Overview of Today's Class

- Classifying numbers
- Explaining the Candy Box Problem
- Examining *mathematical explanation*
  - Extract and identify some features of a "good" mathematical explanation
- Wrap up

Natural Numbers

counting numbers

{1, 2, 3, 4, 5, ...}

(sometimes includes 0)

Whole Numbers

natural numbers and 0

{0, 1, 2, 3, 4, 5, ...}

Integers

whole numbers and their opposites

{..., -3, -2, -1, 0, 1, 2, 3, ...}

Rational Numbers

numbers that can be written as fractions

numbers of the form $\frac{a}{b}$, where a and b are both integers, and $b \neq 0$

More Classes of Numbers

- Real numbers: all numbers that can be represented as a decimal (terminating or infinite)
- Irrational numbers: all real numbers that are not rational numbers

What Are We Using the Candy Box Problem For?

- To develop a sense of the importance of the unit (or whole) with fractions
- To explore ways to represent a situation or problem, and what each offers or obscures
- To listen, interpret, and appreciate others’ ways of representing and solving the problem
- To map correspondences across representations
- To develop criteria for a good mathematical explanations

Candy Box Problem

There was a box of candy on the table. Alyson was hungry because she hadn't had breakfast, so she ate half the candy. Then Rob came along and noticed the candy. He thought it looked good, and had not packed a lunch, so he took two-thirds of what was left in the box.   Jessica came by and decided to take three-fourths of the remaining candies with her to her next class. Then Lani came dashing up and took one piece of candy to munch on. When Lee looked at the candy box, he saw that there was just one piece of candy left. "How many pieces of candy were there in the box to begin with?" he asked Alyson suspiciously.

In your notebook…

- Are you convinced that the only solution to the Candy Box Problem is 48? If so, what has convinced you? If not, what is leaving you skeptical?

- Are there things you would add to your list of features of "good" mathematical explanation? Were there things about others' explanations that were convincing and useful?

Features of “Good” Mathematical Explanations for the Candy Box Problem

1. Makes clear at the outset what is being explained, and why you start there, and carefully connects the explanation to the question or idea being explained
2. Starts from the beginning, and traces the logical flow (step by step) of the reasoning
3. Makes conclusion clear and links back to original question or claim or problem
4. Choosing an effective representation or metaphor or example that makes the ideas clear.
5. Strives to use simple, clear, and accurate language, diagrams or notation. Is as concise as possible, without being too compressed.
6. Defines terms as needed, uses available definitions as needed.
7. Shows what something means or why it is true, and is convincing to the person to whom you are explaining
8. Is calibrated to the context (considers the person to whom you are explaining, and what is already established as true and does not need more explanation) (thinking about what the student already knows)

Reflect in Notebooks

What is unclear? What new insights do you have? What questions do you have about the mathematical ideas?

Anything that stood out from today?

Anything from yesterday?

Wrap-up & Assignments

- Individual assignment due Wednesday
- Reminders:
  - Midterm revisions due by Friday at 5pm
  - Partner assignment due by Friday at 5pm
  - Study Hall in 2400 from 4-6pm
- Announcement:
  - We will be collecting notebooks at the end of class Wednesday