Stochastic Modeling and Approaches for Managing Energy Footprints in Cloud Computing Service

Siqian Shen
Assistant Professor

Industrial and Operations Engineering
University of Michigan

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CC Advantages: Reducing Carbon Emission


How CC Works...

Source: www.veterangeek.com

How CC Works...
Moreover, an idle server consumes 60%+ energy at full mode.
Virtual Machine Consolidation

Large-scale servers with low utilization

Consolidate the work to fewer Cloud servers

Source: Google’s official blog - Energy efficiency in the cloud.

Our data centers use 50% less energy than typical data centers through server (Virtual Machine) consolidation.

— Google.

Other benefits:

- more robust operations schedules
- more idle servers reacting to demand surges

• Stochastic mixed-integer programming models to optimize energy footprints while ensure various Quality of Service (QoS) guarantees for managing servers in Cloud Computing service.
Our Work

- Stochastic mixed-integer programming models to optimize energy footprints while ensure various Quality of Service (QoS) guarantees for managing servers in Cloud Computing service.

- Estimate demand based on distributions of historical data, and dynamically consolidate or distribute jobs on servers through operational scheduling.
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- Vary QoS levels by using joint/multiple chance constraints, to bound chances of job delay and incompleteness.
Our Work

- Stochastic mixed-integer programming models to optimize energy footprints while ensure various Quality of Service (QoS) guarantees for managing servers in Cloud Computing service.

- Estimate demand based on distributions of historical data, and dynamically consolidate or distribute jobs on servers through operational scheduling.

- Vary QoS levels by using joint/multiple chance constraints, to bound chances of job delay and incompleteness.
Outline of Our Research

- Formulations: Stochastic & Chance-Constrained Programs
- Algorithms: the Benders Decomposition and Heuristics
- Computational Design
- Result Analyses
- Conclusions and Future Research
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<td>$N_m$</td>
<td>set of servers in a data center</td>
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Model 1: No Backlogging Parameter

\( \mathcal{N}_m \)  set of servers in a data center

\( \Omega \)  set of finite scenarios for realizing uncertain demand
Model 1: No Backlogging

Parameter

\( \mathcal{N}_m \) set of servers in a data center

\( \Omega \) set of finite scenarios for realizing uncertain demand

\( T \) total number of time periods considered

\( \ell^t \) length of period \( t \) (in hours) for all \( t = 1, \ldots, T \)
Model 1: No Backlogging Parameter

\[ N_m \quad \text{set of servers in a data center} \]
\[ \Omega \quad \text{set of finite scenarios for realizing uncertain demand} \]
\[ T \quad \text{total number of time periods considered} \]
\[ \ell^t \quad \text{length of period } t \text{ (in hours) for all } t = 1, \ldots, T \]
\[ \tilde{d}^t \quad \text{random job requests (demand) received at period } t \]
Model 1: No Backlogging Parameter

$N_m$ set of servers in a data center

$\Omega$ set of finite scenarios for realizing uncertain demand

$T$ total number of time periods considered

$\ell^t$ length of period $t$ (in hours) for all $t = 1, \ldots, T$

$\tilde{d}^t$ random job requests (demand) received at period $t$
Model 1: No Backlogging Formulation

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in N_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \\
\text{s.t.} & \quad \sum_{i \in N_m} e_i \ell^t x_i^t \geq \bar{d}^t \quad \forall 1 \leq t \leq T \\
& \quad \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in N_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in N_m \\
& \quad y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in N_m, 2 \leq t \leq T \\
& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in N_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in N_m, 1 \leq t \leq T
\end{align*}
\]

The basic model consolidates demand on servers to minimize the total energy consumed by all servers over \( t = 1, \ldots, T \).
Model 1: No Backlogging Formulation

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \\
\text{s.t.:} & \quad \sum_{i \in \mathcal{N}_m} e_i \ell_i^t x_i^t \geq \tilde{d}_i^t \quad \forall 1 \leq t \leq T \\
& \quad \ell_i^t x_i^t + s_i y_i^t \leq \ell_i^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \\
& \quad y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \\
& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\end{align*}
\]

\(g_i y_i^t\): energy used for booting machine \(i\) at period \(t\).

\(y_i^t \in \{0, 1\}\): = 1 if server \(i\) is switched to “on” at period \(t\).
Model 1: No Backlogging Formulation

\[ \min \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \]  

(1a)

\[ \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \tilde{d}^t \quad \forall 1 \leq t \leq T \]  

(1b)

\[ \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  

(1c)

\[ y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \]  

(1d)

\[ y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \]  

(1e)

\[ 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  

(1f)

\[ y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  

(1g)

\[ v_i x_i^t: \] energy for job processing in machine \( i \) at period \( t \).

\[ x_i^t \geq 0: \] percentage of server \( i \)'s capacity used at period \( t \).
Model 1: No Backlogging Formulation

\[
\min \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t)
\]

\[
\text{s.t. } \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \tilde{d}^t \quad \forall 1 \leq t \leq T
\]

\[
\ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\]

\[
y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m
\]

\[
y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T
\]

\[
0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\]

\[
y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\]

\[f_i z_i^t:\] energy used at “idle” of machine \(i\) at period \(t\).

\[z_i^t \in \{0, 1\}: \] = 1 if server \(i\) is “idle” at period \(t\).
Model 1: No Backlogging Formulation

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \\
\text{s.t.} & \quad \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \tilde{d}^t \quad \forall 1 \leq t \leq T \\
& \quad \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \\
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& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\end{align*}
\]

Computational time allocated to each period \( t \) is no less than the random demand \( \tilde{d}^t \).
Model 1: No Backlogging Formulation

\[\text{min: } \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \]  
(1a)

s.t. \[\sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \tilde{d}^t \quad \forall 1 \leq t \leq T \]  
(1b)

\[\ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  
(1c)

\[y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \]  
(1d)

\[y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \]  
(1e)

\[0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  
(1f)

\[y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \]  
(1g)

If \(\tilde{d}^t\) is discretely distributed,
Model 1: No Backlogging Formulation

\[ \text{min: } \sum_{t=1}^{T} \sum_{i \in N_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \quad (1a) \]

s.t. \[ \sum_{i \in N_m} e_i \ell_i^t x_i^t \geq \max_{\omega \in \Omega} d^{t\omega} \quad \forall 1 \leq t \leq T \quad (1b) \]

\[ \ell_i^t x_i^t + s_i y_i^t \leq \ell_i^t z_i^t \quad \forall i \in N_m, 1 \leq t \leq T \quad (1c) \]

\[ y_i^1 \geq z_i^1 \quad \forall i \in N_m \quad (1d) \]

\[ y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in N_m, 2 \leq t \leq T \quad (1e) \]

\[ 0 \leq x_i^t \leq 1 \quad \forall i \in N_m, 1 \leq t \leq T \quad (1f) \]

\[ y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in N_m, 1 \leq t \leq T \quad (1g) \]

If \( \tilde{d}^t \) is discretely distributed, and let \( d^{t\omega} \) represent a realization of \( \tilde{d}^t \) in scenario \( \omega \in \Omega \),

- reformulate (1b) as a set of deterministic constraints
Model 1: No Backlogging Formulation

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \\
\text{s.t.} & \quad \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d_t^\omega \quad \forall 1 \leq t \leq T \\
& \quad \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \\
& \quad y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \\
& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T
\end{align*}
\]

The total “on” time of server \(i\) at period \(t\) is no less than computational time plus the time of booting the server (if there is any).
Model 1: No Backlogging Formulation

\[
\text{min: } \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \tag{1a}
\]

s.t.
\[
\sum_{i \in \mathcal{N}_m} e_i \ell_i^t x_i^t \geq \max_{\omega \in \Omega} d_i^{t\omega} \quad \forall 1 \leq t \leq T \tag{1b}
\]
\[
\ell_i^t x_i^t + s_i y_i^t \leq \ell_i^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \tag{1c}
\]
\[
y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \tag{1d}
\]
\[
y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \tag{1e}
\]
\[
0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \tag{1f}
\]
\[
y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \tag{1g}
\]

Server \(i\) is “on” at period 1 if we switch it to “on.”
Model 1: No Backlogging Formulation

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in N_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \\
\text{s.t.} & \quad \sum_{i \in N_m} e_i \ell_i^t x_i^t \geq \max_{\omega \in \Omega} d^t \omega \quad \forall 1 \leq t \leq T \\
& \quad \ell_i x_i^t + s_i y_i^t \leq \ell_i^t z_i^t \quad \forall i \in N_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in N_m \\
& \quad y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in N_m, 2 \leq t \leq T \\
& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in N_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in N_m, 1 \leq t \leq T
\end{align*}
\]

If server \( i \) is “off” at \( t - 1 \) but “on” at \( t \), then it means that

- server \( i \) is switched to “on” at period \( t \)
Model 2: Backlogging with Penalty Setup I

**GOAL:**

- Minimize energy consumption of all servers over $1, \ldots, T$ + the expected penalty cost of backlogging.

Allow **backlogging** such that

- Job $(j, t)$ can be partitioned and processed on multiple servers, at any time that is no more than $L$ periods after period $t$ (“time of submission”).
Model 2: Backlogging with Penalty Setup II

Define Sets:

- $B_1(t)$: backlogging periods such that if $t = 1, \ldots, T - L$, then $B_1(t) = t, \ldots, t + L$; if $t = T - L + 1, \ldots, T$, then $B_1(t) = t, \ldots, T$.
- $B_2(t)$: possible periods for submitting jobs due at $t$, such that if $t \leq L$, then $B_2(t) = 1, \ldots, t$; if $t = L + 1, \ldots, T$, then $B_2(t) = t - L, \ldots, t$.

Additional Parameter:

- $\mathcal{N}_c$: Set of user groups who submit computational demand.
- $\tilde{d}_{jt}$: random job $(j, t)$ submitted by user $j$ at period $t$.
- $p^{tk}_j$: unit penalty of unfinished job $(j, t)$ at period $k$, $\forall k \in B_1(t)$.

New Variables:

- $u^{tk}_{ji}$: percentage of $\ell^t$ for processing job $(j, t)$ on server $i$ in period $k$, $\forall i \in \mathcal{N}_m$, $j \in \mathcal{N}_c$, $t = 1, \ldots, T$, and $k \in B_1(t)$.
- $b^{tk\omega}_j$: unfinished job $(j, t)$ at period $k$ in scenario $\omega$, $\forall k \in B_1(t)$ and $\omega \in \Omega$. 
Model 2: Job-based with Backlogging Formulation

\[ \begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left( \sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^t k b_j^{tk}\omega \right) \\
\text{s.t.} & \quad (1c)-(1g) \Rightarrow \text{Constraints from Model (1)} \\
& \quad \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T \quad (2a) \\
& \quad x_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \quad (2b) \\
& \quad b_j^{tk\omega} = \max \left\{ 0, \tilde{d}_j^{t\omega} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i \ell^l u_{ji}^{tl} \right\} \\
& \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T, k \in B_1(t), \omega \in \Omega \quad (2c) \\
& \quad 0 \leq u_{ji}^{tk} \leq 1, b_j^{tk\omega} \geq 0. \quad (2d)
\end{align*} \]

\( \rho^\omega \): the probability of scenario \( \omega \in \Omega \Rightarrow \) penalize unfinished job requests in the objective, and minimize the expected penalty.
Model 2: Job-based with Backlogging Formulation

\[
\begin{align*}
\min: & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left( \sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_{j}^{tk} b_{j}^{tk\omega} \right) \\
\text{s.t.} & \quad (1c) - (1g) \Rightarrow \text{Constraints from Model (1)} \\
& \quad \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i^k u_{ji}^{tk} \geq \max_{\omega \in \Omega} d_j^{t\omega} \quad \forall 1 \leq t \leq T \quad (2a) \\
& \quad x_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \quad (2b) \\
& \quad b_{j}^{tk\omega} = \max \left\{ 0, d_j^{t\omega} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i^l u_{ji}^{tl} \right\} \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T, k \in B_1(t), \omega \in \Omega \quad (2c) \\
& \quad 0 \leq u_{ji}^{tk} \leq 1, \quad b_{j}^{tk\omega} \geq 0. \quad (2d)
\end{align*}
\]

\(d_j^{t\omega}\): the realization of \(\tilde{d}_j^t\) in scenario \(\omega \in \Omega\) \Rightarrow replace stochastic constraints (2a) by equivalent deterministic constraints.
Model 3: Backlogging with a Joint Chance Constraint

Relax Model (2) by allowing job incompleteness after $L$ backlogging periods, however, **bounded by a certain risk tolerance**.

That is, replace Constraint (2a)

$$
\sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \quad \forall j \in N_c, 1 \leq t \leq T
$$

with

$$
\mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t, \forall j \in N_c, 1 \leq t \leq T \right) \geq \alpha
$$
Model 3: Backlogging with a Joint Chance Constraint

\[ \text{min: } \sum_{t=1}^{T} \sum_{i \in N_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left( \sum_{t=1}^{T} \sum_{j \in N_c} \sum_{k \in B_1(t)} p_{jk}^t b_{jk}^t \right) \]

s.t. (1c)-(1g) ⇒ Constraints from Model (1)
(2b)-(2d) ⇒ Constraints from Model (2)

\[ \mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell_{ji}^k u_{ji}^{tk} \geq \tilde{d}_j^t, \forall j \in N_c, 1 \leq t \leq T \right) \geq \alpha \]
Model 3: Backlogging with a Joint Chance Constraint

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y^t_i + v_i x^t_i + f_i z^t_i) + \sum_{\omega \in \Omega} \rho^\omega \left( \sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p^t_k b^t_k \right) \\
\text{s.t.} & \quad (1c)-(1g) \Rightarrow \text{Constraints from Model (1)} \\
& \quad (2b)-(2d) \Rightarrow \text{Constraints from Model (2)} \\
& \quad \sum_{\omega \in \Omega} \rho^\omega \zeta^\omega \leq 1 - \alpha
\end{align*}
\]

where, for each \( \omega \in \Omega \), binary variables \( \zeta^\omega = 1 \) if \( \forall j \in \mathcal{N}_c, \, 1 \leq t \leq T \), there exists at least one

\[
\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u^t_{ji} < d^t_j,
\]

and 0 otherwise.
Model 3: Backlogging with a Joint Chance Constraint

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left( \sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_{jk}^t b_{jk}^t \right) \\
\text{s.t.} & \quad (1c)-(1g) \Rightarrow \text{Constraints from Model (1)} \\
& \quad (2b)-(2d) \Rightarrow \text{Constraints from Model (2)} \\
& \quad \sum_{\omega \in \Omega} \rho^\omega \zeta^\omega \leq 1 - \alpha \\
& \quad \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^t + M_j^t \zeta^\omega \geq d_j^t \\
& \quad \forall \omega \in \Omega, j \in \mathcal{N}_c, 1 \leq t \leq T \\
& \quad \zeta^\omega \in \{0, 1\} \quad \forall \omega \in \Omega.
\end{align*}
\]

where \( M_j^t \) is set as the maximal standard time for processing job \((j, t)\), e.g.,
\[
M_j^t = \max_{\omega \in \Omega} d_j^{t \omega}, \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T.
\]
Model 4: Backlogging with Multiple Chance Constraints

Instead of a joint chance constraint

$$
\mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t, \ \forall j \in \mathcal{N}_c, \ 1 \leq t \leq T \right) \geq \alpha,
$$

we formulate a series of job-based constraints, each of which is associated with a risk tolerance $\alpha_j^t$, for job $(j, t)$, $\forall j \in \mathcal{N}_c$ and $1 \leq t \leq T$.

$$
\mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \right) \geq \alpha_j^t \ \forall j \in \mathcal{N}_c, \ 1 \leq t \leq T.
$$
Computational challenges from:

- Large-Scale Time Intervals \((1, \ldots, T)\)
- Large Number of Users and Servers \(\left| \mathcal{N}_c \right|\) and \(\left| \mathcal{N}_m \right|\)
- Large Number of Scenarios \(\left| \Omega \right|\) for Describing the Uncertainty \(\tilde{d}\)
- Binary Server Operational Decisions \(y\) and \(z\)
Benders Decomposition

Example: Model 2

\[
\begin{align*}
\text{min:} & \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \text{Prob}^\omega \left( \sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_{j}^{tk} b_{j}^{tk\omega} \right) \\
\text{s.t.} & \quad \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m \\
& \quad y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m, 2 \leq t \leq T \\
& \quad 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
& \quad \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T \\
& \quad \chi_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m, 1 \leq t \leq T \\
\end{align*}
\]

Scenario-based Decisions (Second Stage Subproblem)

\[
\begin{align*}
& \quad b_{j}^{tk\omega} = \max \left\{ 0, \quad d_j^{tw} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i \ell^l u_{ji}^{tl} \right\} \\
& \quad \forall j \in \mathcal{N}_c, 1 \leq t \leq T, \ k \in B_1(t), \ \omega \in \Omega \\
& \quad 0 \leq u_{ji}^{tk} \leq 1, \ b_{j}^{tk\omega} \geq 0. \ \text{Continuous!}
\end{align*}
\]
A Heuristic Approach

Idea: fix schedules of a subset of servers. Then optimize schedules for the rest of servers using math modeling.
We pre-determine a subset of servers’ schedule by setting

\[ x^1_i = 1 - s_i / \ell^t \quad \forall i = 1, \ldots, \chi(1), \]
\[ x^t_i = 1 \quad \forall 2 \leq t \leq T, \ i = 1, \ldots, \chi^{-}(t), \]
\[ x^t_i = 1 - s_i / \ell^t \quad \forall 2 \leq t \leq T, \ i = \chi^{-}(t) + 1, \ldots, \chi^{-}(t) + \chi^{+}(t) \quad \text{if} \ \chi^{+}(t) > 0, \]

where for \( t = 1, \ldots, T, \)

\[ \chi(t) = \left\lfloor \sum_{j \in N_c} \max_{\omega \in \Omega} d_{ij}^t / \ell^t \right\rfloor, \]
\[ \chi^{-}(t) = \min\{\chi(t - 1), \chi(t)\}, \text{and} \ \chi^{+}(t) = \max\{\chi(t) - \chi(t - 1), 0\}. \]
\[ |\mathcal{N}_c| = 2 \text{ (two types of users) and } |\mathcal{N}_m| = 5, 10, \text{ and } 20. \]

Set \( T = 24 \text{ hours}. \)

Average energy consumption of Off, Idle, Processing, and Booting for a 3.0 Ghz server to be, respectively, 0W, 150W, 250W, and 250W (i.e., \( v_i = 100W, f_i = 150W \)).
\( \mathcal{I} = 10\% \) | \( \mathcal{I} = 30\% \) | \( \mathcal{I} = 50\% \) \\
\hline
\( \mathcal{N}_m \) & \( E[\sum_{t=1}^{T} \tilde{d}_t] \) (hours) & Bk (kWh) & \( E[\sum_{t=1}^{T} \tilde{d}_t] \) (hours) & Bk (kWh) & \( E[\sum_{t=1}^{T} \tilde{d}_t] \) (hours) & Bk (kWh) \\
\hline
5 & 12 & 19.2 & 36 & 21.6 & 60 & 24 \\
10 & 24 & 38.4 & 72 & 43.2 & 120 & 48 \\
20 & 48 & 76.8 & 144 & 86.4 & 240 & 96 \\
\hline

- \( \mathcal{I} \): computational intensity
- \( E[\sum_{t=1}^{T} \tilde{d}_t] = \mathcal{I} \times |\mathcal{N}_m| \times 24 \) (hours)
- \( \text{Bk} \): gives benchmark energy consumption (objective) by having servers first “on” then “idle.”
### Computational Design Benchmark

<table>
<thead>
<tr>
<th>$N_m$</th>
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<th>$I = 30%$</th>
<th>$I = 50%$</th>
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<td>$E[\sum_{t=1}^{T} \tilde{d}^t]$ (hours)</td>
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<td>24 38.4</td>
<td>72 43.2</td>
<td>120 48</td>
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<tr>
<td>20</td>
<td>48 76.8</td>
<td>144 86.4</td>
<td>240 96</td>
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</tbody>
</table>

- \( I \): computational intensity
- \( E[\sum_{t=1}^{T} \tilde{d}^t] = I \cdot |N_m| \cdot 24 \) (hours)
- \( Bk \): gives benchmark energy consumption (objective) by having servers first “on” then “idle.”
Computational Design

Demand Patterns

(a) Type 0 Demand Curve

(b) Type 0 Job Demand Sample

(c) Type 1 Demand Curve

(d) Type 1 Job Demand Sample

Types 0 ∼ 3: Homogeneous. Types 4 & 5: Heterogeneous.
Computational Design

- CPLEX 12.4 via ILOG Concert Technology with C++
- HP Workstation Z210 with CPU 3.20 GHz and 8GB memory
- CPU time limits = 1800 seconds for each instance
- Test five instances for each parameter combination
## Results of Model 1

<table>
<thead>
<tr>
<th>( N_m )</th>
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<td><strong>30%</strong></td>
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Results of Model 1

Microsoft Exchange
On-premise vs. Cloud Comparison, CO2e per user

Microsoft Dynamics CRM
On-premise vs. Cloud Comparison, CO2e per user
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Results of Model 1

Figure: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, Type 3 Demand
A Revisit of Models

Model (1): \[ \sum_{i \in N_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d^{t \omega} \quad \forall 1 \leq t \leq T. \]

Model (2): \[ \sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell^k u^t_{ji} \geq \max_{\omega \in \Omega} d^{t \omega}_j \quad \forall 1 \leq t \leq T. \]

Model (3): \[ \mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell^k u^t_{ji} \geq \tilde{d}^t_j, \forall j \in N_c, 1 \leq t \leq T \right) \geq \alpha. \]

Model 4: \[ \mathbb{P} \left( \sum_{k \in B_1(t)} \sum_{i \in N_m} e_i \ell^k u^t_{ji} \geq \tilde{d}^t_j \right) \geq \alpha^t_j \quad \forall j \in N_c, 1 \leq t \leq T. \]
Energy Use in Models 1-4 ($N_m = 20$, $I = 50\%$)

Unit penalty $p_{jk}^t = 100$ for penalty case, $\forall j \in N_c$, $1 \leq t \leq T$, and $k \in B_1(t)$.

Shen, Wang (UMich)
## Solution Approach Comparison

<table>
<thead>
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<th>No.</th>
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<th>B-Time</th>
<th>B-Total</th>
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### Table: $N_m = 20$, $I = 50\%$, and Five Instances

“C-”, solving Model (2) by directly solving its MIP.

“B-”, employing Benders decomposition.

“H-”, using the approximation approach.
The performance of the Benders approach varies among instances and is unstable.
Solution Approach Comparison

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</table>

Table: $N_m = 20, \ T = 50\%$, and Five Instances

For Model 2, the differences between H-Total and C-Total are within 0.3% gaps for all instances.
## Solution Approach Comparison

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Table: $N_m = 20$, $I = 50\%$, and Five Instances
98: $\alpha_{t_j} = 98\%$, $\forall j \in \mathcal{N}_c$; 1-96: $\alpha_0^t = 100\%$, $\alpha_1^t = 96\%$, $\forall 1 \leq t \leq T$. 

Conclusions

- Effectively managing energy footprints and QoS via stochastic optimization models.
- Yield respective 80%, 50%, and 30% of energy savings for 10%, 30%, and 50% demand intensity regardless of demand patterns.
- Backlogging and chance constraints provide additional flexibility in server scheduling and reduce energy use.
- The Benders decomposition and the heuristic approach are faster and yield good results.
- User prioritization via multiple chance constraints can effectively reduce consumed energy.
Thank You!

Questions?