Linking Stocks to Flows by Strategies: On the Strategic Foundations of Competitive Equilibria.*

Extended Abstract

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> "A strategic foundation for competitive equilibrium must show how strategic interaction by rational agents leads to competitive, price-taking behavior." (Gale, 2000, p.1)

In this paper we reexamine the strategic foundations of competitive equilibria. We do so in the context of a dynamic matching and bargaining model with a time invariant inflow of new traders. We endorse the view expressed in Gale (2000, p.3) that a strategic foundation should investigate a model in which all endogenous variables are determined by choices of the agents in the model and any feasible profiles of strategies chosen by the agents determines a unique feasible outcome. As in Gale (1987, section 6) and most of the literature building on his contribution, we restrict attention to stationary strategy profiles and identify outcomes with steady-states.

We contend that Gale's steady state model does not provide strategic foundations for competitive equilibria in this sense and propose a simple modification aimed to rectify this problem: rather than assuming that agents are infinitely lived and discount future payoffs, we assume that agents face a strictly positive death rate. As we show this modification ensures that every stationary strategy profile determines a unique steady state. This is in contrast to Gale's model. With infinitely lived agents only stationary strategy profiles in which the same numbers

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(formally: measures) of buyers and sellers enter the market and trade with probability one are consistent with a steady state and, in addition, whenever they exist steady states are not uniquely determined by the strategy profile.

Having established that in our model strategy profiles do indeed determine outcomes (and also having shown existence of equilibria), we next turn to the question whether the consideration of the frictionless limit provides foundations for competitive equilibria. (We find it convenient to formulate our model in continuous time and consider the limit as the matching rate goes to infinity rather than the limit as the death rate goes to zero. This does not affect the substance of our conclusions but simplifies the interpretation of some of them.)

We begin with a simple example demonstrating that the convergence properties of equilibria in our model are more subtle than one might expect. In this example there exists a sequence of equilibrium outcomes converging to a limit in which the law of one price fails (i.e., not all trades take place at the same price) and convergence to a competitive equilibrium thus does not obtain. At the same time there also exists a sequence of equilibria satisfying the law of one price with the limit price satisfying (as in Gale) the flow market-clearing conditions *and* (in contrast to Gale) the stock market-clearing conditions with respect to the endogenous limit stocks.

A closer examination of our example suggests that assuming costs and valuations to be drawn from the same finite grid and imposing a genericity assumption might suffice to ensure the law of one price in the limit. Our main convergence result shows that this is indeed the case. Furthermore the limit price clears both flows and stocks. Our model thus not only provides strategic foundations for competitive equilibrium, but resolves the tension between the flow concept and stock concept of market clearing present in both Gale (1987) and Rubinstein and Wolinsky (1985).