## PROBLEM SET 7

1. Let $S$ be the set $\{1,2, \ldots, m n\}$. Partition $S$ into $n$ sets $A_{1}, A_{2}, \ldots, A_{n}$ each of size $m$. Let $B_{1}, B_{2}, \ldots, B_{n}$ be a second partition of $S$ into $n$ sets of size $m$. Show that we can reorder the $B_{i}$ so that $A_{i} \cap B_{i}$ is nonempty for $i=1,2, \ldots, n$.
2. Give an example of a flow network, and a flow $F$ through it which is not optimal, such that, for any optimal flow $F^{\prime}$, we have $F^{\prime}(e)<F(e)$ for some edge $e$. In other words, we can't improve the flow $F$ just by increasing throughput everywhere.
3. Let $G$ be a flow network with $k$ edges and let $F$ be an optimal flow. Show that there is a sequence of flows $F_{0}=0, F_{1}, F_{2}, \ldots, F_{k}=F$ so that each $F_{i+1}$ is obtained from $F_{i}$ by increasing along an augmenting path. (Hint: make one edge correct at a time.)
4. Consider a $(2 k+1) \times(2 k+1)$ checker board, with $2 k^{2}+2 k+1$ black squares and $2 k^{2}+2 k$ white squares. Remove one black square from the board. Show that the remainder of the board can be covered with dominos. (Hint: a direct approach is easier than using Hall's marriage theorem)
5. Find a maximal flow and a minimum cut in the network below:

6. Deal a deck of cards into 13 piles, with 4 cards in each. Show that it is possible to pick one card from each deck so that you pick exactly one card of each rank. (One ace, one deuce, and so forth, up to one king.)
