# Private Information in Bilateral Trade and in Markets

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**Abstract**

I consider a simple bilateral trading game between a seller and a buyer who have private valuations for an indivisible good. The seller makes a price offer which the buyer can either accept or reject. If the seller can observe the valuation of the buyer (if information is symmetric), then the trading outcome is trivially efficient. If the seller cannot observe the valuation (if information is asymmetric), then the outcome must be inefficient, as is known from the Myerson-Satterthwaite Impossibility Theorem. This bilateral trading game between a single buyer and a single seller is embedded into a matching market with a continuum of traders. I show that in this market the relation between information and efficiency is reversed. In particular, if information is symmetric, trading in the market is, in fact, inefficient. Thus, in markets, private information has a surprising, efficiency enhancing role.

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# 1 Introduction

Asymmetric information makes bilateral trade inefficient, as is known from the Myerson-Satterthwaite impossibility theorem. With symmetric information, however, bilateral trade is efficient. Embedding a bilateral trading game into a larger market, I show that this connection between efficiency and information is reversed: with symmetric information, the market outcome is bounded away from the efficient one, even when trading frictions are small. In the same model, trading with asymmetric information becomes efficient once frictions vanish, demonstrated in Lauermann (2007).

To model a decentralized market, I use a steady state, dynamic matching and bargaining game with an exogeneous inflow, similar to the model by Douglas Gale (1987). He considers bilateral trade between one buyer and one seller and embeds it into a larger dynamic market game as follows: There is a continuum of buyers and sellers who are matched into pairs at the beginning of each period. Within each pair, they bargain over the terms of trade. The pairs are connected by allowing an unsuccessful trader to be matched with another partner in a new pair in the next period. However, there is a friction that makes waiting for the next period costly, so the integration of the market is not perfect. Here, I follow McAfee (1993), and in particular Satterthwaite and Shneyerov (2008), and assume that this friction is an exogeneous probability $\delta \in (0, 1)$ that a trader cannot enter the next period and exits (dies).

The equilibrium outcome of versions of the dynamic matching and bargaining game have been shown to be efficient if frictions are small and any of the following three market clearing forces is present: if bargaining power is symmetric between buyers and sellers (for example, Gale (1987)); if there is a chance that one buyer receives price offers from several competing sellers (Satterthwaite and Shneyerov (2008)); or if information is asymmetric (Lauermann 2008). Formally, when the exit rate $\delta$ converges to zero, equilibrium outcomes of these models become competitive and, therefore, efficient. Here I show the opposite: If sellers have all the bargaining power, if bargaining takes place only in pairs, and if information is symmetric, then outcomes do not become efficient, even for small frictions when $\delta$ converges to zero. In addition to the limit, I characterize the unique steady state equilibrium for every level of the exit rate $\delta \in (0, 1]$ and provide comparative statics result for the surplus that is realized in equilibrium.

The intuition for the surprising result is the following: If sellers can observe buyers' willingness to pay and have all the bargaining power, sellers can engage in perfect price discrimination. This allows sellers to make strictly positive profits - even in the limit for small $\delta$. Thus, sellers are not willing to trade with those buyers whose valuations are just above their cost. Therefore, marginal buyers who should trade in the efficient allocation do not find a trading partner, and the allocation is inefficient.

The first step of the intuition (the possibility of perfect price discrimination) is well-known from the Diamond (1971) paradox. But price discrimination by itself does not create inefficiencies:[1] In every single buyer-seller pair, the full surplus is realized. Inefficiencies arise only because bilateral bargaining is part of a market and sellers can wait and make profits on future buyers. This is illustrated by *the main comparative static result* about the relation between the level of frictions and the realized surplus: inefficiencies become *larger* when the exit rate is smaller, that is, when the market is better integrated; inefficiencies are maximal when the exit rate is close to zero. On the other hand, trading is efficient when the exit rate is close to one, the market is not integrated, and bargaining is essentially bilateral.

The result suggests that it is harmful when sellers can use information about the buyers' willingness to pay to engage in price discrimination. This emphasizes the economic importance of private information or "consumer privacy" as discussed, for example, in Varian (1996); consumer privacy aims at withholding information from the seller. Conventional economic intuition derived solely from the detrimental effects of asymmetric information for bilateral trade might give the misleading impression that there should be as little privacy as possible from an overall welfare perspective. The current paper demonstrates, however, that rents generated by price discrimination can lead to welfare reducing distortions. (In addition, price discrimination can lead to "productive" distortions because it allows inefficient, high cost sellers to trade; see Section 3.)

This paper is also related to the literature on embedding problems of "contract design" into (matching) markets (for example, Inderst (2001, 2004) or Felli and Roberts (2002)). As in their models, a property of exchange between a small set of agents in isolation is fundamentally altered when considered as part of the equilibrium of a market.

Furthermore, perfect price discrimination by sellers implies that trading outcomes are not competitive, even when the friction (the exit rate) is small. Therefore, the current model is an instance of a dynamic matching and bargaining game in which outcomes do not become competitive for vanishing frictions; other examples are Serrano (2002) and De Fraja and Sakovics (2001). Lauermann (2009) contains a general discussion of the

[1] In Diamond's original paper, equilibrium outcomes are inefficient because of his assumption that sellers must use linear prices while individual buyers have elastic demand (rather than unit demand). Thus, monopolistic pricing leads to inefficiency.

conditions under which the limit outcome of a dynamic matching and bargaining game is (or is not) competitive. The focus of the current paper is on the potential inefficiency of the outcome (which is not implied by its non-competitiveness) and on the provision of comparative statics of the equilibrium outcome for *all* levels of the exit rate (rather than the characterization of only the limit).

The next section introduces the model and the result. In Section 3, I discuss the result and the assumptions. In particular, I compare the results with symmetric and asymmetric information. With asymmetric information, perfect price discrimination is not feasible. The discussion of asymmetric information is informal and draws on results from Lauermann (2008).[2] In the last section, I also consider a situation in which buyers have bargaining power and make price offers as well; I argue that bargaining power can substitute asymmetric information and I discuss the implications for the robustness of the result. While the limit result is not robust to the introduction to bargaining power, the general observation that symmetric information is detrimental to efficiency is robust.

## 2 Model and Analysis

**The Model.** There is a continuum of buyers and sellers. Sellers are endowed with one unit of an indivisible good, and their cost of selling are $c = 0$. Buyers want to buy one unit of the good, and their valuations for the good are given by $v \in [0, 1]$. Buyers and sellers interact in a repeated market over infinitely many periods, with time running from minus to plus infinity. At the beginning of each period, there is a stock of buyers and sellers. All traders from the stock are matched into pairs consisting of one seller and one buyer. Within each pair, the seller observes the type of the buyer and then announces a price $p$. The buyer announces whether he accepts or rejects the offer. If he accepts, the seller receives a payoff $p$, while the buyer receives $v - p$; if he rejects, both traders receive nothing. Subsequently, those agents who have already traded leave the market together with a share $\delta$ of those who have not. After that, new players enter the market. The inflow of buyers and the inflow of sellers has mass one each. The distribution of valuations among entering buyers is given by a cumulative distribution function $G(\cdot)$. With the inflow of new traders, the period ends, and the next period starts according to the same rules. The stock is described by $M$, and $\Phi^B(\cdot)$: $M$ denotes the mass of buyers in the stock at the beginning of each period. This mass is equal to the mass of sellers. The function $\Phi^B(\cdot)$ describes the distribution of buyers' types $v$ in the stock. This distribution $\Phi^B(\cdot)$ is, of

[2] Ideally, one would have liked to have a full analysis of the polar case with asymmetric information in the current paper as well but analyzing both models is beyond the scope of a note.

course, endogenous and depends on how agents trade. The distribution of types in the inflow, $G(\cdot)$, on the other hand, is exogeneous.

The mass of entering buyers with valuations above some given $v$, $(1 - G(v))$, can be interpreted as demand. The mass $(1 - G(v))$ is assumed to be strictly decreasing and its density $g(\cdot)$ exists and is strictly positive. Supply, that is, the mass of entering sellers, is equal to one. Let $p^w$ be the Walrasian price at which demand is equal to supply,$1 - G(p^w) = 1$, clearly $p^w = 0$. (If sellers are heterogeneous, the Walrasian price is interior, see Section 3.)

I restrict attention to equilibria in which sellers use symmetric and stationary pricing strategies $P(\cdot, \cdot)$, where $P(p', v)$ is the probability of offering a price $p \leq p'$ to a buyer having type $v$; $P(\cdot, v)$ is a cumulative distribution function. The payoff to a seller who uses a pricing strategy $P(\cdot, \cdot)$ can be derived as follows. Denote by $D(P(\cdot, \cdot))$ the probability of trading in any given period,[3] and denote by $q^S(P(\cdot, \cdot))$ the probability that a seller is able to trade at some time during his lifetime. Given $D(P(\cdot, \cdot))$, one can derive $q^S(P(\cdot, \cdot))$ recursively from $q^S(P(\cdot, \cdot)) = D(P(\cdot, \cdot)) + (1 - D(P(\cdot, \cdot)))(1 - \delta) q^S(P(\cdot, \cdot))$. Since there is no discounting, the seller does not care about when he conducts a trade but only about whether he is able to trade before he must exit and what price he gets. (Including a discount rate would not change results.) Denote by $E[p|P(\cdot, \cdot)]$ the expected price conditional on being able to trade.[4] The expected profit is $\Pi(\cdot)$, where $\Pi(P(\cdot, \cdot)) = q^S(P(\cdot, \cdot)) E[p|P(\cdot, \cdot)]$ .

To derive the optimal search strategy of a buyer, observe that he is essentially sampling without recall from a known and constant distribution of prices. For this problem, it is well known that the optimal solution can be described by a threshold, a reservation price $r$, such that a price $p$ is accepted if and only if $p \leq r$ (see McMillan and Rothschild (1994)). The payoff to a buyer of type $v$ with a reservation price $r$ depends on the expected price offer, $E[p|p \leq r, v]$,[5] and the probability of trading some time during his lifetime, that is, to receive an acceptable offer $p \leq r$ before being forced to exit, denoted by $q^B(r, v)$; expected payoffs are $U^B(r, v) = q^B(r, v)(v - E[p|p \leq r, v])$. Let $V(v) = sup_r U^B(r, v)$ be the maximized expected lifetime payoff. At the reservation price $r(v)$, buyers must be indifferent between acceptance and rejection, so $v - r(v) = (1 - \delta) V(v)$. Rewriting yields

$$r(v) = v - (1 - \delta) V(v) . \tag{1}$$

[3] Buyers accept $p$ if $p$ is below their reservation price $r(v)$ as defined below. Given $r$, $D(P(\cdot, \cdot)) = \int_0^1 P(r(\tau), \tau) d\Phi^B(\tau)$. Here and throughout the paper I abstract from measurability problems.

[4] Let $E[p|P(\cdot, \cdot)] = 0$ if $q^S(P(\cdot, \cdot)) = 0$.

[5] Let $E[p|p \leq r, v] = r$ if $q^B(r, v) = 0$.

The price offer $P(\cdot, v)$ shall be optimal for every possible type $v$. For this, let $U^S(p, v|P(\cdot,\cdot))$ denote the profit of a seller who offers a price $p$ to a buyer of type $v$ and continues according to the strategy $P(\cdot,\cdot)$, where $U^S(p, v|P(\cdot,\cdot)) = p$ if the price is accepted (if $p \leq r(v)$) and $U^S(p, v|P(\cdot,\cdot)) = (1-\delta)\Pi(P(\cdot,\cdot))$ if the price is rejected. Every price in the support of $P(\cdot, v)$ is optimal if for all $p \in \text{supp} P(\cdot, v)$, the price is such that $p \in \arg\max U^S(p, v|P(\cdot,\cdot))$ for all $v$.

The market is in a steady state if the inflow is equal to the outflow. The inflow of buyers with valuations below $v$ is $G(v)$. The outflow consists of all buyers who trade plus those buyers who die. Equality of in- and outflows holds if

$$G(v) = M \int_0^v [P(r(\tau), \tau) + \delta(1 - P(r(\tau), \tau))] \, d\Phi^B(\tau),$$

and similarly for sellers, $1 = M[D(P(\cdot,\cdot)) + \delta(1 - D(P(\cdot,\cdot)))]$.

A steady state equilibrium consists of a pair of two strategies, $P(\cdot,\cdot)$ and $r(\cdot)$, the steady state distribution $\Phi^B(\cdot)$, and the mass $M$ of traders, such that the price offer strategy $P(\cdot,\cdot)$ maximizes ex ante expected profits $\Pi(\cdot)$, every price in the support of the offer strategy is optimal for every $v$, reservation prices satisfy (1), and the steady state conditions hold.

**Result.** For every exit rate $\delta$, the pricing strategy is characterized by a unique cutoff $\bar{v}(\delta)$. The price offer is unacceptable for all buyers with a valuation below $\bar{v}$ (the probability of being offered a price at or below the valuation $v$, $P(v, v)$ is zero for all $v < \bar{v}$). For all other buyers the price offer is merely acceptable ($P(v, v) = 1$ for all $v > \bar{v}$). The cutoff type is decreasing in $\delta$: more buyers can trade when frictions are large and the equilibrium outcome is therefore more efficient. The share of buyers who trade is minimal when the exit rate vanishes. In particular, the cutoff does not converge to its efficient level, which would be zero. Equilibrium is essentially unique for all $\delta$; only the price offer to the cutoff type $v = \bar{v}$ and the exact unacceptable offers to types $v < \bar{v}$ are not determined.

**Proposition 1** *For every exit rate, there exists an equilibrium described by a unique cutoff* $\bar{v}(\delta)$ *such that*

$$\bar{v}(\delta) = (1-\delta) \int_{\bar{v}}^1 v g(v) \, dv \qquad \text{and} \qquad P(v, v) = \begin{cases} 0 & \text{if } v < \bar{v} \\ 1 & \text{if } v > \bar{v}. \end{cases}$$

*and* $P(p, v) = 0$ *for all prices* $p$ *below valuation* $v$. *Thus, buyers having types above the cutoff trade with probability one at a price equal to their valuation; types below the cutoff do* *not trade. The cutoff* $\bar{v}(\delta)$ *is lower when the exit rate is higher; at the extremes* $\bar{v}(1) = 0$ *and* $\lim_{\delta \to 0} \bar{v}(\delta) \equiv \bar{v}_* > 0$.

The proposition is proven in the remainder of the section. Take a sequence of strictly decreasing exit rates $\{\delta_k\}_{k=1}^{\infty}$ with $\lim_{k\to\infty} \delta_k = 0$. For each $k$ at least one equilibrium exists (see below); fix one equilibrium for each $k$. Given the equilibrium, let $l_k(v)$ denote the lowest price offered to a type $v$, defined as $l_k(v) \equiv \inf\{p : P_k(p, v) > 0\}$. It follows from reasoning familiar from the Diamond paradox that price offers are at least equal to the willingness to pay, $l_k(v) \geq v$. Suppose not, so that there is some equilibrium with a pricing strategy $P(\cdot, \cdot)$ with $l_k(v) < v$ for some $v$ and $\delta_k$. Because of the probability of exiting while waiting, a buyer of type $v$ is willing to pay a premium of $\delta_k(v - l_k)$ and his reservation price $r_k$ is above $l_k(v)$: $r_k = v - (1 - \delta_k) V_k(v) > l_k$, from $V_k(v) \leq (v - l_k(v))$. Hence, by definition of $l_k(v)$, there is some $p' \in suppP(\cdot, \cdot)$ such that $p' < r_k(v)$. However, offering a price $p$ equal to the reservation price would strictly increase profits, that is,

$$U^S\left(p', v|P(\cdot, \cdot)\right) = p' < r_k(v) = U^S\left(r_k(v), v|P(\cdot, \cdot)\right),$$

and thus there is some $p'$ is the support of $P(\cdot, v)$ that fails to be optimal. Therefore, $P(\cdot, v)$ cannot be an equilibrium strategy unless $l_k(v) \geq v$, which implies that $P(p, v) = 0$ for all prices $p$ below valuations $v$. Therefore, in every equilibrium sellers either offer an acceptable price $p = v$ or some unacceptable price above the valuation. In both cases payoffs to the buyer are zero; hence, $r_k(v) = v$.

Let $\Pi_k^*$ denote the equilibrium profit of sellers. The profit determines the optimal offer strategy $P$. Consider a seller who is matched with a type $v'$: If this valuation is strictly above the continuation value of the seller, that is, if $v' > (1 - \delta_k)\Pi_k^*$, then the optimal offer is clearly a price $p = v'$; in this case, the seller makes more revenue from trading than from waiting further. If the valuation is below the continuation value, then any unacceptable price $p > v'$ is optimal. Hence, profits determine a cut-off type $\bar{v}_k^*$ with $\bar{v}_k^* \equiv (1 - \delta_k)\Pi_k^*$ such that every strategy in equilibrium is characterized by $l_k(v) \geq v$ (from before) and

$$P_k^*(v, v) = \begin{cases} 0 & \text{if } v < \bar{v}_k^* \\ 1 & \text{if } v > \bar{v}_k^*. \end{cases} \tag{2}$$

Note that the exact unacceptable offer is not specified uniquely, and the optimal offer to the type $\bar{v}$ may or may not be acceptable.

Take any pricing strategy that has the necessary equilibrium structure derived before, that is, take any $P(\cdot, \cdot)$ such that the lowest price offer is $v$, $l(v) \geq v$ and such that (2) holds for some $\bar{v}$, $P(v,v)=1$ if $v>\bar{v}$ and $P(v,v)=0$ if $v<\bar{v}$. Denote this strategy by a subscript $\bar{v}$, $P_{\bar{v}}(\cdot,\cdot)$. If sellers use this strategy, it is possible to calculate the expected profit, denoted by $\Pi_k(P_{\bar{v}}(\cdot,\cdot))$. A strategy $P_{\bar{v}}(\cdot,\cdot)$ is part of an equilibrium if and only if

$$\bar{v}=(1-\delta_k)\,\Pi_k(P_{\bar{v}}(\cdot,\cdot))\,. \tag{3}$$

To derive $\Pi_k(P_{\bar{v}}(\cdot,\cdot))$, one needs to know the trading probability of a seller and the expected price he receives. For the former, note that all entering buyers with valuations above $\bar{v}$ trade immediately; no other buyer can trade. Thus, the total mass of buyers who enter the market and who trade is $(1-G(\bar{v}))$. In a steady state, this mass of trading buyers must be exactly equal to the mass of trading sellers. Therefore, a mass $(1-G(\bar{v}))$ of all sellers in the inflow trade, and hence

$$q_k^S(P_{\bar{v}}(\cdot,\cdot))=1-G(\bar{v})\,. \tag{4}$$

The expected price a seller receives conditional on trading, $E[p|P_{\bar{v}}(\cdot,\cdot)]$, is simply the expected valuation given $v\geq\bar{v}$. Note that buyers with such a valuation remain in the market only for one period; consequently, the distribution of their types is given by the original distribution in the inflow, $G(\cdot)$. Thus:

$$E[p|P_{\bar{v}}(\cdot,\cdot)]=\frac{1}{1-G(\bar{v})}\int_{\bar{v}}^{1}vg(v)\,dv,$$

and profits are

$$\Pi_k(P_{\bar{v}}(\cdot,\cdot))=q_k^S(P_{\bar{v}}(\cdot,\cdot))\,E[p|P_{\bar{v}}(\cdot,\cdot)]=\int_{\bar{v}}^{1}vg(v)\,dv. \tag{5}$$

Using this observation and condition (3), a pricing strategy $P_{\bar{v}}(\cdot,\cdot)$ is optimal if and only if $\bar{v}$ satisfies

$$\bar{v}=(1-\delta_k)\int_{\bar{v}}^{1}vg(v)\,dv. \tag{6}$$

A solution to this equation exists by the intermediate value theorem. Both sides of the equation are continuous, at $\bar{v}=0$ the right-hand-side is strictly above zero, while at $\bar{v}=1$ the right-hand-side is zero. Hence, an equilibrium exists: If all other sellers use the pricing strategy $P_{\bar{v}}$, then $P_{\bar{v}}$ is optimal. Reservation prices are $r_k(v)=v$. Given $\bar{v}$, the equilibrium stock can be determined as follows. The mass of buyers in the stock with valuations above $\bar{v}$ is $1-G(\bar{v})$. The mass of buyers with valuations below $\bar{v}$ is $G(\bar{v})/\delta_k$, since only then is the mass of their inflow, $G(\bar{v})$, equal to the mass of the outflow due to exit, $\delta_k(G(\bar{v})/\delta_k)$. Thus, the total mass of buyers and the total mass of sellers is

$M = (1 - G(\bar{v})) + G(\bar{v})/\delta_k$. The distribution of buyer's types follows easily from their original distribution in the inflow: conditional on $v \geq \bar{v}$, types are distributed according to $G(\cdot)$; likewise for $v < \bar{v}$. Furthermore, the solution to (6) is unique because the right-hand-side is strictly decreasing in $\bar{v}$. Hence, the equilibrium stock of buyers and sellers, equilibrium payoffs, and the acceptable price offers are uniquely determined as claimed.

For the comparative statics, note that condition (6) defines the cutoff $\bar{v}$ as an implicit function of the exit rate $\delta$. Taking the total derivative on both sides of (6) shows that the solution $\bar{v}$ is strictly decreasing in $\delta_k$,

$$\frac{\partial}{\partial \delta}\bar{v}(\delta) = -\int_{\bar{v}(\delta)}^{1} vg(v)\,dv - (1-\delta)\,\bar{v}g(\bar{v}) < 0.$$

Thus, the higher the exit rate, the more buyers can trade and the more surplus is realized. Finally, let $\bar{v}_*$ be the limit of $\bar{v}_k$ as $\delta_k$ becomes zero, $\bar{v}_* = \lim_{k\to\infty} \bar{v}_k$ with $\delta_k \to 0$. This limit exists by $\bar{v}_k$ being decreasing in $\delta_k$. In the limit, (6) becomes $\bar{v}_* = \int_{\bar{v}_*}^{1} vg(v)\,dv$; clearly $\bar{v}_* = 0$ is not a solution because the expected willingness to pay of a buyer in the inflow is positive, $E[v] > 0$. This implies $\bar{v}_* > 0$, completing the proof of the proposition.

## 3 Remarks and Conclusion

**Asymmetric Information and Price Discrimination.** As noted in the introduction, the equilibrium becomes efficient with $\delta \to 0$ if information is asymmetric, that is, if sellers do not observe the valuation of buyers before they make an offer. Recall that the source of the inefficiency in the model is the possibility of sellers making strictly positive profits. With symmetric information this is true even if sellers are willing to trade with all buyers since the expected profit from price discrimination is $E[v|v \geq 0]$, which is strictly positive (see Equation 5). This is different with asymmetric information: If the sellers are willing to trade with all buyers, they must be offering a uniform price $p = 0$. This implies that their profits are zero. For this reason, asymmetric information is important for efficiency: If valuations are private information, (perfect) price discrimination becomes impossible. Therefore, profits when trading with all types above some threshold $\bar{v}$ are, at most, $\bar{v}$ itself. With symmetric information, profits when trading with all buyers having a valuation above some threshold $\bar{v}$ might be strictly larger than $\bar{v}$.

Asymmetric information can be modeled in the current framework by restricting the possibility of sellers to condition their price offer on the type of the buyer. Then, in a symmetric, pure strategy equilibrium,[6] for all sellers the price offer strategy $P$ must put probability one on some common price $\bar{p}$, independent of the type of the buyer. The offer is accepted by all buyers with $v \geq \bar{p}$ and each seller's trading probability is $q^S(\bar{p}) = 1 - G(\bar{p})$ (see the derivation of seller's trading probability before, equation (4)). The profit of a seller is equal to the trading probability times the price,

$$\Pi^S(\bar{p}) = (1 - G(\bar{p})) \quad \bar{p} \qquad \text{for all } \delta. \tag{7}$$

Unless $\bar{p} = 0$, the trading probability is less than one, that is, in basic economic terms, *sellers are rationed if prices are too high.* Furthermore, for a given $\bar{p} > 0$ set by the other sellers, consider a single seller who offers some $p'$ strictly below $\bar{p}$; as shown in Lauermann (2008), the trading probability for the single seller becomes one when the exit rate is small

$$q^S(p') \to 1 \qquad \text{for all } p' < \bar{p},\ \delta \to 0.$$

This is a simple consequence of the fact that buyers with valuations below $\bar{p}$ do not find any trading opportunity, making up a strictly positive share of the stock. Thus, when a seller can sample more and more buyers before exiting, a seller who offers $p'$ trades with probability approaching one. As shown in Lauermann (2008), this implies that it cannot be the case that prices are positive in the limit: Each seller would have an incentive to lower his price to some $p'$ close to $\bar{p}$ and trade with probability one rather than being rationed at $\bar{p}$ itself. The critical difference between symmetric information and asymmetric information can be seen by comparing the profit with asymmetric information from equation (7) for $\bar{p} > 0$ with the profit with symmetric information from equation (5) for $\bar{v} > 0$,

$$\begin{aligned} \Pi^S(\bar{p}) &= (1 - G(\bar{p})) \quad \bar{p} && \text{(Asymmetric Information)} \\ \Pi^S(\bar{v}) &= (1 - G(\bar{v})) \quad E[v|v \geq \bar{v}] && \text{(Symmetric Information)} \end{aligned}$$

noting that $E[v|v \geq \bar{v}] > \bar{v}$. Therefore, with symmetric information, sellers do not necessarily have an incentive to trade with types below $\bar{v}$ - despite being rationed.

**Bargaining Power.** Increasing the bargaining power of buyers makes perfect price discrimination impossible. Suppose buyers themselves can make offers as in Gale (1987): Within each buyer-seller pair, with probability $\beta$ the buyer is the proposer of a take-it-or-leave-it price offer and with probability $(1 - \beta)$ the seller is the proposer. When $\beta > 0$, the limit with $\delta \to 0$ is efficient; this follows from Lauermann (2009) or Gale (1987).

[6] The analysis in Lauermann (2008) covers not only symmetric *pure* strategy but also allows for symmetric *mixed* strategies.

As discussed in Lauermann (2009), letting buyers make offers with positive probability $\beta > 0$ is similar to introducing asymmetric information; although it is still true that the offer that is *received* by a buyer depends on the type, all types can now *make* the same offers when chosen as the proposer. Importantly, in equilibrium, payoffs depend only on the offers made as a proposer. (If a trader is chosen to be the responder, the proposer makes an offer that makes the responding trader indifferent between accepting and rejecting.) Therefore, a buyer of a given type can mimic the strategy of another type similar to a setting with asymmetric information. Without perfect price discrimination, when the exit rate becomes small, trade happens at a price of zero and the limit is competitive. Thus, the discussion suggests that bargaining power can substitute asymmetric information.

**Robustness.** Although the limit is competitive when $\beta > 0$ and $\delta \to 0$, convergence to the competitive outcome is not uniform in $\beta$: For every $\delta$, there is some $\beta$ small enough such that buyers receive almost no surplus. Therefore, given any level of frictions $\delta$, the trading outcome can be far away from the competitive outcome in terms of buyer's payoffs and, by the same reasoning, given any $\delta$ close to zero, the trading outcome can be far away from efficiency in terms of realized surplus. Since equilibrium converges uniformly with asymmetric information, this observation implies the following: there is an open set of parameter values for the exit rate and the bargaining power, $\delta$ and $\beta$, such that the equilibrium with full information is less efficient than any equilibrium with asymmetric information. Therefore, the fact that asymmetric information can be good for welfare is robust in the parameter space. (The limit result as such is not.)

**Constrained Efficiency.** In the current model, the constrained efficient outcome is first best efficient: If sellers offer a price of zero then buyers can trade immediately and the full surplus relative to the inflow is realized.[7] In fact, the equilibrium itself is first best efficient if $\delta = 1$. Therefore, all inefficiencies stem only from the strategic behavior of the agents. The possibility of strategic behavior, in turn, is a result of the exit rate which gives sellers local market power and allows them to charge a price above costs.

In standard search models, frictions can affect the trading outcome through two channels: indirectly through the impact on strategic behavior and directly through the fact that the first best efficiency is physically not feasible. To the best of my knowledge, this is the only search model in which the indirect effect of strategic behavior can be studied in isolation from the direct effect.

[7] The reason why the current matching technology allows the first best trading outcome is the simplifying assumption that sellers are homogeneous with costs $c = 0$; see the discussion of heterogeneity below.

**Heterogeneous Sellers.** If sellers are heterogeneous with costs distributed according to some smooth distribution function $G^S(\cdot)$, then trading is still not efficient in the limit. First, to define efficiency, recall that the market-clearing price $p^w$ is such that $G^S(p^w) = 1 - G(p^w)$. In an efficient allocation all buyers having valuations above $p^w$ should trade with sellers having costs below $p^w$ - and no one else. Second, by the same reasoning as in the main model, buyers make zero payoffs and accept all prices below their valuation. An immediate consequence is this: sellers with costs above $p^w$ can trade (although they should not) with all buyers who have a valuation above their own costs, that is, with all $v \geq c > p^w$. Price discrimination allows unproductive sellers to trade, which makes the outcome inefficient. This is yet another reason for inefficiencies arising from symmetric information, which is distinct from the one analyzed in the main model.

**Conclusion.** Information has a counterintuitive role in dynamic matching and bargaining games: If traders are well informed, then the market outcome might not be efficient even for small frictions. This result is despite the fact that with symmetric information bargaining between all traders is efficient. Considering a market with symmetric information is an insightful thought experiment. One can use the current result, by way of contrast, to understand better existing models of decentralized markets in which the trading outcome becomes efficient.

## References


[1] De Fraja, Gianni; Sakovics, Jozsef (2001): "Walras Retrouve: Decentralized Trading Mechanisms and the Competitive Price," *Journal of Political Economy*, Vol 109, 842-863.

[2] Diamond, Peter A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 158-68.

[3] Felli, Leonardo; Roberts, Kevin (2002): "Does Competition Solve the Hold-up Problem?," CEPR Discussion Papers 3535.

[4] Gale, Douglas (1987): "Limit Theorems for Markets with Sequential Bargaining," *Journal of Economic Theory*, Vol 43, 20-54.

[5] Inderst, Roman (2001): "Screening in a Matching Market," *Review of Economic Studies*, Vol. 68, 849-68.

[6] Inderst, Roman (2004): "Matching Markets with Adverse Selection," *Journal of Economic Theory*, 145-166.

[7] Lauermann, Stephan (2008): "Price Setting in a Decentralized Market and the Competitive Outcome," MPI Collective Goods Preprint, No. 6, Mimeo, Bonn.

[8] Lauermann, Stephan (2009): "Dynamic Matching and Bargaining Games: A General Approach," Mimeo, University of Michigan.

[9] McMillan, J. and M. Rothschild (1994): "Search," in: RJ Aumann and S. Hart (eds.), *Handbook of Game Theory*, Vol 2, Elsevier.

[10] Satterthwaite, Mark; Shneyerov, Artyom (2007): "Dynamic Matching, Two-Sided Incomplete Information and Participation Costs: Existence and Convergence to Perfect Competition," *Econometrica*, Vol 75, 155 - 200.

[11] Satterthwaite, Mark; Shneyerov, Artyom (2008): "Convergence of a Dynamic Matching and Bargaining Market with Two-sided Incomplete Information to Perfect Competition," *Games and Economic Behavior*, Vol 63, 435-467.

[12] Serrano, Roberto (2002): "Decentralized Information and the Walrasian Outcome: A Pairwise Meetings Markets with Private Information," *Journal of Mathematical Economics*, Vol 38, 65-89.

[13] Varian, Hal (1996): "Economic Aspects of Personal Privacy," *U.S. Dept. of Commerce*, Privacy and Self-Regulation in the Information Age, 1996.