors drop out are determined by two "exhaustion" equations:

$$\int_0^S Q^c(u, P^*) du = I^c. \quad (3)$$

$$\int_0^S Q^m(u, P^*) du + \Delta(P^*) = I^m. \quad (4)$$

Equation (3) states that P^* and S must be chosen so that the competitors sell their entire inventory during the first phase; equation (4) states that P^* and S must be chosen so that the cartel sells its entire inventory during the two phases.

A sufficient condition for the existence of a noncooperative equilibrium (with nonrandomized strategies) is that the consumer demand curve have a point of unit elasticity ($\underline{P}$) and that elasticity along the curve increase

FIG. 1.—The equilibrium price path in the dominant extractor model

strictly with price. The proof is relegated to Appendix A. All linear⁶ or concave demand curves and many convex curves satisfy this sufficient condition.

Whenever an equilibrium exists, the price path will have the appearance as shown in figure 1. As is indicated in the figure, the price begins at some level, P'_0 , grows for S periods at the rate of interest, and then grows more gradually for T additional periods until the choke price is reached.

During the entire first phase, price and marginal revenue remain in the same proportion. For this to occur, the elasticity of excess demand (η^{ex}) must, in equilibrium, be the same at each point chosen by the cartel. If the consumer demand curve satisfies the sufficiency condition mentioned above, the point elasticity of a *given* excess demand curve increases as the price rises. So the competitive sales must fall as price rises in order to *shift* the excess demand curve to the right and restore the elasticity of excess demand to its previous value. The elasticity of aggregate demand (η^{ag}),⁷ that of excess demand multiplied by the cartel's market share, rises during the first phase with the cartel's share.⁸ Since the sales of the competitive fringe diminish as the first phase progresses, so does the flow of its discounted profits. In contrast, sales and therefore the discounted profit flow of the dominant firm may increase during the first phase.⁹

⁶ E.g., in the case of the linear demand curve $P = F - a(Q^m + Q^c)$, equations (1) and (2) imply $Q^m(u, P^*) = (1/a)(F - P^*)e^{-ru}$ and $Q^c(u, P^*) = (F/a)(1 - e^{-ru})$. Since, in this case, competitive sales are independent of the termination price, (3) uniquely determines the duration of the first phase; (4) may then be used to determine the termination price of that phase.

⁷ Since the cartel always receives a positive marginal revenue, it always operates in the elastic region of the *excess* demand curve it faces. However, the *aggregate* demand curve may be inelastic early in the first phase. A failure to distinguish these two elasticities has led some to infer, merely because estimated aggregate demand for oil is inelastic, that OPEC's pricing policy is suboptimal.

⁸ In principle, therefore, data about market shares of a dominant extractor could be used to update any previous estimate of the elasticity of aggregate demand obtained during the first phase.

⁹ Since $\dot{Q}^m \geq 0$ as $(P/a)(\partial a/\partial p) \geq 1$, the sales of the cartel will increase as the first phase progresses unless the aggregate demand curve is very convex.

II. A Comparison of Different Market Structures

Consider the transition from a competitive world of many small extractors to a world where some of these extractors form a collusive cartel which dominates the remaining small firms. In figure 2, the equilibrium price path (A) prior to the formation of the cartel is portrayed. Price rises from its initial level (P_0) at the rate of interest until the world inventory ($I^m + I^c$) is exhausted; then the price remains at the choke level (F). Since selling a barrel of oil in any period has the same discounted value, each firm (including prospective members of the cartel) receives a total discounted profit of P_0 multiplied by its initial stock.

If the firms owning I^m barrels decide to collude while the owners of the remaining I^c barrels continue to act competitively, the equilibrium price path changes to that described in the previous section. The new path (B) rises at the rate of interest during the first phase and at a slower rate during the second. Moreover, since cumulative sales along the new path must remain equal to the unchanged world inventory, the two paths must cross. This can happen only if the new price path begins above P_0 (at P'_0), rises at the rate of interest during its first phase and then crosses the competitive path in the second phase as indicated in figure 2.

The formation of the cartel raises the sum of discounted profits of each extractor.¹⁰ However, the profits of every competitive firm jump by an *even larger percentage* than those of the cartel. This result arises here for exactly the same reason as in the theory of cartels selling an ordinary producible good. The competitive firms get the full benefit of the higher prices without themselves having to contort their sales in order to generate those prices.

To verify diagrammatically that the competitors get a "free ride," note that discounted profits of the competitive sector change from $P_0 I^c$ to $P'_0 I^c$ when the cartel forms. Profits of the firms which join the cartel, however, rise by a smaller percentage since the cartel must sell some of its stock during the second phase at discounted values below P'_0 .

This situation, of course, gives individual members of the cartel an incentive to "chisel" or defect *provided* the others would maintain the cartel discipline. For, if only a single member broke ranks, it could earn P'_0 per barrel by selling all its oil before the second phase. If, however, its chiseling led to the defection of the other members, the price would drop to the competitive level (P_0) and all extractors would be worse off. Threats of quitting may be used to affect the division of proceeds within the cartel. However, members presumably recognize that maintenance of the cartel is in their collective interest. As Stigler has put it, "a large

¹⁰ In standard welfare analyses of extraction, it is shown that only the intertemporal allocation associated with the competitive price path is Pareto-optimal. Since the presence of a dominant firm benefits all extractors, consumers must be the only losers.

Fig. 2.—Comparison of price paths under different market structures

group of free riders will find that the street car won't run."¹¹ In the case of a firm (or cartel) selling an ordinary producible, another reason for the possible loss of a dominant position is that entrepreneurs attracted by the profits of the competitive fringe learn how to produce the good and eventually enter the industry. In the case of an exhaustible resource, however, mere knowledge of the extraction technology does not permit entry. The entrepreneur cannot enter without acquiring an additional amount of the resource, an impossibility in a model without new discoveries.

If all the firms were to join the cartel, the monopoly path (C) would result. Since cumulative sales along this path must equal those along the other two, the monopoly path must cut each of the others. For this to occur, it must begin above the other two (at P''_0), cut each of them in the first¹² phase, and terminate last. Pure competition may be regarded as one extreme of the dominant extractor model where the cartel owns virtually nothing. Similarly, pure monopoly may be viewed as the other extreme where the cartel owns virtually everything. In the former case, the entire path consists of a first phase, while in the latter case, the entire path consists of a second phase. The dominant extractor model is intermediate between the extremes of pure monopoly and pure competition in the sense that, given the *same* world stock, the price lies below the high monopoly price, but above the low competitive one; and, with a dominant firm, the fixed world stock is exhausted more rapidly than under monopoly, but less quickly than under competition.¹³

¹¹ See Stigler (1966, p. 233) for a discussion of cartels and free riders producing an inexhaustible flow good.

¹² The price path of the monopolist cannot cross the path of the dominant firm in its *second* phase. For if the two paths touched there, the marginal revenues would then be equal. And, since both would be growing at the rate of interest, they would remain equal. But this implies that the two price paths would have to stick together rather than cross.

¹³ If the dominant firm initially owned a smaller amount of the fixed world oil stock, it can be shown that the price path would change in a predictable way provided the path were uniquely determined by the initial stocks. The more the competitive sector owned, the lower would be the initial price and the more quickly the world's oil would be depleted. The first phase of each lower path would cut the second phase of each upper path.

III. Extensions to the Case of Increasing Marginal Costs

If the marginal cost of extracting at each plant is upward sloping, not only does the acquisition of several plants by the cartel provide it with a larger stock of oil than any other firm, but also it enables the cartel to extract at the same rate for a lower cost. Specifically, when the oil stock of the cartel is X times that of any other firm, the cartel will be able to produce at X times the rate of another firm for the same marginal cost. That is when

$$\bar{I}^m = \left(\frac{\bar{I}^c}{n} \right) X,$$

$$\phi'(Q^m) = C' \left(\frac{Q^m}{X} \right),$$

where $C'(q^c)$ is the marginal cost of extraction at each of the n competitive plants, and $\phi'(Q^m)$ is the marginal cost of extraction for the cartel.

Prior to the formation of the cartel, each firm acts competitively, taking the price path as given and maximizing discounted profits. The optimal extraction policy is one where the difference between price and marginal cost is increased at the rate of interest until supplies are exhausted. Since each firm is identical, all extract at the same rate at any instant and earn the same overall discounted profit. If the firms which will comprise the cartel are considered as a group, they optimally extract at X times the rate of an individual firm at any instant and exhaust their stocks (which are X times as large) at the same moment.¹⁴ Prior to the formation of the cartel, the group earns X times the discounted profits of a typical firm.

Once the cartel forms, the equilibrium¹⁵ shifts. If the cartel is very large relative to any other extractor, it sets the price path for the competitive fringe. In equilibrium, the excess of price over marginal cost at each competitive firm must grow at the rate of interest until supplies at each firm are exhausted; afterward, it grows at a smaller rate. Similarly, the difference between marginal revenue (to the excess demand curve) and marginal costs of the cartel must grow at the rate of interest until its supplies are exhausted. As will be shown, the equilibrium will have the same basic properties as in the case of constant marginal costs.

First, any equilibrium must again involve a coexistence phase followed by a phase where the cartel takes over. To prove this, note that once either sector operates alone, the other will never wish to sell later. If the competitive firms operate alone, price less their marginal cost must rise at the

The three paths in figure 2 can be considered as special cases from this entire field of price paths. Such a transfer of the world's oil stock from the cartel to the competitive sector would actually occur if some members of the cartel defected.

¹⁴ A concise analysis of the case where competitive firms have different initial stocks and extraction costs has been conducted by Goldsmith (1974).

¹⁵ In contrast to the case of constant marginal costs, the general existence of equilibrium has not yet been formally established for the case of increasing marginal costs.

rate of interest. But then, the marginal profits of the cartel [$P - \phi'(0)$] will rise at a slower rate, making it suboptimal for the cartel to have waited to sell in the future. Similarly, if the cartel operates alone, marginal revenue less marginal cost must rise at the rate of interest. But then price less the competitor's marginal cost [$P - C'(0)$] will grow at a smaller rate,¹⁶ making it suboptimal for a competitive extractor to have waited to sell in the future. Therefore, since it is optimal for both sectors to exhaust their initial stocks, any equilibrium must involve a coexistence phase followed perhaps by the operation of one sector.

Moreover, the competitive extractors cannot be the ones to operate last. If the competitive firms continued to sell after the cartel dropped out, then at the termination of the first phase, the marginal cost of each firm would exceed that of the cartel. To satisfy the first order conditions, marginal revenue less the marginal cost of the cartel would also have to exceed price less the marginal cost of each competitive firm at *all* previous moments. However, for the marginal cost of each competitive firm to exceed that of the cartel at every previous instant, each competitive extractor would have to operate at more than $1/X$ th the rate of the cartel until the cartel exhausted its supplies—an impossibility, since each firm has only $1/X$ th the stock of the cartel. Thus, in any equilibrium it is the cartel which must outlast the competitive extractors.

Second, as in the case of constant marginal costs, the real price of oil increases monotonically. For this result, it is sufficient that the cost and demand functions be stationary. For—assuming stationarity—suppose that the real price ever failed to rise between one period and the next. Then, for price less marginal cost to rise at the rate of interest, the output at each competitive firm would have to fall (assuming stationarity of costs). Similarly, for the marginal profits of the cartel to rise, its output would also have to fall. But, if the output of each sector fell, the price would (assuming stationarity of demand) rise—contradicting the assumption.

Third, a comparison of the competitive and dominant extractor equilibria reveals that—even in this more general case—the formation of the cartel raises the discounted profits of each competitive firm by an even greater percentage than those of the cartel.¹⁷ Each firm initially earns $1/X$ th the profits of the group which forms the cartel. When the cartel forms, the price path changes. The competitive firms devise a new sales path to maximize the value of their output along the new price path. If the new price path were fixed, the maximizing strategy for the cartel would be to extract in each period X times as much as each competitive extractor. Given the new price path, this strategy would restore the profits

¹⁶ For this proposition to be valid, it is sufficient to assume that elasticity of demand increases with price along the demand curve.

¹⁷ The corresponding analysis in a Cournot model of a dominant firm selling an ordinary producible good may be found in Shubik (1959, pp. 134–35).

of the cartel to X times the profits of each firm. Of course, this strategy is infeasible since the price path would not then remain fixed, and so the cartel cannot choose this unconstrained optimum. The profit it gets from its optimal choice constrained to generate the new equilibrium price path will, therefore, be less than X times that of the typical competitive firm. Thus, when the cartel forms, a disproportionate share of the increase in discounted profits goes to the competitive fringe.

Appendix A

Existence of Equilibrium

Equilibrium in the dominant firm model occurs when, given the price path of the cartel, the competitors cannot make greater discounted profits by selecting a different, feasible sales path, and, given the sales path of the competitors, the cartel cannot make greater profits by altering its price path. Any solution with the following five properties is, therefore, an equilibrium for the case of zero marginal costs.¹⁸

1. As long as both sectors sell, price and marginal revenue grow at the rate of interest.
2. After the competitors drop out, marginal revenue continues to grow at the rate of interest until the cartel stops selling; it then remains at the choke level (F).
3. After the competitors drop out, price never grows faster than the rate of interest.
4. Over the entire horizon, the cumulative sales of each sector exactly match its initial inventory.
5. At no intermediate point does the partial sum of sales of either sector exceed its initial stock (the requirement for a feasible sales path).

A condition sufficient for the existence of equilibrium is that the consumer demand curve have a point of unit elasticity ($\underline{P}$) and that elasticity of demand increase strictly with price.

To prove that this condition is sufficient for the existence of equilibrium given arbitrary initial stocks, we show: (A) If this condition holds, *any* solution to equations (1)–(4) has the five properties above. (B) If this condition holds, there will *always* exist a solution to equations (1)–(4).

A. By construction, any solution to equations (1)–(4) will possess properties 1, 2, and 4. Furthermore, the condition mentioned above insures that any solution to (1)–(4) will have property 3. It must be shown, however, that any solution to (1)–(4) satisfies property 5 if the sufficiency condition holds. This follows since (1)–(4) implies a *nonnegative*¹⁹ sales path for each sector and each sector ultimately exhausts its initial supplies.

¹⁸ These require trivial modification for the case of constant marginal costs.

¹⁹ That $Q^m(u, P^*) \geq 0$ follows from equation (2). The proof that $Q^c(u, P^*) \geq 0$ relies on the sufficiency condition. Since

$$\eta^{ex} = \frac{(Q^m + Q^c)}{Q^m} \eta^{ag}(P), \quad \frac{\partial \eta^{ex}}{\partial P} = 0 \quad \text{for } \frac{\partial \eta^{ag}}{\partial P} > 0 \text{ only if } \frac{Q^m + Q^c}{Q^m}$$

decreases as P increases. This requires that Q^c/Q^m fall monotonically as the first phase progresses. Since $Q^c(0, P^*) = 0$, competitive sales must be strictly positive at each moment of the first phase and must decrease as that phase progresses.

FIG. 3.—Determination of P^*

B. Next, we show that there will always exist a solution to equations (1)–(4) provided the sufficiency condition holds. Since (1) and (2) can always be solved for the sales function of each sector, what is required is a proof that a (P^*, S) combination always exists satisfying the two exhaustion equations. Since competitive sales are positive and decrease as the first phase progresses, there exists for *any* termination price (P^*), a unique duration (S) for the first phase satisfying equation (3).²⁰ Hence, equation (3) implicitly determines a positive function $S(P^*)$.

Substituting $S(P^*)$ into (4) converts it to a single equation in one unknown (P^*):

$$H(P^*) = \int_0^{S(P^*)} Q^m(u, P^*) du + \Delta(P^*) = \bar{I}^m. \quad (5)$$

A solution to this equation always exists since $H(P^*)$ is a continuous function equal to zero at the choke price and approaching infinity as P^* approaches $\underline{P}$ (the point of zero marginal revenue) from above.²¹ Somewhere in between lies a termination price solving the exhaustion equations. Figure 3 illustrates the determination of P^* .

Appendix B

The Dominant Extractor Model as a Limiting Case of Asymmetric Cournot Oligopoly

Assume the oil industry to be composed of firms in two sectors extracting oil at zero cost and selling it to consumers at the same price. One sector consists of the cartel, which acts in the coordinated manner of a single firm. Each of the n oligopolists in the second sector is assumed to own an equal fraction of the sector's oil. None, however, owns as much as the cartel [$\bar{I}^m > (\bar{I}^c/n)$].

²⁰ If $Q^c(u, P^*) \leq 0$ for some termination price, there might exist no duration which would exhaust the competitive stock. The same problem might arise if competitive sales, although positive, decreased toward zero as we moved back further from termination of the first phase. In either case, the function $S(P^*)$ might not exist.

²¹ $H(F) = 0$ since $\Delta(F) = Q^m(u, F) = 0$.

$$\lim_{P^* \downarrow \underline{P}} H(P^*) = \infty \quad \text{since } Q^m(u, P^*) \geq 0, \quad \text{while } \lim_{P^* \downarrow \underline{P}} \Delta(P^*) = \infty$$

—the cumulative sales are infinite along a path which begins where marginal revenue is zero and continues forever with the marginal revenue “growing” from that point at the rate of interest.

Every firm in the oil industry (including the cartel) is assumed to take the aggregate sales path of the other n firms as given and, taking account of the effects of its own sales on price, to devise its optimal, intertemporal sales strategy. The price path is determined by the aggregate sales paths of the $n + 1$ extractors. The characteristics of the optimal strategy for each extractor (including the cartel) are that: (1) the marginal revenue rises at the rate of interest until the firm exhausts its supplies, and then rises at an equal or smaller rate; and (2) each firm exhausts its initial inventory.

Since the marginal revenue of either type of firm is simply the price it would receive for selling an additional unit of oil less the revenue it would lose (because of the resulting slight reduction in price) on the inframarginal units it was selling, the firm selling the larger number of inframarginal units has the lower marginal revenue.

At the end of the first phase, the marginal revenue of the type of firm which drops out will exceed that of the firm selling to the entire market. From the marginal conditions, it is clear that this same relationship in marginal revenues must hold at any moment of the coexistence phase. This implies that the sales of the firm destined to drop out are smaller than the sales of the firm which will take over at each moment of the first phase. This, in turn, implies that, in equilibrium, the cartel cannot be the first to exhaust its supplies.

For, suppose the cartel dropped out first. Then, regardless of the length of the first phase, the marginal conditions would imply that cumulative sales of the cartel would be smaller than cumulative sales of each smaller firm. But then the exhaustion equations would never be satisfied, since any first phase long enough to exhaust the cartel's supplies would more than exhaust the stock of each smaller extractor. Since assuming the cartel is the first to drop out leads to an inconsistency between the marginal equations and the exhaustion equations, no equilibrium will have this characteristic. In any equilibrium the cartel will continue to sell after the smaller oligopolists have exhausted their supplies. The following equations must then hold:

$$P(u, P^*) = P[nq^c(u, P^*) + Q^m(u, P^*)], \quad (6)$$

$$P^*e^{-ru} = P(u, P^*) - q^c(u, P^*) \cdot a[P(u, P^*)], \quad (7)$$

$$MR(P^*) \cdot e^{-ru} = P(u, P^*) - Q^m(u, P^*) \cdot a[P(u, P^*)], \quad (8)$$

$$\int_0^S q^c(u, P^*) du = \frac{\bar{I}^c}{n}, \quad (9)$$

$$\int_0^S Q^m(u, P^*) du + \Delta(P^*) = \bar{I}^m. \quad (10)$$

Equation (7) states that the marginal revenue of a representative smaller firm u moments before it exhausts its supplies at P^* is equal to its discounted marginal revenue at termination. Equation (8) states that, u moments prior to termination of the first phase, the marginal revenue of the cartel will equal the discounted value of its marginal revenue when the n smaller firms depart.

The new system (6)–(10) is very similar to the model (1)–(4) studied in sections I and II. Except for notation, the only difference is between (1) and (7). The smaller firms no longer drive the current price down as far as the discounted value of the termination price because they consider the losses on their inframarginal units. Hence, price rises at less than the rate of interest during the first phase of the oligopoly model.

As the number of firms (n) dividing the noncartel oil stock grows, however, the sales of an individual firm at each moment of the first phase become negligible. If the slope of the demand curve is bounded, the last term of equation (7) vanishes in the limit and our system (6)–(10) converges²² to (1)–(4). Thus, the price taking behavior of the competitive fringe emerges as the limiting case of the Cournot oligopoly model.

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²² For the case of the linear demand curve of n. 6, eqq. (6)–(10) imply that

$$nq^c(u, P^*) = \frac{nF(1 - e^{-ru})}{a(n + 2)}$$

and

$$Q^m(u, P^*) = \frac{F + n(F - P^*)e^{-ru} - (2P^* - F)e^{-ru}}{a(n + 2)}.$$

As $n \rightarrow \infty$, these sales functions converge to those obtained in the dominant firm case of n. 6.