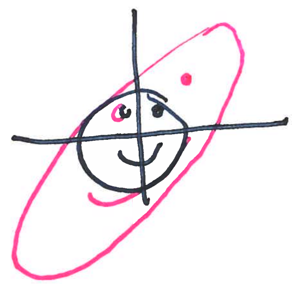
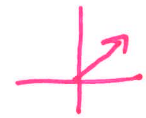
9/14/22

Last time

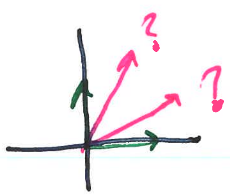


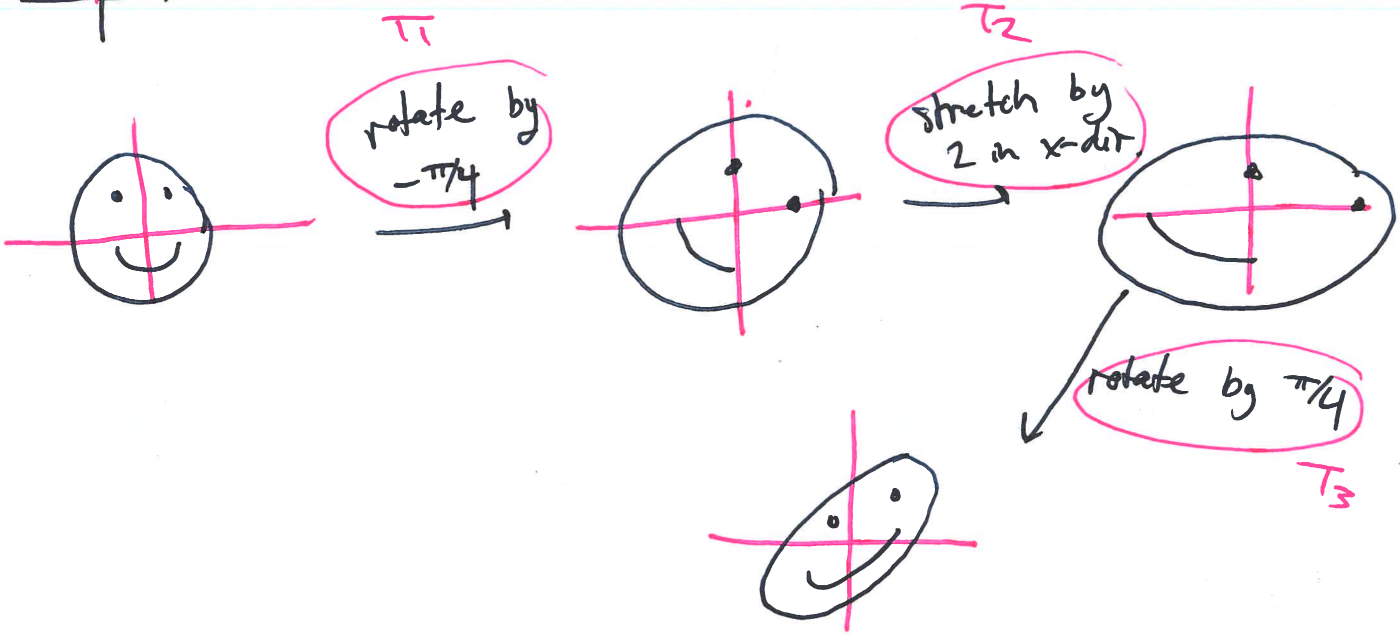
$T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ stretches by a factor of 2 in the diagonal direction (in the direction of the vector $\begin{bmatrix}1\\1\end{bmatrix}$)



Q Matrix for T?

Usual way: $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = ?$ $\quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = ?$



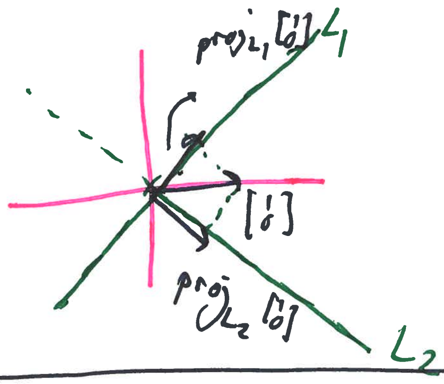


$T = T_3 \circ T_2 \circ T_1$

Matrix for T is: $\begin{bmatrix}\cos\pi/4 & -\sin\pi/4\\ \sin\pi/4 & \cos\pi/4\end{bmatrix} \cdot \begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}\cos\frac{-\pi}{4} & -\sin\frac{-\pi}{4}\\ \sin\frac{-\pi}{4} & \cos\frac{-\pi}{4}\end{bmatrix} \vec{x}$

$= \begin{bmatrix}3/2 & 1/2\\ 1/2 & 3/2\end{bmatrix}$

A2 $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = ?$ $\quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = ?$

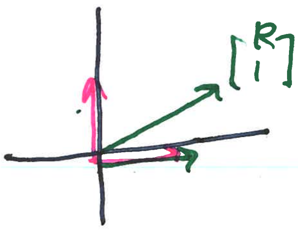


$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = 2\,\text{proj}_{L_1}\begin{bmatrix}1\\0\end{bmatrix} + \text{proj}_{L_2}\begin{bmatrix}1\\0\end{bmatrix}.$$

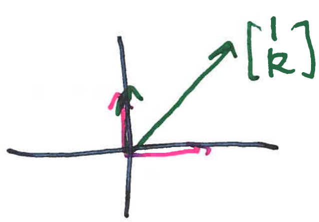
etc.

More 2.3 practice.

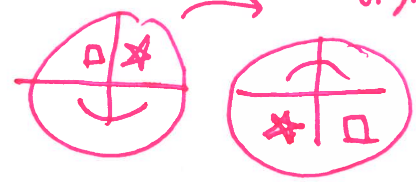
One more type of linear transform: shear



Matrix: $\begin{bmatrix}1 & k\\0 & 1\end{bmatrix}$ (horizontal shear)



Matrix: $\begin{bmatrix}1 & 0\\k & 1\end{bmatrix}$ (vertical shear) rot 180° ⇒ refl across origin



For the matrices A in Exercises 33 through 42, compute $A^2 = AA$, $A^3 = AAA$, and A^4. Describe the pattern that emerges, and use this pattern to find A^{1001}. Interpret your answers geometrically, in terms of rotations, reflections, shears, and orthogonal projections.

33. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **34.** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **35.** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

36. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ **37.** $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ **38.** $\frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$

39. $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ **40.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

41. $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ **42.** $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

In Exercises 43 through 48, find a 2×2 matrix A with the given properties. Hint: It helps to think of geometrical examples.

43. $A \neq I_2$, $A^2 = I_2$ **44.** $A^2 \neq I_2$, $A^4 = I_2$

45. $A^2 \neq I_2$, $A^3 = I_2$

46. $A^2 = A$, all entries of A are nonzero.

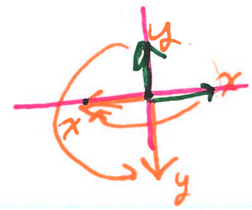
47. $A^3 = A$, all entries of A are nonzero.

48. $A^{10} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

33. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}^{1001} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$



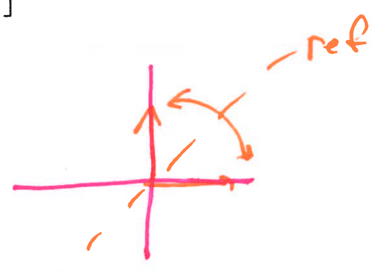
A is rotation by π

$\Rightarrow A^2 =$ rotation by $2\pi =$ id

35. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

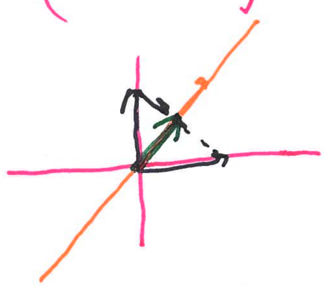
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{1001} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



42. $\left(\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right)^2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\left(\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right)^{1001} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

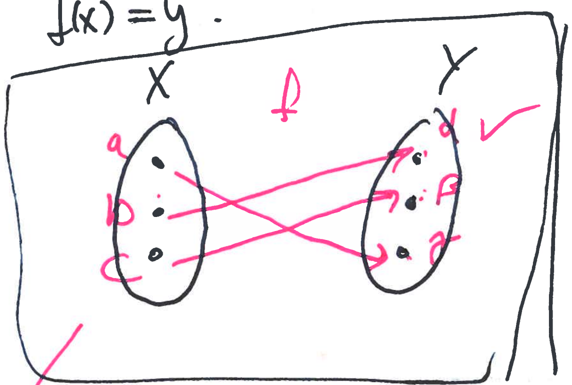
$\text{proj}_{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$ "idempotent": $A \stackrel{?}{=} A^2$

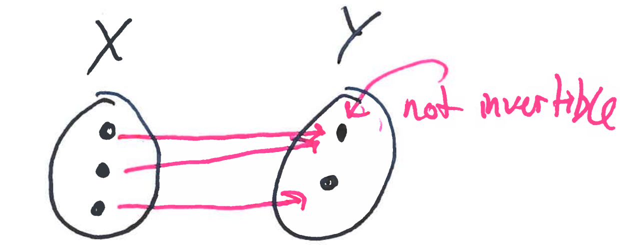


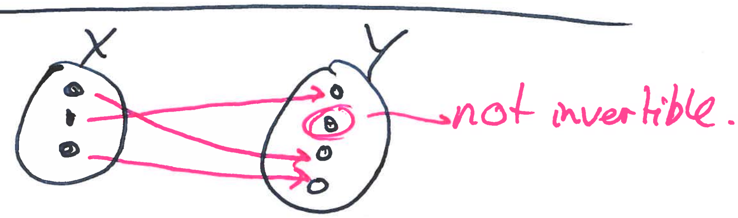
§2.4 Inverse transformations

A function $f: X \longrightarrow Y$ is called <u>invertible</u> if for all $y \in Y$ there exists a <u>unique</u> $x \in X$ such that $f(x) = y$.

"bijection"







$f(a) = \gamma$,

$f(b) = \alpha$

$f(c) = \beta$

We can define the inverse function of f:

$$f^{-1}: Y \longrightarrow X$$

$f^{-1}(\alpha) = b$, $f^{-1}(\beta) = c$, $f^{-1}(\gamma) = a$

In general, $f(x) = y \iff f^{-1}(y) = x$