EECS 661

Modeling and Optimal Control of Hybrid Systems: Two Case Studies

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Objective

Investigate hybrid system modeling approaches that can be easily used for control design.

ANSWER:

Mixed Logical Dynamic Systems (Bemporad and Morari, 1999)

- What are Mixed Logical Dynamic (MLD) Systems?
  - Integrates dynamics and logic as linear difference equations and linear inequalities
- Why is MLD useful?
  - Optimal control results naturally from MLD formulation
  - Control via numerical optimization routines (e.g. mixed integer quadratic programming (MIQP))
  - Limitations of hybrid automata
    - Servo-level control

Outline

- Objective
- Mixed Logical Dynamical (MLD) systems
- Timed Automata to MLD
- Case study 1
  - On/off controller for MEMS system
- Case study 2
  - Battery charge equalization
- Summary

MLD Representation

$$x(t+1) = Ax(t) + B_1u(t) + B_2\delta(t) + B_3z(t)$$

$$y(t) = Cx(t) + D_1u(t) + D_2\delta(t) + D_3z(t)$$

$$E_2\delta(t) + E_3z(t) \leq E_1u(t) + E_4x(t) + E_5$$

x: States

y: Outputs

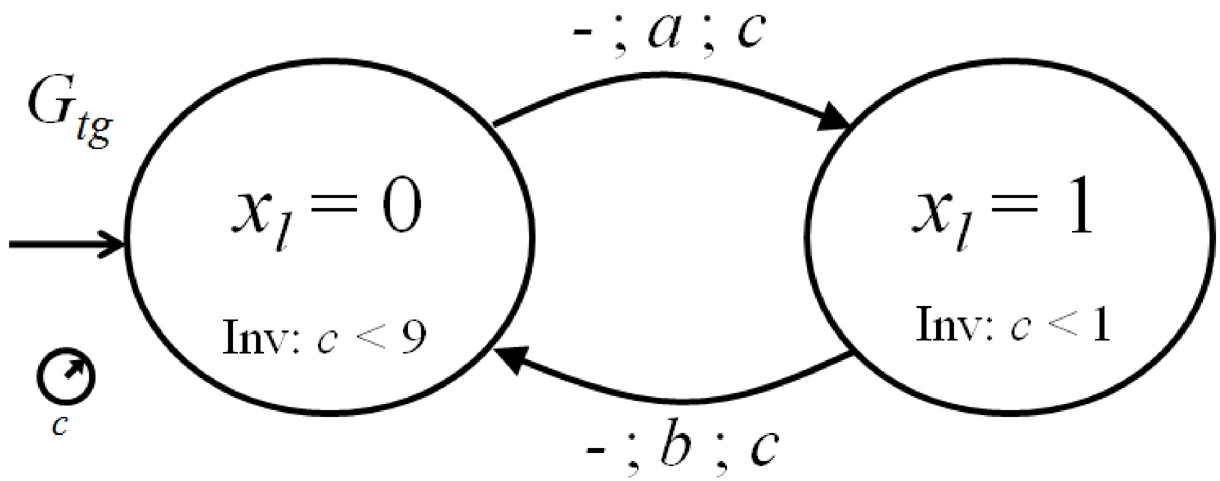
u: Inputs

} Continuous and discrete components

z: Continuous auxiliary variables

δ: Discrete auxiliary variables

Timed automata to MLD : an example



$$c(t+1) = c(t) + 1$$

$$(0,0) \xrightarrow{3} (0,3) \xrightarrow{a} (1,0) \xrightarrow{b} (0,0) \xrightarrow{8} (0,8) \xrightarrow{a} (1,0) \xrightarrow{b} (0,0)\ldots$$

$$(a,4),(b,5),(a,14),(b,15),(a,24),(b,25)\ldots$$

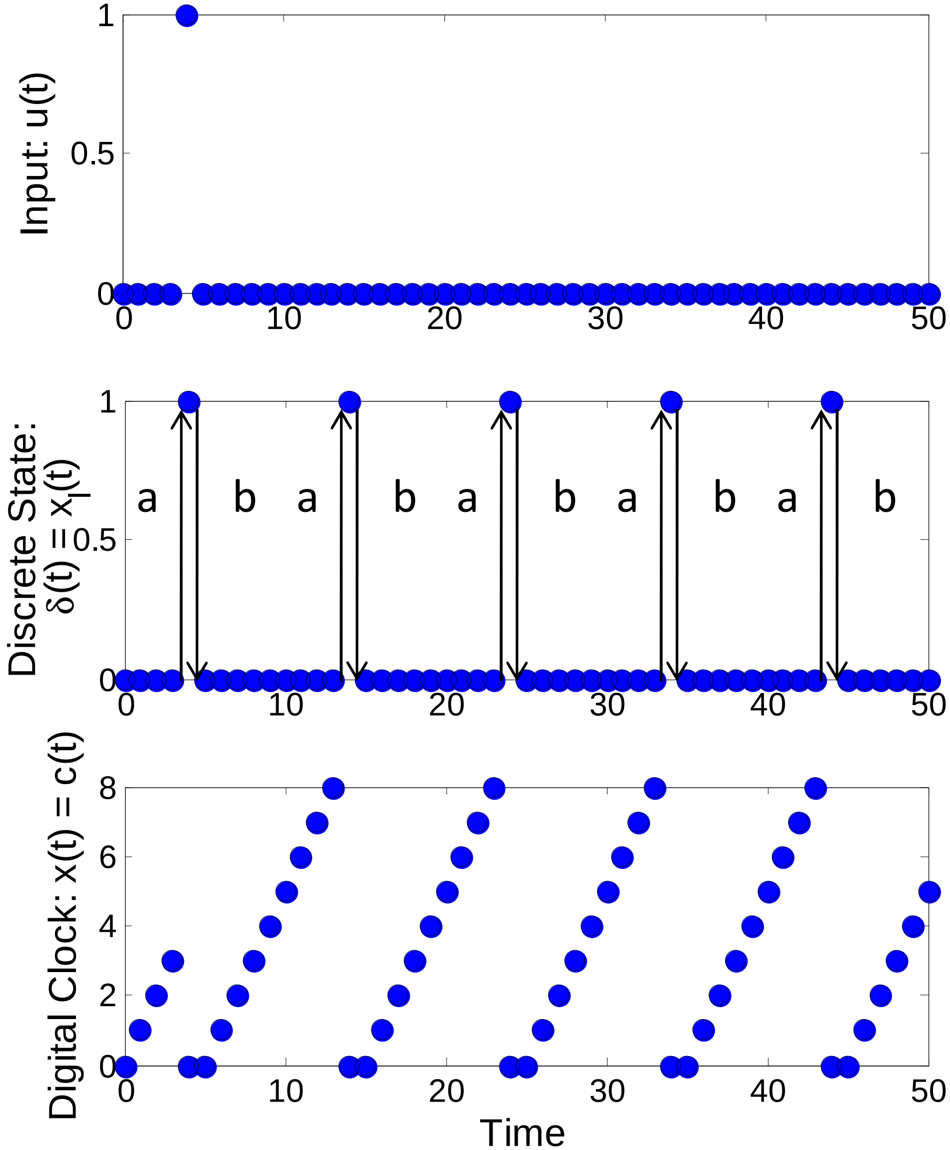
Response from MLD

$$x(t+1) = z(t)$$

$$E_2\delta(t) + E_3 z(t) \leq$$

$$E_1 u(t) + E_4 x(t) + E_5$$

- Output equivalent to run of G_{tg}
- Models clock dynamics & reset



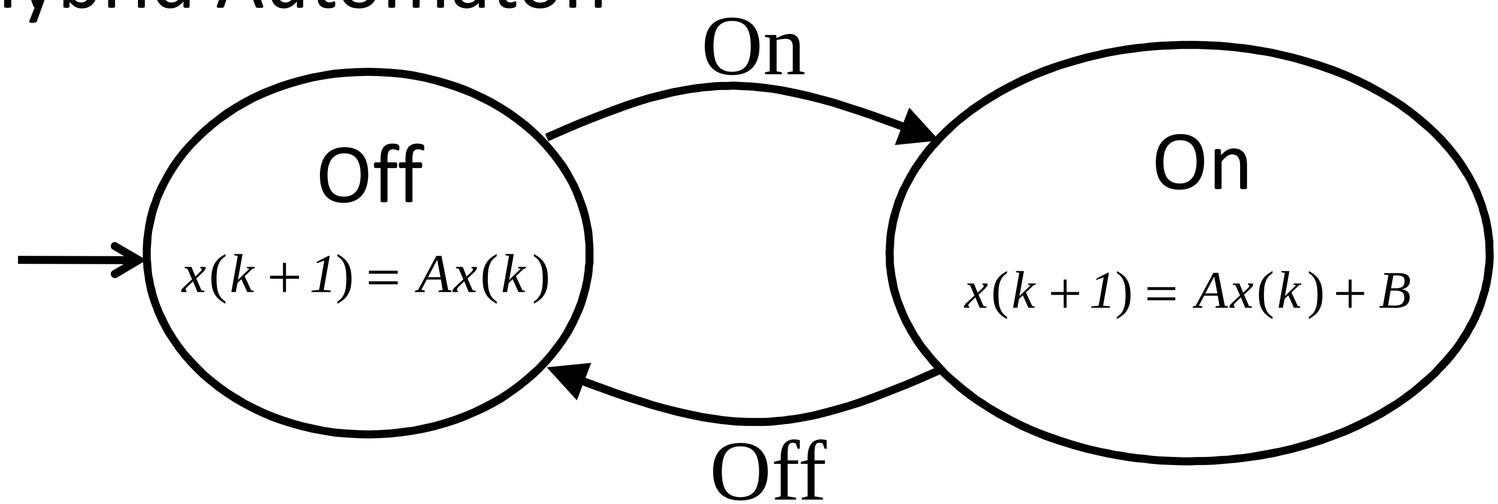
$$(a,4),(b,5),(a,14),(b,15),(a,24),(b,25)\ldots$$

Case study 1: Optimal On-Off Controller for an autonomous MEMS system

- Problem
  - Reach the neighborhood of a final desired state at a prescribed time with the use of minimal energy from a given initial state
- Motivation
  - Energy budget is limited in autonomous MEMS systems
  - High energy loss in analog amplifiers used for piezo-electric actuators
  - Energy loss during charging and discharging in Pulse Width Modulation (function of switching frequency)

Hybrid system representation

- Hybrid Automaton



- MLD

$$x(k+1) = Ax(k) + Bu(k),\ u(k) \in \{0,1\}$$

Details of system and energy costs

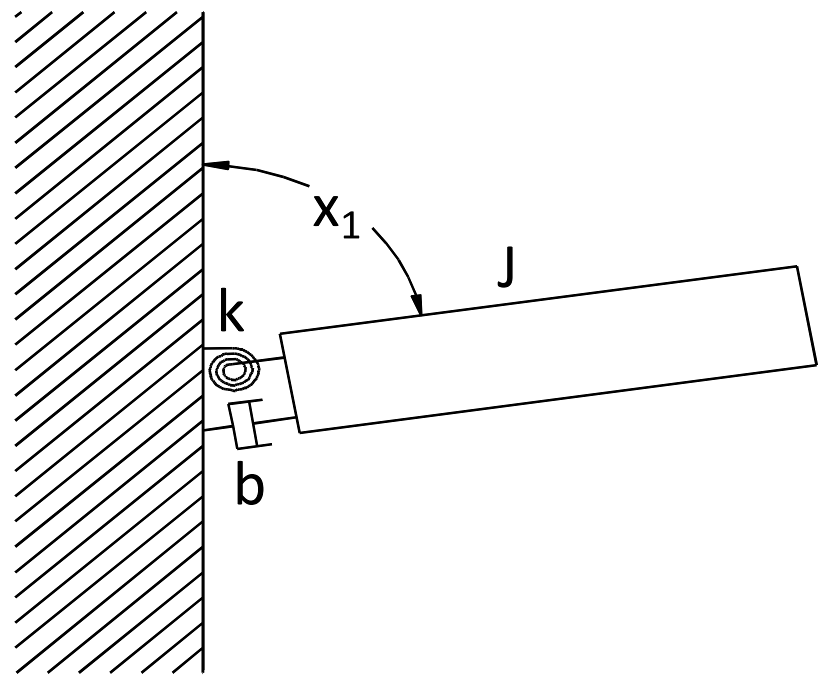
- states of the system
  - x1 : angle (rad)
  - x2 : angular velocity (rad/sec)
- Energy requirement
  - J_c : transition cost

$$J_C = \frac{1}{2} C U_{\max}^2 \left[\sum_{k=1}^{n} (u_k - u_{k-1})^2 + u_0^2 \right]$$

$$C = \text{Capacitance}$$

  - J_R : resistive cost

$$J_R = \sum_{k=0}^{n} \frac{U_{\max}^2}{R} T_s u_k, \; R = \text{Resistance}$$

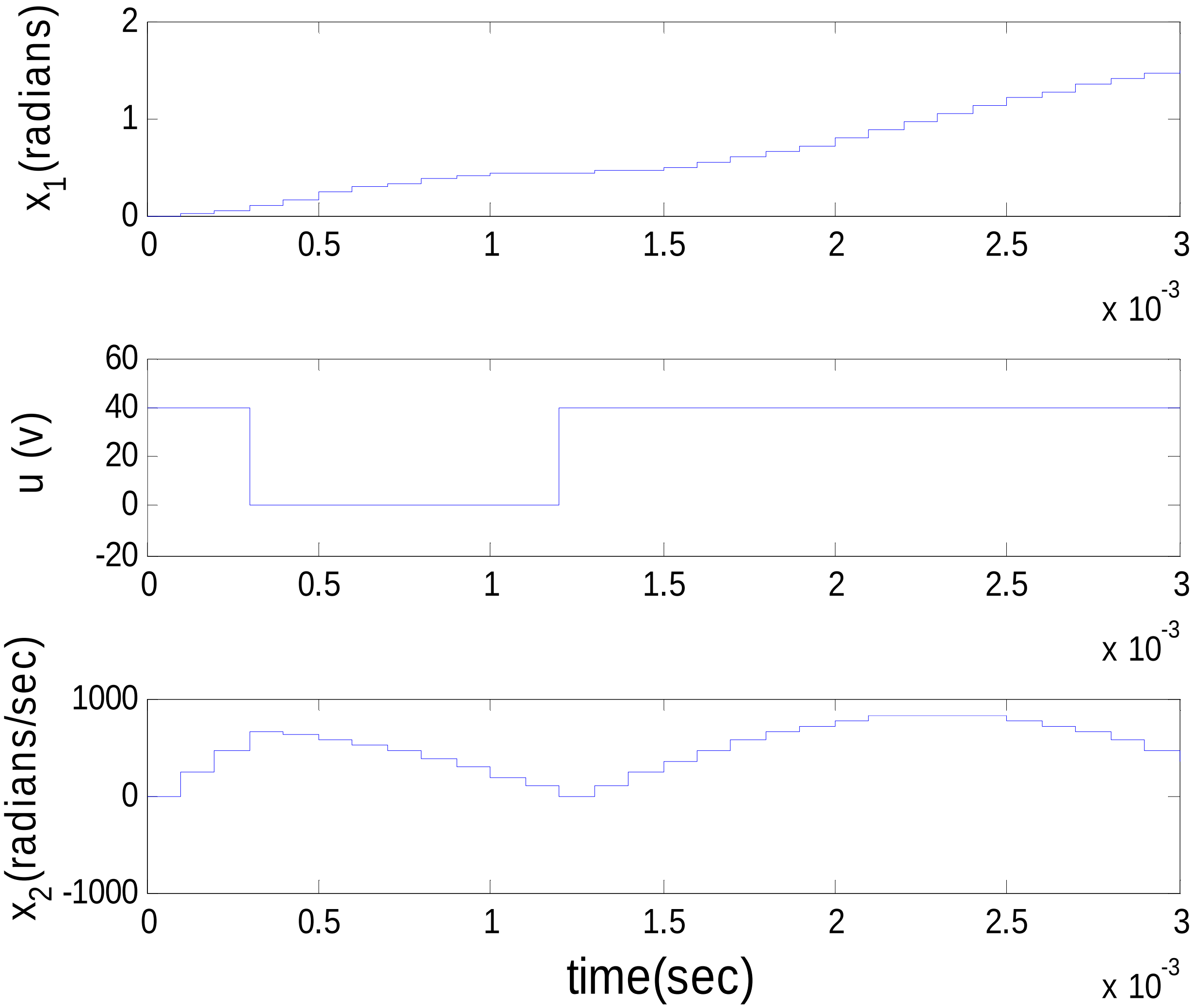


Result : Optimal sequence using MIQP

$$x_d = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}, \varepsilon = \begin{bmatrix} 0.01 \\ 1000 \end{bmatrix}$$

$$U_{\max} = 40V$$

- Achieved the target with two transitions
- Successfully implemented MIQP on a MLD system to obtain an optimal on/off sequence



Case study 2 : Switched Capacitor Circuits for Battery Charge Equalization

Research Question:

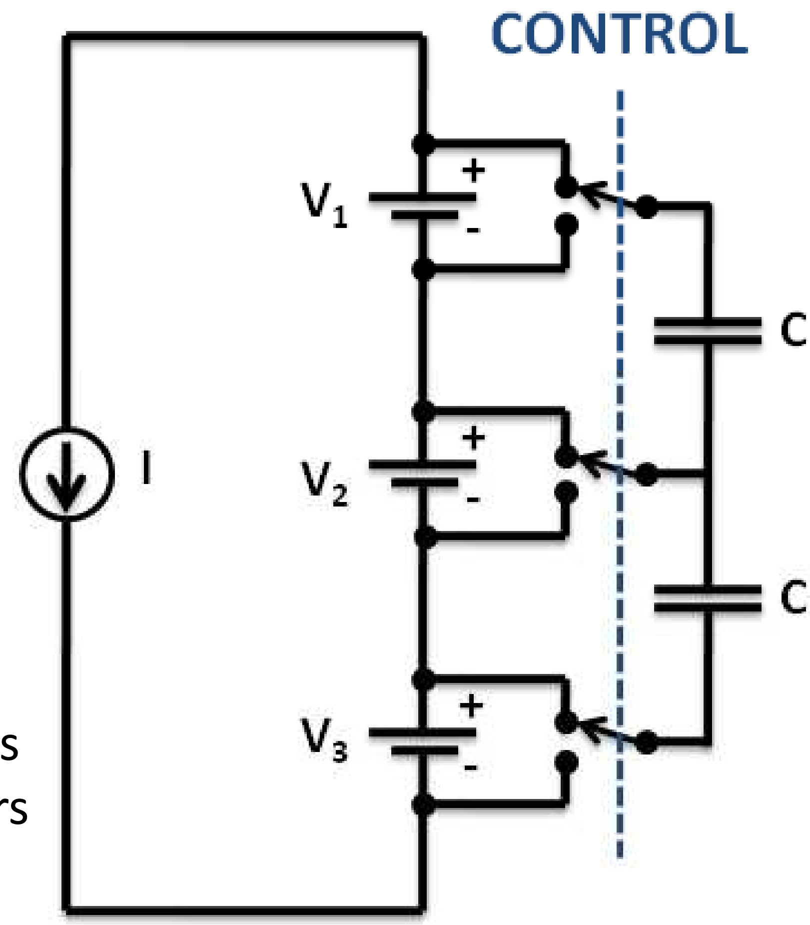
How do we control battery packs to minimize health degradation?

Key Challenge:

Varying charge levels in battery cells arranged in series may cause damage from
– Over/under charging individual cells
– Over/under discharging individual cells

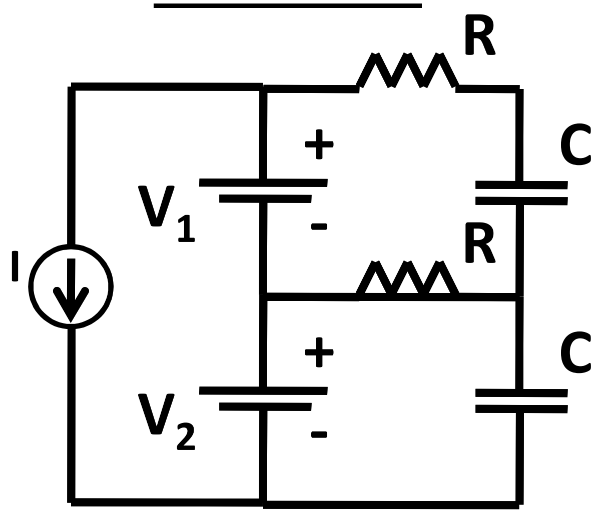
Solution:

Equalize charge using switched capacitor circuits
– Shuttle charge from cell to cell using capacitors
– Switch in unison at constant frequency
– Requires no closed-loop sensing or actuation!

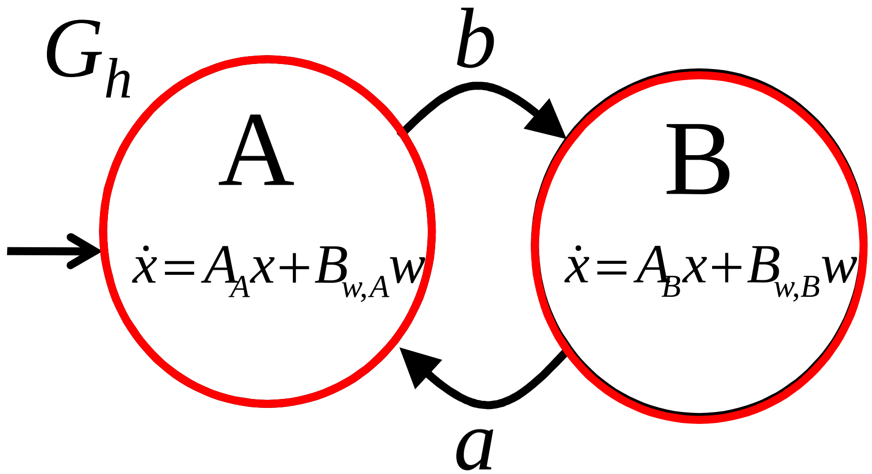


Case Study 2: Modeling

Circuit Model



Hybrid Automaton Model



State-Space Model

Mode A

$$\begin{bmatrix} \dot{V}_1(t) \\ \dot{V}_2(t) \\ \dot{V}_3(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} & 0 & \frac{1}{RC_1} \\ 0 & 0 & 0 \\ \frac{1}{RC} & 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_1} \\ -\frac{1}{C_2} \\ 0 \end{bmatrix} I(t)$$

Mode B

$$\begin{bmatrix} \dot{V}_1(t) \\ \dot{V}_2(t) \\ \dot{V}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{RC_2} & \frac{1}{RC_2} \\ 0 & \frac{1}{RC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \end{bmatrix} + \begin{bmatrix} -\frac{1}{C_1} \\ -\frac{1}{C_2} \\ 0 \end{bmatrix} I(t)$$

Case Study 2: Model Analysis

MLD System:

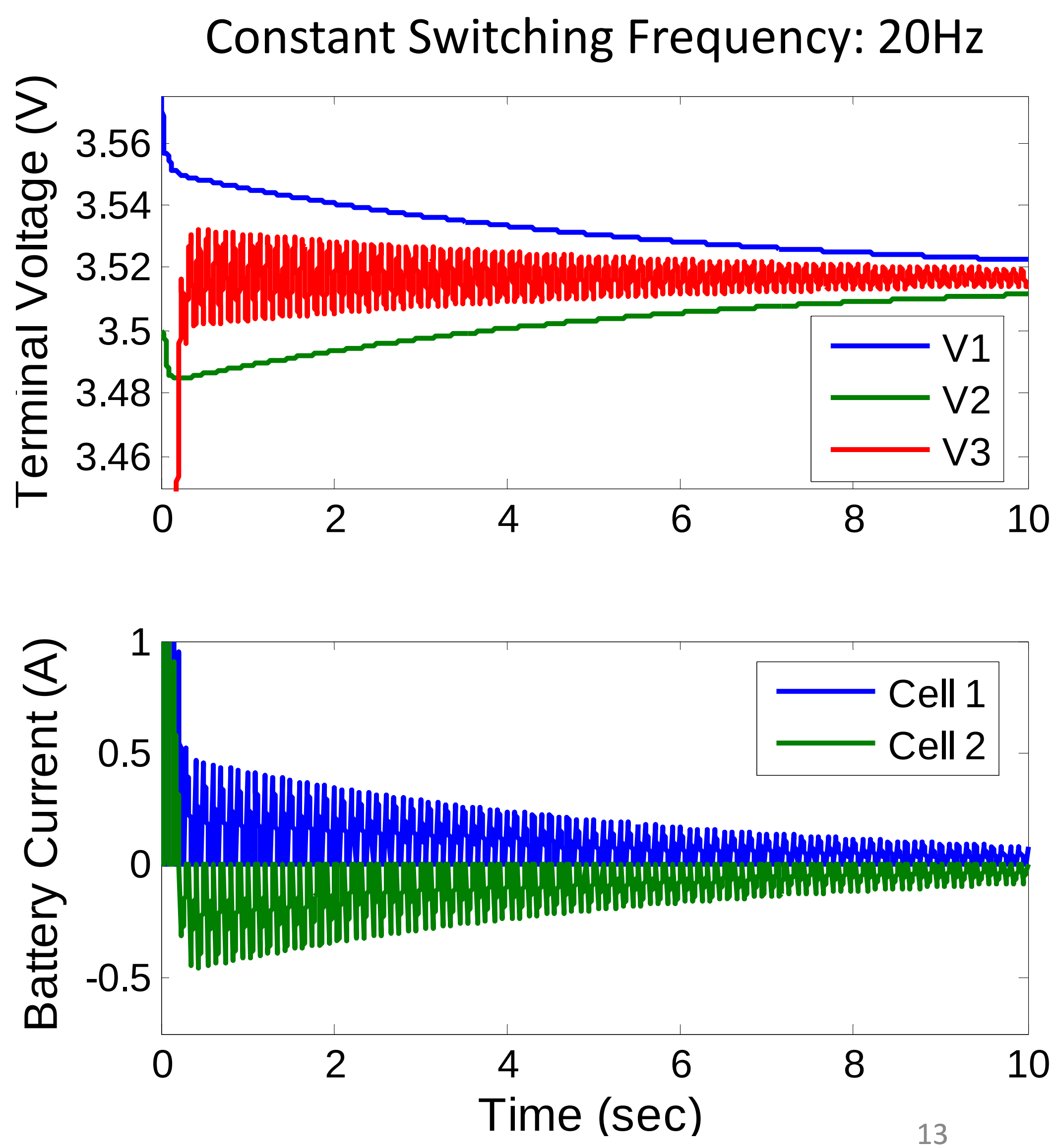
$$x(t+1) = z(t)$$

$$E_3 z(t) \le E_1 u(t) + E_4 x(t) + E_5$$

- Constant, synchronous switching equalizes cell voltage
- Cell 1 : positive current (discharge)
- Cell 2 : negative current (charge)

Next Step:

Use control theory to develop optimal switching sequence



Summary

- Investigated extension of models for hybrid systems discussed in class - MLD
- MLD integrates dynamics and logic into system of linear dynamics and inequalities
- Amenable to control design

Examples

- Timed automaton to MLD transformation
- Two case studies
  - On-Off Control for Autonomous MEMS Systems
  - Battery Charge Equalization

Thank you for your attention!

Questions?