1. Let $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \in \mathbb{R}^3$ and let $T \colon \mathbb{R}^3 \to \mathbb{R}^3$ be orthogonal projection onto the line containing $\vec{v}$.

   (a) Find the matrix for T.

   (b) Is the matrix for T invertible?

2. Consider the following matrix and its reduced row-echelon form:

$$M = \begin{bmatrix} 3 & 2 & 7 & 1 \\ 4 & 1 & 6 & 5 \\ -1 & 2 & 3 & 3 \end{bmatrix}, \quad RREF(M) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Find a basis for $\ker M$

(b) Find three different bases for the image of M.

3. Let A be a 3×3 matrix with

$$A \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = A \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

Find a basis for $\ker A$.

Question 2 (10 points)
Consider the chemical reaction

$$a\text{NO}_2 + b\text{H}_2\text{O} \longrightarrow c\text{HNO}_2 + d\text{HNO}_3,$$

where $a, b, c,$ and d are unknown positive integers. The reaction must be **balanced**; that is, the number of atoms of nitrogen (N), oxygen (O), and hydrogen (H) must be the same before and after the reaction. The term bH_2O refers to b water molecules, which consists of $2b$ hydrogen and b oxygen atoms. As customary, give the smallest possible positive integer solution.

(a) (4 points) Set up a system in the unknowns.

(b) (2 points) Label each equation with a unit. (What type of thing is being equated to what?)

(c) (4 points) Solve the system to balance the reaction.

System (including "=" and right-hand side) | Units

Balanced reaction:

$a =$ | $b =$ | $c =$ | $d =$

Question 2 (11 points)

(a) (5 points) Determine if the vectors below are linearly independent.

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

(b) (6 points) Let $\vec{w}$ be the vector below, and let $\vec{v}_1$ and $\vec{v}_3$ be as above. For which value(s) of b are the vectors $\vec{v}_1$, $\vec{w}$, and $\vec{v}_3$ linearly *dependent*?

$$\vec{w} = \begin{bmatrix} 1 \\ -1 \\ b \\ 2 \end{bmatrix}$$