Abstract—Temperature monitoring is a critical issue for lithium ion batteries. Since only the surface temperature of the battery can be measured, a thermal model is needed to estimate the core temperature, which can be higher and hence more critical. In this paper, an on-line parameter identification scheme is designed for a cylindrical lithium ion battery thermal model, by which the parameters of the thermal model can be identified automatically. An adaptive observer is designed based on the on-line parameterization methodology and the closed loop architecture. A linear battery thermal model is explored first, where the internal resistance is assumed to be constant. The methodology is later extended to address temperature dependent internal resistance with non-uniform forgetting factors. The capability of the methodology to track the long term variation of the internal resistance is beneficial for battery health monitoring.

I. INTRODUCTION

Lithium ion batteries have been widely considered as an energy storage device for hybrid electric vehicles (HEV), plug-in hybrid electric vehicles (PHEV) and battery electric vehicles (BEV). Thermal management is a critical issue for onboard lithium ion batteries due to their narrow window of operating temperatures. An accurate prediction of the battery temperature is the key to an effective thermal management system and to maintain safety, performance, and longevity of these Li-Ion batteries.

Some of the previous works on thermal modeling and management predict the detailed temperature distribution throughout the cell [1], [2], [3], [4], but are not suitable for onboard application due to high computational intensity. Others use one single temperature to capture the lumped thermal behavior of the cell [5], [6], [7]. Even though the single temperature approximation is computationally efficient, it might lead to over-simplification since the temperature in the core of the cell can be much higher than in the surface [8]. It is in the core where major battery thermal breakdown and degradation occurs.

Lumped thermal models capturing both the surface and the core temperatures of the cell have also been studied in [8] and [9]. Such simplified models are efficient for onboard application due to their limited number of states. In addition to the higher fidelity of the two-state model, the prediction of the surface temperature can be compared with the measured value, and the errors can be fed back to correct the core temperature estimation. The accuracy of the model parameters is of great importance since it determines the precision of the core temperature estimation. Model parameters can be approximated by correlating to the geometry of the battery and the physical properties of all its components [9], but such approximation may not be accurate due to the complicated layered structure of the cell and the interface resistance between the layers. The parameters can also be determined by fitting the model to the data obtained from designed experiments [8], [9]. However, some of the thermal parameters, such as the internal resistance, may change over the battery lifetime due to degradation, and thus need to be identified continuously.

An online parameterization scheme is designed in this paper to automatically identify the thermal model parameters based on the commonly measured signals in vehicle battery systems. Based on the online identifier, an adaptive observer is then designed for core temperature estimation. A linear battery model with constant internal resistance is investigated first, where the pure least square algorithm is sufficient for identification. When the internal resistance of the battery is non-constant, e.g. temperature dependent [5], [10], a non-uniform forgetting factor is utilized to identify the time-varying resistance. The internal resistance of the lithium ion battery may increase over lifetime due to degradation as the solid electrolyte interphase (SEI) grows in thickness [11], [12]. The least square algorithm with non-uniform forgetting factors is also explored to track the long term growth of the internal resistance. The growth of the internal resistance greatly affects the power capability, and can be viewed as an indication of the battery state of health (SOH).

II. LUMPED THERMAL MODEL OF A CYLINDRICAL LI THIUM ION BATTERY

A cylindrical battery is modeled with two states [9], namely the surface temperature \( T_s \) and the core temperature \( T_c \), as shown in Fig. 1. The governing equations for the single cell thermal model are defined as [9],

\[
C_c \dot{T}_c = I^2 R_c(T_c) + \frac{T_s - T_c}{R_c}, \quad C_s \dot{T}_s = \frac{T_f - T_s}{R_d(V)} - \frac{T_s - T_c}{R_c}. \quad (1)
\]

In this model, heat generation is approximated by a concentrated source of Joule loss in the battery core, computed as the product of the current I squared and an internal resistance \( R_c \). The internal resistance \( R_c \) is modeled as temperature dependent[5], [10], and described here as

\[
R_c = -0.00027T_c^2 + 0.032T_c^2 - 1.22T_c + 19.8, \quad (2)
\]

X. Lin, H. Perez, J. Siegel and A. Stefanopoulou are with the Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109, USA. E-mail: xflin@umich.edu, heperez@umich.edu, siegeljb@umich.edu and annastef@umich.edu

Y. Li and R. D. Anderson are with the Vehicle and Battery Controls Department, Research and Advanced Engineering, Ford Motor Company, Dearborn, MI 48121, USA. E-mail: yli19@ford.com and rander34@ford.com
where \( R_c \) is in \( m\Omega \) and \( T_c \) is in °C. Heat exchange between the core and the surface is modeled by heat conduction over a thermal resistance, \( R_c \), which is a lumped parameter including both the conduction and contact thermal resistance. A convection resistance \( R_u \) is modeled between the surface and the surrounding coolant to account for convective cooling. The value of \( R_u \) is a function of the coolant flow velocity \( V \), as described in [13], [14]

\[
R_u = \frac{D}{kNUT}, \quad Nu = qRe^mPr^{0.36}, \quad Re = \frac{VD}{v},
\]

where \( D \) is the diameter of the battery, \( A \) is the surface area of the battery, \( k \) is the thermal conductivity of the coolant, \( Nu \) is the Nusselt number, \( Re \) is the Reynolds number, \( Pr \) is the Prandtl number and \( v \) is the kinematic viscosity. These quantities are related to the physical properties of the coolant and the geometries of the battery pack, such as the spacing between cells. They can be calculated for specific type of coolant and battery pack configuration. Values of coefficients \( m \) and \( q \) for various \( Re \) ranges can be found in [13], [14]. The rate of temperature change of the surface and the core depends on their respective lumped heat capacities. The parameter \( C_c \) is the heat capacity of the jelly roll inside the cell, and \( C_s \) is related to the heat capacity of the battery casing.

The complete parameter set for this model includes \( C_c \), \( C_s \), \( R_c \), \( R_s \), and \( R_u \). Model identification techniques will be developed to obtain parameter values based on measurable inputs and outputs of the model. The thermal model in Eq. (1) is a nonlinear model since \( R_c \) is a function of \( T_c \), and \( R_u \) depends on \( V \). Such nonlinearity, especially in \( R_c \), complicates the parameter identification. For simplicity, a thermal model with constant \( R_c \) is investigated first, and the methodology will then be extended to account for the full nonlinear model.

### III. Parameterization Methodology

For model identification, a parametric model

\[
z = \Theta^T \phi
\]

is derived first by applying Laplace transformation to the model, where \( z \) is the observation, \( \Theta \) is the parameter vector and \( \phi \) is the regressor [15]. Both \( z \) and \( \phi \) should be measured or can be generated from measured signals.

With a parametric model, various algorithms can be chosen for parameterization, such as the gradient search and the least squares. The method of least squares is preferred here due to its advantages in noise reduction [15].

The recursive least squares algorithm is applied in an online fashion, where parameters are updated continuously [15]

\[
\dot{\Theta}(t) = \frac{\epsilon(t) \phi(t)}{m^2(t)}, \quad \dot{P}(t) = -P(t) \frac{\phi(t) \phi^T(t)}{m^2(t)} P(t)
\]

\[
\epsilon(t) = z(t) - \Theta^T(t) \phi(t), \quad m^2(t) = 1 + \phi^T(t) \phi(t),
\]

where \( m(t) \) is the normalization factor to enhance the robustness of parameter identification.

In some cases, to make the observation \( z \) and the regressors \( \phi \) proper (or causal), a filter \( \frac{1}{N} \) will have to be designed and applied. The parametric model will then become

\[
\frac{z}{\Lambda} = \frac{\Theta^T}{\Lambda} \phi .
\]

### IV. Parameterization of the Thermal Model with Constant \( R_c \) and Adaptive Observer Design

In this section, a parameterization scheme and adaptive observer is designed for the battery thermal model with constant internal resistance \( R_c \).

#### A. Parameterization Design

The inputs are the current \( I \), the coolant temperature \( T_f \), and the coolant velocity \( V \). The measurable output is the battery surface temperature \( T_s \). A parametric model can be derived by performing Laplace transformation on Eq. (1), and substituting unmeasured \( T_c \) by measured \( I, T_f, V \) and \( T_s \).

\[
s^2 T_s - s T_{s,0} = \frac{R_c}{C_s C_r R_s} f^2 + \frac{1}{C_r C_s R_s R_u(V)} T_f - T_s - C_c + C_s \frac{1}{C_r C_s R_u(V)} s(T_f - T_{s,0}) + \frac{1}{C_r} \frac{T_f - T_s}{R_u(V)},
\]

where \( T_{s,0} \) is the initial surface temperature. It is noted that the initial core temperature is considered as equal to the initial surface temperature as if the battery starts from rest.

For the parametric model in Eq. (7), we have

\[
\frac{z}{\Theta} = \frac{\alpha}{\beta} \frac{\gamma}{\mu} \Theta, \quad \phi = [f^2 \frac{T_f - T_s}{R_u(V)} s T_{s,0} \frac{T_f - T_s}{R_u(V)}]^T
\]

where \( \alpha = \frac{R_c}{C_r C_s R_u(V)} \), \( \beta = \frac{1}{C_r C_s R_u(V)} \), \( \gamma = -\frac{C_c + C_s}{C_r C_s R_u(V)} \), and \( \mu = \frac{1}{C_r} \). It is noted that \( \frac{1}{R_u(V)} \) is treated as a whole as a regressor, since \( T_f \) and \( T_s \) can both be measured, and \( R_u \) can be calculated based on knowledge of \( V \) using Eq. (3). With \( \alpha, \beta, \gamma \) and \( \mu \), \( C_c, C_s, R_s \), and \( R_c \) can be obtained by

\[
C_c = \frac{\gamma}{\beta} - \frac{1}{\mu} C_s, \quad C_s = \frac{1}{\mu} C_r R_c = \frac{\alpha}{\beta} - R_s = -\frac{\mu^2}{\gamma \mu + \beta}
\]

A second order filter should be applied to the signals in Eq. (7) to make them proper. The filter takes the form

\[
\frac{1}{\Lambda(s)} = \frac{1}{(s + \lambda_1)(s + \lambda_2)},
\]

where \( \lambda_1 \) and \( \lambda_2 \) are designed based on the input and system dynamics. The least squares algorithm in Eq. (5) can then be applied for parameter identification.
B. Adaptive Observer Design

It is a common practice to design a closed loop observer to estimate the unmeasurable states of a system based on the measurable outputs. The observer for a linear system

\[ \dot{x} = Ax + Bu, \quad y = Cx + Du \]  

takes the form [16]

\[ \dot{x} = A\hat{x} + Bu + L(y - \hat{y}), \quad \hat{y} = C\hat{x} + Du, \]  

where \( x \) and \( y \) are the actual system states, \( \hat{x} \) and \( \hat{y} \) are estimated states and output, \( L \) is the observer gain, and \( A, B, C \) and \( D \) are model parameters. The difference between the measured and the estimated output is used as the feedback to correct the estimated states.

Comparing with an open loop observer (observer without output injection), the closed loop observer can accelerate the convergence of the estimated states to that of the real plant under uncertain initial conditions, e.g. a Luenberger observer [16], or optimize the estimation by balancing the effect of process and measurement noises, e.g. a Kalman filter [17].

For the cylindrical battery thermal model in Eq. (1),

\[ x = [T_c \ T_s] ^T, \quad y = T_s, \quad u = [I]^T \frac{T_f - T_s}{R_c(V)}, \]

\[ A = \begin{bmatrix} \frac{1}{R_c C_c} & \frac{1}{R_s C_s} \\ \frac{1}{R_c C_c} & \frac{1}{C_c R_c} \end{bmatrix}, B = \begin{bmatrix} \frac{R_c}{C_c} & 0 \\ 0 & \frac{1}{C_s} \end{bmatrix}, C = [0 \ 1], D = 0. \]  

An adaptive observer is designed based on certainty equivalence principle [15], where the estimated parameters from on-line identification in Eq. (5) are adopted for the observer. The structure of the whole on-line identification scheme and adaptive observer is shown in Fig. 2.

![On-line Identification Scheme and Adaptive Observer Structure](image1)

As shown in Fig. 2, when the thermal management system is operating in real time, the input current \( I \), coolant temperature \( T_f \) and the measured surface cell temperature \( T_s \) are fed into the parameter identifier to estimate model parameters \( R_{in}, R_c \) and \( R_s \). The adaptive observer, on one hand, adopts the estimated parameters for temperature estimation, and on the other hand, takes the errors between the measured and the estimated \( T_s \) as a feedback to correct its core temperature and surface temperature estimation. The estimations for both parameters and temperatures are updated at each time step.

### Nominal Values of Parameters and Initial Guess for Identification

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( C_c (JK^{-1}) )</th>
<th>( C_s (JK^{-1}) )</th>
<th>( R_{in} (m\Omega) )</th>
<th>( R_s (KW^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Values</td>
<td>268</td>
<td>18.8</td>
<td>3.5</td>
<td>1.266</td>
</tr>
<tr>
<td>Initial Guess</td>
<td>100</td>
<td>50</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

V. Simulation Verification

Simulation has been conducted to verify the designed parameterization scheme and adaptive observer. A cylindrical battery thermal model with parameters for an A123 32157 \( LiFeP04 \) graphite battery is used to generate data for verification of the methodology. Parameters are assumed by scaling up values from [8] and [18]. Nominal values of the model parameters are listed in Table I.

The main purpose of the simulation here is to check whether the designed algorithm can be applied to identify those assumed parameters and estimate core temperature \( T_c \), and thus reasonable values of the assumed model parameters are sufficient.

A driving cycle with high power excursion, the Urban Assault Cycle (UAC) [19], is adopted as the current excitation for the simulation. The UAC cycle and the coolant flow velocity profile are shown in Fig. 3. The output of the model, \( T_c \), is also plotted in the bottom plot of Fig. 3. The air flow temperature is fixed at 25°C.

![Fig. 3. Simulated Current and Coolant Velocity Inputs and Surface Temperature Output for Identification](image2)

The generated signals \( I, V \) and \( T_s \) are used for on-line least squares parameterization. The four parameters to be identified, \( C_c, C_s, R_s \) and \( R_c \), are initialized with the values in Table I, which are quite away from the nominal values listed in the same table.

The on-line identification results are plotted in Fig. 4. It can be seen that all the 4 parameters converge to the nominal values in Table I.

The response of the adaptive observer, which adopts the identified parameters by online parameterization, is plotted in Fig. 5. In Fig. 5, the \( T_c \) and \( T_s \) simulated by the model are
presented and the estimated $T_e$ and $T_s$ are plotted to evaluate the performance of the adaptive observer. The simulated core temperature $T_c$ and surface temperature $T_s$ are initialized to be $25^\circ$C and the adaptive observer is preset to start from $10^\circ$C for both the surface and the core temperatures. It can be seen that the convergence rate of the surface temperature $T_s$ is independent of that of the parameters because it is directly measured and fed back to the observer. However, the convergence of the unmeasured core temperature $T_c$ depends on the convergence of the parameters. As can be seen in Fig. 4, the identified $T_c$ converges to the simulated $T_c$ after the identified parameters converge to the right values.

VI. PARAMETERIZATION OF THE BATTERY THERMAL MODEL WITH TEMPERATURE DEPENDENT $R_e$

When the battery internal resistance $R_e$ is a function of the core temperature $T_c$, such as in Eq. (3), the parametric model in Eq. (7) will no longer be linear and thus direct application of the identification algorithm in Eq. (5) will result in biased estimation of the parameters, as shown in Fig. 6.

It can be seen from Fig. 6 that when $R_e$ is a function of $T_c$, it will be time-varying since $T_c$ is fluctuating all the time. The least square algorithm in Eq. (5) can only address parametric models with constant parameters, and its estimation can only converge to constant values if the stability conditions are satisfied. As a result, although the real $R_e$ is varying, the value identified by Eq. (5) tends to track its average value. This will not only introduce errors in $R_e$ estimation but will also affect the estimation of other constant parameters. As shown in Fig. 6, significant errors can also be observed for the estimation of the constant parameters $C_c$, $C_s$ and $R_c$. Such errors are introduced because the least square algorithm aims at minimizing the errors in the model output estimation by finding a set of optimal parameters. However, in this case, since the errors in $R_e$ identification are inevitable, the other parameters will also have to be biased to minimize the overall errors in $T_s$ estimation. Such biased parameterization will corrupt the estimation of the core temperature $T_c$ without causing large errors in the estimated surface temperature $T_s$, as shown in Fig. 7.
as the covariance matrix dynamics in Eq. (5), where \( \eta \) is the 
forgetting factor matrix \([15]\).

The least square identification algorithm tries to find the 
optimal parameters that best fit the inputs and outputs over 
the whole data set. A pure least square algorithm treats each 
data point with equal weight, no matter if it is acquired most 
recently, or obtained some time earlier. However, when a 
forgetting factor is applied, the data points will be weighted 
differently. Specifically, the newly acquired data are favored 
over the older ones. In the form shown in Eq. (14), the weight 
of the data will decay exponentially with the time elapsed, 
and the larger the forgetting factor is, the faster such decay 
will be. Consequently, the least square algorithm will update 
its results of identification primarily based on the recent data 
fed into it and thus can track the parameters when they are 
time-varying.

The least square algorithm with forgetting factors can be 
applied directly to the original linear parametric model in 
Eq. (7). Of the four lumped parameters, namely \( \alpha, \beta, \gamma \) 
and \( \mu \) in Eq. (7), since only \( \alpha \) is related to time varying 
\( R_c \), and all the others are constant, non-uniform forgetting 
factors should be adopted here. The \( \eta \) matrix is designed as 
\( \text{diag}(\eta_1,0,0,0) \), where \( \eta_1 \) is the forgetting factor associated 
with \( \alpha \) (and hence \( R_c \)).

Simulation is conducted with \( \eta_1 = 0.35 \), and the results 
of identification are shown in Fig. 8. It is noted that the 
identified \( R_c \) can now follow the real varying \( R_c \), and as a 
result, there is no bias in the estimation of the other constant 
parameters. Consequently, with the identified parameters, 
the adaptive observer can now estimate the battery core 
temperature \( T_c \) accurately even when the internal resistance 
\( R_c \) is temperature dependent, or a function of other variables, 
such as \( \text{SOC} \), as shown in Fig. 9.

Different from the variation of the internal resistance 
caused by the fluctuation in the core temperature of the bat-
tery, the growth of the internal resistance due to degrada-
tion is a process that occurs slowly over the battery lifetime. The 
internal resistance might increase substantially over hundreds 
of current cycles or days according to [11], [20] and [12].

In this paper, the growth in internal resistance due to 
degradation is simulated and used to test the capability of 
the identification algorithm to detect the slow increase of the 
resistance. The internal resistance \( R_c \), originally a function 
of the core temperature \( T_c \), is now augmented with a term 
which is linearly increasing over time. The drive cycle used 
for simulation is the same UAC cycle shown in Fig. 3, but is 
repeated for 500 times and the rate of growth in internal 
resistance is set at 0.17%/cycle. The rate of degradation 
may also increase with the temperature according to [11], 
[20] and [12]. This effect is not considered here since the 
main purpose of the simulation is to test the identification 
algorithm.

The results of the online identification are shown in 
Fig. 10. It can be seen from Fig. 10 that the real internal 
resistance (simulated) gradually increases over time and is 
subject to short-term variation due to the fluctuation of the 
battery core temperature. The identified \( R_c \) follows both the 
long-term and short-term variation of the real one with a 
small delay as shown in the inset of Fig. 10. In real vehicle 
operation, since \( R_c \) is varying all the time, it is difficult to 
evaluate \( \text{SOH} \) by the instantaneous value of \( R_c \). Therefore, 
the averaged \( R_c \) might be a better choice instead. The mean 
value of \( R_c \) for each UAC cycle is plotted in the lower half 
of Fig. 10. It is noted that the averaged \( R_c \) can capture the 
long-term increase of the internal resistance and the identified 
value is a good estimation of the real one.

VIII. CONCLUSIONS AND FUTURE WORK

The core temperature of a lithium ion battery, which 
is usually not measurable, is of great importance to the 
onboard battery management system, especially when the 
batteries are subject to high C-rate. The core temperature 
can be estimated by a two states thermal model, and the
parameters of the models are critical for the accuracy of the estimation. In this paper, an online parameter identification scheme based on least square algorithm is designed for the cylindrical lithium ion battery thermal model. The online identification scheme can automatically identify the model parameters based on the commonly available onboard signals and update the observer for adaptive monitoring. When the internal resistance of the battery is temperature dependent, which is a more realistic situation, the least square algorithm can be augmented with non-uniform forgetting factors. The algorithm with forgetting factors can not only track the time-varying internal resistance, but also guarantee unbiased identification of the remaining constant parameters. The online parameterization also shows the capability to track the long-term variation (over cycles and days) of the internal resistance due to aging or degradation/abuse. The growth in internal resistance can be used for the SOH monitoring of the batteries. The methodology developed has been verified with simulations and is to be validated with experiments in the immediate future.

Applications, such as HEV, BEV and PHEV, usually have hundreds, or even thousands, of battery cells in series to meet their high power and energy requirements. Hence the vehicle level battery thermal management will be performed on a module basis, instead of on a cell basis. The single cell thermal model used in this paper can be scaled up to a pack model by considering cell to cell thermal interaction, and the parameterization methodology and the adaptive observer design will be investigated for the pack level model. Initial results of this pack level work can be found in [21].

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