

Explaining Returns with Loss Aversion

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Abstract

I develop and test an equilibrium asset pricing model based on loss averse investors. The model specifies a pricing kernel that is a nonmonotonic function of the market return. It also implies that investors demand a higher risk premium for risk associated with negative market returns than for positive market returns. The model assumes rational expectations and is consistent with no-arbitrage pricing.

Estimates of the model's parameters are similar to values reported elsewhere. As the loss aversion literature predicts, the accuracy of the model depends on the frequency with which data is observed. Consistent with Benartzi and Thaler (1995), the model explains annual returns better than competing models, but it does not explain monthly, quarterly, or half-year returns. The model fits both returns that reflect the equity premium and stock returns alone.

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1 Introduction

In a series of influential articles, Kahneman and Tversky (1979, 1986, 1992) suggest that economic agents do not maximize the expected value of Von Neumann-Morgenstern utility functions, but behave according to a set of rules collectively known as prospect theory. Among other things, prospect theory conjectures that agents exhibit loss aversion, or greater sensitivity to losses than to gains. Several researchers have presented experimental evidence to support loss aversion, and a few have argued that loss aversion can explain some of the empirical anomalies in academic finance.¹

However, economists have not yet used loss aversion to describe the expected returns of a cross section of assets. This is surprising because the loss aversion hypothesis is plausible, testable, and important. The hypothesis is plausible because it is based on psychological observation rather than mathematical tractability. It is testable because it has strong implications for asset pricing data. Most significant, loss aversion has important consequences for economic theory. If people are loss averse, much of what we think we know about the economics of uncertainty may be incorrect. Given the recent advances in econometric technology and the richness of financial data available, it seems logical to subject loss aversion to rigorous testing.

In this study, I develop and test an equilibrium asset-pricing model that incorporates loss aversion into the behavior of agents. The model that I propose is simple to derive and test. It produces first order conditions that are generally consistent with no-arbitrage pricing and it assumes that agents are rational maximizers of the expectation of their value functions. I test the model with the generalized method of moments. The tests I perform confirm that the loss aversion hypothesis helps to explain asset returns.

Like the traditional CAPM, the loss aversion model specifies a pricing kernel that is a function of the market return. However, the loss aversion model deviates from the CAPM

¹Two recent papers that describe experimental evidence about loss aversion include Thaler, Tversky, Kahneman and Schwartz (1997) and Gneezy and Potters (1997). Tversky and Kahneman (1992) cite a number of other papers that describe experimental evidence. Benartzi and Thaler (1995) argue that loss aversion can explain the equity premium puzzle.

in two important ways. First, the loss aversion model's pricing kernel is the absolute value of the market return raised to some negative power while the CAPM's pricing kernel is just a linear function of the market return. Thus, the loss aversion model implies that small market returns are weighed much more heavily than most alternative models. By making small losses and gains more significant than large market returns, the loss aversion model adds some variance to the pricing kernel and improves its fit to the data.

The second way in which the loss aversion model differs from the CAPM is that it treats covariances with the pricing kernel conditional on negative and positive market returns differently. In other words, it specifies a higher risk premium for "downside" risk than for "upside" risk. If assets have different downside betas (on the pricing kernel) than upside betas, they may not be priced correctly by models that treat risk symmetrically, like the CAPM.

Tests of the model confirm that the loss aversion hypothesis is plausible. As pointed out by Benartzi and Thaler (1995), loss aversion models depend critically on the frequency with which investors monitor their returns and determine whether they have made losses or gains, or investors' *evaluation period*. While a loss aversion model should fit returns measured at the representative investor's evaluation period, it will not generally fit returns measured over other intervals.

Test results are sensitive to the evaluation period assumed. For evaluation periods less than one year, GMM estimates produce large pricing errors and unexpected parameter values. Using annual returns, however, the loss aversion model fits the data very well. GMM estimates of the loss aversion parameters are close to the estimates previously reported by Kahneman and Tversky (1992), and the loss aversion model produces pricing errors that are 40% smaller than the CAPM's errors. The loss aversion model even explains the returns to stock portfolios in the absence of bond returns. The GMM test of overidentifying restrictions does not reject the model.

The empirical results of this study demonstrate that both bonds and stocks appear to be priced more correctly by their downside betas than by their upside betas. Bonds have

large downside betas (on the pricing kernel) relative to stocks, but their upside betas are fairly similar to stock betas. Thus, the loss aversion model provides an intuitive explanation for the equity premium. Bonds have low expected returns because they pay off well in bad states of the world. The downside risk of bonds is significantly less than their upside risk. By allowing risk to be priced asymmetrically, bonds and stocks can be priced by the same pricing kernel.

The next section derives the loss aversion asset pricing model. Section 3 describes the data used in the tests of the model and section 4 contains test results. Section 5 concludes.

2 Asset Pricing with Loss Aversion

This section derives the loss aversion asset pricing model. It defines the loss aversion hypothesis, discusses the aggregation of loss averse investors, and derives both stochastic discount factor and ordinary factor model representations of the model.

2.1 The Loss Aversion Hypothesis

The model incorporates three of the conjectures that make up the myopic loss aversion hypothesis described in Benartzi and Thaler (1995). The first conjecture is that the welfare of an economic agent is a function of the agent's change in wealth or consumption rather than her level of wealth or consumption. Thus, the object of interest to a loss averse investor, or the argument of her value function, is her portfolio return.

The second conjecture is that agents are risk averse over gains but risk loving over losses. For example, a loss averse agent will prefer receiving \$100 with certainty to receiving an equal chance to receive \$200 or nothing. However, faced with the prospect of losing \$100 with certainty, he will prefer a lottery with equal probability of either no loss or a loss of \$200. A loss averse agent will be willing to make a bet "double or nothing" conditional on losing the first round of betting, but he will not accept a second bet if he wins the first round. An example of two possible loss averse value functions is plotted in Figure 1. In

mathematical terms, a loss averse value function is concave for positive net returns and convex for negative net returns.

The third conjecture is called first-order risk aversion (Segal and Spivak, 1990). Standard economic models assume that agents maximize expected utility subject to appropriate constraints. To optimize, agents set the first derivative of their constrained utility function equal to zero. Since Von Neumann-Morgenstern utility functions are smooth, agents are almost indifferent between miniscule losses and gains in standard models. By contrast, agents that exhibit first-order risk aversion care about very small risks. The value functions of first-order risk averse agents have steeper slopes over losses than they do over gains. Graphically, a value function that reflects first order risk aversion has a kink, or a point where the slope suddenly changes, that is typically at zero.

First-order risk averse agents create some new problems for economic models. For example, models must define losses and gains. Investors might feel that net returns greater than zero are gains and negative returns are losses. They might alternatively define returns less than the Treasury-bill rate or the market return as losses. Defining losses and gains is equivalent to defining a “threshold” return, the only return which is neither a gain nor a loss. While in principle the threshold return could be estimated from the data, I set the threshold return to zero for simplicity.

Another new problem is that models of loss aversion are not invariant to the frequency with which returns are observed. If a loss averse investor only examines his portfolio once each year, the investor cares about whether the annual portfolio return is negative or positive. He ignores whether or not his portfolio return has been negative over some months, behaving in a myopic fashion. Benartzi and Thaler (1995) estimate that the representative investor’s evaluation period, the period over which returns are measured, is approximately one year. I examine the correct evaluation period by fitting the model with returns measured at alternative frequencies. The model should fit returns measured at the correct evaluation period, but it may not returns measured at other frequencies.

There is one conjecture in Benartzi and Thaler’s myopic loss aversion hypothesis that

I do not implement. Tversky and Kahneman (1992) suggest that people do not base decisions on unbiased expectations of future outcomes. They propose a model in which agents maximize *prospective* value rather than expected value. Prospective value is defined in terms of the rank of the cumulative probabilities of alternative realizations. It can be approximated with a simple nonlinear transformation of the expectation operator. Given the large consequences of incorporating the conjectures described above, the marginal effect of introducing prospective value seems small. Furthermore, while the data contain enough information to identify a first-order risk aversion parameter and a convexity/concavity parameter, it seems unlikely that they can also identify two prospective value parameters. I assume that agents maximize expected value rather than prospective value. Since agents are maximizing expected value, the model can be considered a rational expectations model.

The simplest way to price assets with loss averse investors is to posit a value function for agents to maximize. To capture the essential features of loss aversion, the value function must exhibit the properties described above. One value function suggested by Tversky and Kahneman (1992) is the following:

$$\begin{aligned} V(x) &= x^\alpha & x \geq 0 \\ &= -\lambda(-x)^\beta & x < 0, \end{aligned} \tag{1}$$

where α and β are between zero and one and λ is positive. They prefer this function because it exhibits preference homogeneity,² which is consistent with their experimental evidence. Tversky and Kahneman estimate that both α and β have a value of 0.88 and they estimate λ to be 2.25. Based on this evidence, it seems reasonable to set $\alpha = \beta$ to reduce the number of parameters to be estimated. This makes the function's relative risk aversion coefficient over losses equal to the negative of its relative risk aversion coefficient

²A value function that exhibits preference homogeneity has a certainty equivalent function, $C(\cdot)$, with the property that $C(kf) = kC(f)$ for any constant k and lottery f .

over gains. Thus, the value function is,

$$\begin{aligned}
V^*(x) &= x^\alpha & x \geq 0 \\
&= -\lambda(-x)^\alpha & x < 0 \\
0 &< \alpha < 1 \\
\lambda &> 0.
\end{aligned} \tag{2}$$

While it would be nice to allow investors to be more risk averse over gains than they are risk loving over losses, the data do not contain enough information to identify more parameters than α and λ .

2.2 Conditions for Maximization

Before aggregating investors into a representative agent, it is important to understand the conditions for maximization for each investor. Each investor is assumed to maximize the expectation of the value function described in (2) over his evaluation period, the period over which he evaluates returns as either losses or gains. The simplest way to model a loss averse investor is to assume that his evaluation period is equal to the frequency at which asset returns are observed.

Assuming that the evaluation and observation periods are equal, each loss averse agent maximizes expected value given the constraint that his portfolio weights sum to one. Each agent holds a portfolio with gross return $R_p = \sum_{i=1}^n w_i R_i$. The return net of the threshold return (i.e. one) is written r_p . Forming the lagrangian function,

$$\mathcal{L} = EV^*(r_p) + \theta \left(\sum_{i=1}^n w_i - 1 \right), \tag{3}$$

where θ is the lagrange multiplier on the portfolio constraint. Using a stochastic version of Leibniz' rule in Amemiya (1985), the first order condition for maximization is

$$E \left[\left(\frac{\alpha |r_p|^{\alpha-1}}{\theta} I_+ + \frac{\lambda \alpha |r_p|^{\alpha-1}}{\theta} I_- \right) R_i \right] = 1, \tag{4}$$

where I_+ is an indicator function that equals one if $r_p \geq 0$, and equals zero otherwise; $I_- = 1 - I_+$. This first order condition can also be written as

$$E \left[|r_p|^C (AI_+ + BI_-) R_i \right] = 1, \quad (5)$$

where $A = \frac{\alpha}{\theta}$, $B = \lambda A$, and $C = \alpha - 1$. Each investor is assumed to adjust his portfolio weights to make this condition hold for each asset, $i = 1, 2, \dots, n$, subject to the constraint that his portfolio weights sum to one.

In standard economic problems with risk averse agents, second order conditions are almost always satisfied by construction. If investors are loss averse then their value functions are not globally concave and second order conditions are no longer guaranteed to hold. The second order condition for each investor's problem is the familiar condition that the bordered Hessian matrix be negative definite. The bordered Hessian matrix has the form

$$H = \begin{bmatrix} 0 & -1 & -1 & \dots & -1 \\ -1 & \frac{\partial^2 \mathcal{L}}{\partial w_1^2} & \frac{\partial^2 \mathcal{L}}{\partial w_1 \partial w_2} & \dots & \frac{\partial^2 \mathcal{L}}{\partial w_1 \partial w_n} \\ -1 & \frac{\partial^2 \mathcal{L}}{\partial w_2 \partial w_1} & \frac{\partial^2 \mathcal{L}}{\partial w_2^2} & \dots & \frac{\partial^2 \mathcal{L}}{\partial w_2 \partial w_n} \\ -1 & \dots & \dots & \dots & \dots \\ -1 & \frac{\partial^2 \mathcal{L}}{\partial w_n \partial w_1} & \frac{\partial^2 \mathcal{L}}{\partial w_n \partial w_2} & \dots & \frac{\partial^2 \mathcal{L}}{\partial w_n^2} \end{bmatrix}. \quad (6)$$

Each element of the interior of the matrix has the form

$$\{H_{ij}\} = \frac{\partial^2 \mathcal{L}}{\partial w_i \partial w_j} = \alpha(\alpha - 1)E \left[|r_p|^{\alpha-2} (I_+ - \lambda I_-) R_i R_j \right]. \quad (7)$$

Proving that the bordered Hessian matrix is negative definite requires showing that successive principal minors of the matrix alternate in sign. Rather than compute expressions for each of these principal minors, I examine the second order conditions for the two-asset case and assume that second order conditions hold in the more general n -asset case. For second order conditions to be satisfied in the two-asset case, it must be the case that the

determinant

$$|H_2| = \begin{vmatrix} 0 & -1 & -1 \\ -1 & \frac{\partial^2 \mathcal{L}}{\partial w_1^2} & \frac{\partial^2 \mathcal{L}}{\partial w_1 \partial w_2} \\ -1 & \frac{\partial^2 \mathcal{L}}{\partial w_2 \partial w_1} & \frac{\partial^2 \mathcal{L}}{\partial w_2^2} \end{vmatrix} \quad (8)$$

is greater than zero. This quantity can be written as

$$|H_2| = \alpha(\alpha - 1) \left\{ E \left[|r_p|^{\alpha-2} (I_+ - \lambda I_-) 2R_1 R_2 \right] - E \left[|r_p|^{\alpha-2} (I_+ - \lambda I_-) (R_1^2 + R_2^2) \right] \right\}. \quad (9)$$

Since α is less than one, the condition can be written as

$$E \left[|r_p|^{\alpha-2} (I_+ - \lambda I_-) (R_1^2 - 2R_1 R_2 + R_2^2) \right] > 0. \quad (10)$$

By completing the square, this translates into

$$E \left[|r_p|^{\alpha-2} (I_+ - \lambda I_-) (R_1 - R_2)^2 \right] > 0. \quad (11)$$

Finally, the second order condition can be expressed as an upper bound on λ ,

$$\frac{E[|r_p|^{\alpha-2} I_+ (R_1 - R_2)^2]}{E[|r_p|^{\alpha-2} I_- (R_1 - R_2)^2]} > \lambda. \quad (12)$$

The upper bound on λ should be greater than one since returns are positive more often than they are negative and negative returns are bounded below by -1.

It makes sense that the second order condition is an upper bound on λ . To some degree, λ measures the importance of the portion of the value function that is convex. If the convex portion of the value function sufficiently dominates the concave part then the expectation of the value function will be convex. As long as λ is sufficiently small the second order conditions for the two-asset case are satisfied. I assume that second order conditions for the case of many assets are similar.

2.3 Aggregation

Having derived the conditions for optimization of a single agent, it is important to ask when these conditions will hold in aggregate. Under certain conditions, a loss averse representative agent can be constructed. I assume that financial markets are complete and that the second order conditions of all agents are satisfied. Under these conditions, loss averse investors fit into the aggregation framework of Constantinides (1982) and there exists a representative agent whose first order conditions for utility maximization describe market equilibrium. The representative agent's value function is a weighted average of the value functions of the agents in the economy.

As long as all the loss averse agents in the economy have the same threshold return and evaluation period, the representative agent will exhibit first-order risk aversion even if only a fraction of the investors are loss averse. Even if investors start investing at different times and have unique values of α and λ in (2), the representative agent's value function over his evaluation period will still display first-order risk aversion. This follows from the fact that the representative agent's value function is a weighted average of the value functions of individual investors. If enough agents are loss averse, then the representative agent's value function will also be concave above and convex below the threshold return.

If agents have either different threshold returns or different evaluation periods then the representative agent's value function will not necessarily display loss aversion. If the representative agent is not loss averse then loss aversion should matter little in aggregate and models that ignore loss aversion should perform well. In order to develop a model in which loss aversion is important for asset pricing, I assume that all agents have the same evaluation period and a threshold return of zero. In fact, I assume that the value function (2) is the correct function for the representative agent.

Assuming that (2) is the representative agent's value function implies that (5) is the representative agent's first order condition. Since the representative agent holds all securities, the portfolio of interest in (5) is the market portfolio. Thus, the loss aversion asset

pricing model is

$$E \left[|r_m|^C (AI_+ + BI_-) R_i \right] = 1, \quad i = 1, 2, \dots, n, \quad (13)$$

where r_m is the net return of the market portfolio. This first order condition is expressed in the familiar stochastic discount factor form of

$$E [m R_i] = 1 \quad \forall i, \quad (14)$$

where the stochastic discount factor, m , is $[|r_m|^C (AI_+ + BI_-)]$.

This is a model for evaluation period returns only. It can be used to estimate the representative agent's evaluation period by determining at what frequency of returns it fits; it should only fit returns calculated at a frequency equal to the representative agent's evaluation period. Deriving a model for returns measured more frequently than the representative agent's evaluation period is a difficult problem. Such a model is beyond the scope of this paper.

2.4 Consistency with No-Arbitrage Pricing

By the fundamental theorem of asset pricing (Dybvig and Ross, 1992), there are no arbitrage opportunities if and only if the term corresponding to m in (14) is strictly positive. As long as both A and B are positive values, m will be strictly positive. Kahneman and Tversky's estimates of α and λ in (1) imply that there are no arbitrage opportunities. In the empirical results of this paper, the values of A and B are generally positive as well.

The loss aversion model is consistent with no arbitrage because the slope of the loss averse value function is positive everywhere. As in all utility based models, the loss aversion pricing kernel in (13) is marginal value. Although the loss averse value function is concave for gains and convex over losses, as long as A and B are positive, it is increasing everywhere. Since marginal value is always positive, loss aversion does not imply arbitrage.

However, if assets are priced by loss averse agents then investors that are not loss averse can earn high profits and achieve a high level of utility by purchasing assets that are heavily

discounted by the loss aversion value function. This is analogous to the fact that people with reservation prices that are above the market clearing price of a commodity receive some surplus by consuming the commodity. From an econometrician's viewpoint, using the wrong value function to discount random returns leads to profit opportunities that appear to have little risk. If people are loss averse then a loss aversion model may be able to explain some asset pricing anomalies by correctly accounting for risk.

2.5 A Factor-Based Model

The loss aversion model in (13) can be represented as a simple factor model. Using the fact that any stochastic discount factor of the form (14) can be written as

$$E(R_i) = \frac{1}{E(m)} - \beta_{r_i, m} \frac{\sigma^2(m)}{E(m)}, \quad (15)$$

and the linearity of the covariance operator, the risk aversion model can be expressed as a two-factor APT-type model with factors $m_+ = I_+ |r_m|^C$ and $m_- = I_- |r_m|^C$. Specifically,

$$E(R_i) = \gamma_0 - \beta_{r_i, m_+} A \gamma_1 - \beta_{r_i, m_-} B \gamma_2, \quad (16)$$

where γ_0 , γ_1 and γ_2 are all positive risk premium terms, and both β_{r_i, m_+} and β_{r_i, m_-} are univariate betas (e.g., $\beta_{r_i, m_-} = \frac{\text{COV}(r_i, m_-)}{\text{VAR}(m_-)}$). The beta associated with positive market movements, β_{r_i, m_+} , is referred to as an upside beta in this study. Similarly, β_{r_i, m_-} is called a downside beta. This factor model is used to interpret the economic meaning of the model's parameter estimates below.

2.6 Economic Interpretation of the Model

The factor model (16) illustrates clearly how the loss aversion model deviates from standard asset pricing models. Since m_+ and m_- both contain the absolute value of the market return raised to a negative power, small market returns are weighted more heavily in the pricing

kernel than large market returns. Figure 2 plots the pricing kernel for different values of the market return. Two features of the plot are particularly striking. First, the limit of the pricing kernel as the market return approaches zero is infinity regardless of which side the limit is taken from. This feature of the pricing kernel is specific to the loss averse value function (2) assumed above. Other loss averse value functions might specify a maximum value for the pricing kernel. Nevertheless, since the second derivative of the value function must change sign at zero, all loss averse value functions will tend to weigh small market returns more heavily than large market returns.

The second striking feature of Figure 2 is the fact that the pricing kernel is declining over positive returns but it is increasing over negative returns. Ordinarily, the pricing kernel is thought to be decreasing in market returns. The increasing and then decreasing characteristic of the pricing kernel is not specific to the value function chosen here; all loss averse value functions will specify a pricing kernel that is not monotonic in the market return. The loss aversion model's pricing kernel is increasing over negative returns because the loss averse value function is convex over losses. The pricing kernel function in Figure 2 is remarkably similar to the nonlinear pricing kernel estimated with monthly data by Bansal and Viswanathan (1993), which is reproduced in Figure 3.

The factor model implies that risk associated with positive market movements, as measured by the beta on m_+ , is compensated at a different rate than risk associated with negative market movements. This is an intuitively attractive proposition. An asset that happens to covary a great deal with the market return when the market is up (high upside beta) should not necessarily be considered a risky asset even if its high covariance gives it a high beta. Similarly, an asset with a high covariance in negative market states and no covariance in positive market states might be considered riskier than its unconditional beta suggests.

The loss aversion asset pricing model developed here is an excellent candidate for econometric testing because it has strong implications for the data. The next two sections describe econometric tests of the model and the results of the tests.

3 Test Method and Data

The first order conditions stated above are tested with the generalized method of moments (GMM) of Hansen (1982) and Hansen and Singleton (1982). Because the loss aversion model is nonlinear, there is no simple way to estimate the exponent, C , with standard linear estimation techniques. Even if C were known, it would be difficult to estimate and test the model with a cross-sectional technique. Since the factors in (16) are not portfolio returns, ordinary intercept tests are not applicable. Furthermore, because of the serial correlation inherent in overlapping returns, techniques like that in Fama and MacBeth (1973) may not estimate parameters with sufficient accuracy.

All of the estimates reported below are second stage GMM estimates. GMM minimizes a weighted sum of pricing errors for a number of assets. The pricing error for asset i is

$$g_i(\theta) = 1 - \frac{1}{T} \sum_{t=0}^T m_t(\theta) R_{it}. \quad (17)$$

In the first stage of GMM, the sum of squared pricing errors is minimized. In the second stage, a variance-covariance matrix for the first stage pricing errors is estimated. The inverse of this variance matrix is called the weighting matrix, W . In the second stage, GMM minimizes the quantity $g(\theta)' W g(\theta)$ where $g(\theta)$ is the vector of the model's pricing errors. In this paper, the weighting matrices used by GMM are estimated by the Newey and West (1987) method.

Given the model and the test method, it remains to specify a set of asset returns to explain. The test assets should exhibit a fairly large dispersion of average returns for statistical power. To examine the conclusions of Benartzi and Thaler (1995), the assets should also capture the equity premium. The number of assets chosen should not exceed the number of moment conditions that GMM can estimate efficiently.

Thirteen portfolios are used in the tests. Ten of the thirteen portfolios are the value-weighted NYSE/AMEX size decile portfolios tracked by the Center for Research in Security Prices (CRSP). The size portfolios are used because they have sufficiently different average

returns, and Berk (1995) argues that smaller stocks should be riskier than larger stocks in some undiversifiable sense. The other three portfolios are bond portfolios followed by CRSP: the long-term corporate bond portfolio, the Treasury-bill portfolio and the long-term government bond portfolio. Including these portfolios makes the inferences relevant to the equity premium. The tests use returns from the beginning of 1963 to the end of 1995 (396 months).

Tests require specification of the evaluation period of the representative investor. To determine how sensitive the model is to different evaluation periods, tests are performed with returns measured over one month, one quarter, six months, and one year. To increase statistical power, the models are tested with overlapping quarterly, semi-annual, and annual returns that are observed monthly. For example, the first return used in a test of the CAPM with quarterly data is the return from the beginning of January, 1963, to the end of March of the same year. The second observation is the return from the beginning of February of 1963 to the end of April. Overlapping returns are serially correlated by construction. GMM accounts for this correlation in forming parameter and variance estimates. To adjust for the serial correlation, the number of lags used to estimate the weighting matrix must increase with the evaluation period. The tests use five lags for monthly returns, six lags for quarterly returns, twelve lags for six-month returns, and eighteen lags for annual returns. Some summary statistics for the annual overlapping portfolio returns appear in Table 1.

The loss aversion model is compared to two benchmark models in the tests below. The benchmark models are the ordinary CAPM and a two-factor model that includes the market return and a size-based factor called *SMB* (small minus big). The *SMB* portfolio used here is based on that used in Fama and French (1993). It is constructed by subtracting the equal-weighted average return of CRSP's five largest size decile portfolios from the equal-weighted average return of CRSP's five smallest portfolios. Since *SMB* is created by sorting stocks by market size, it should help explain the dispersion of average returns across size portfolios in the data.

Tests performed with GMM sometimes use instrumental variables to improve statistical

power. In several unreported GMM tests, various instrumental variables seemed to add noise rather than power to the results. Tests with instrumental variables usually produce larger p-values for the test of overidentifying restrictions and higher standard errors for the parameter estimates than equivalent tests without instruments. This probably indicates that the instruments are only weakly correlated with pricing errors, and are thus not useful for improving power (Stock and Wright, 1996). No instruments are used for the tests reported.

4 Test Results

This section presents the results of tests of the loss aversion model. Interpreting the results of these tests is difficult because both the magnitude of pricing errors and the accuracy of parameter estimates may change with the length of returns measured (Handa, Kothari, and Wasley, 1993). In order to compare the loss aversion model’s accuracy to a familiar benchmark, tests of both the CAPM and a two-factor model with a market factor and the *SMB* factor described above are reported. The two-factor model is a useful benchmark because it has the same number of degrees of freedom as the loss aversion model. The pricing errors of the two factor model can be compared directly to the errors of the loss aversion model in a “horse race” to determine which model fits the data best.

In both Tables 2 and 3 all of the regular GMM summary statistics are reported, including parameter estimates, standard errors, and χ^2 tests of overidentifying restrictions. One reported statistic that is less common is the average absolute error of the model, labelled “Ave |Err| %” in the tables. This statistic is intended to measure the overall goodness of fit of the model. It is the average of the absolute values of the moment conditions that the GMM algorithm is minimizing. Thus, in Tables 2 and 3, it is an average of 13 numbers. When the GMM algorithm achieves its minimization by decreasing the components of the weighting matrix rather than by pushing the moment conditions toward zero, the average absolute error statistic is large.

4.1 Tests of the Benchmark Models

Tests of the two benchmark models are reported in Table 2. The accuracy of these benchmark models is compared to the accuracy of corresponding loss aversion models below. In Panel A, tests of the CAPM with overlapping returns of different periodicities are analyzed. Consistent with Handa, Kothari, and Wasley (1993), the CAPM fits the data better when it is estimated with longer-horizon returns. While the parameter estimates vary little, the χ^2 test of overidentifying restrictions is smallest for annual returns. The average absolute error (per month) of the model is also smallest for annual returns. The model is rejected at the 10% significance level for all horizons less than one year.

Panel B reports tests of the two-factor model that includes both the market return and *SMB* as factors. As expected, this model fits much better than the CAPM. It has smaller pricing errors than the CAPM for each return horizon. Like the CAPM, it seems to fit annual returns better than returns measured at shorter horizons. Unlike the CAPM, the two-factor model's coefficients decrease monotonically with the return horizon. This may indicate that *SMB* and the market return are more highly correlated at longer return horizons. While the two-factor model fits the data fairly well, it is still rejected by the data at the 10% significance level for all return horizons less than one year.

4.2 Tests of the Evaluation Period Loss Aversion Model

Tests of the loss aversion model for evaluation period returns, (13), are reported in Table 3. Tests are again performed with evaluation periods of one month, three months, six months, and one year. The last two lines of Table 3 report the result of the GMM test of the loss aversion model when the evaluation period is one year, which is what Benartzi and Thaler (1995) estimate the representative agent's period to be. The model fits annual returns better than the benchmark models. Its absolute average error is 40% smaller than the error of the CAPM and 15% smaller than the error of the two-factor model at the same frequency. The model cannot be rejected by the test of overidentifying restrictions. Furthermore, the value of the loss aversion parameter, $\lambda = A/B$, implied by these parameter estimates is

3.11. Given the precision with which the parameters are estimated, 3.11 is fairly close to 2.25, the value found by Tversky and Kahneman (1992) in their experiments. The value of α implied by these estimates is $C + 1 = 0.758$, which is also close to 0.88, the experimental estimate previously reported by Tversky and Kahneman. Figure 1 plots the value function (2) for both Tversky and Kahneman's parameter values and the parameter values in Table 3. There appears to be little economic difference between the two sets of parameter estimates.

Since the loss aversion model performs well for annual returns, it should not be expected to fit returns measured over other periods. In fact, it does not perform well for evaluation periods less than one year. For an evaluation period of one month, the loss aversion model's parameter estimates imply risk loving behavior over gains and risk aversion over losses ($\alpha = 1.367$). For three month returns, the model's parameter estimates indicate that investors care more about gains than they do about losses, or that λ is less than one. The exponent parameter, C , in the estimates calculated with six-month returns is very close to zero, indicating that the representative agent's value function is nearly linear with a kink at zero.

Tests of the model for evaluation periods that are less than one year all produce average absolute errors that are large. In fact, for all of the returns horizons less than one year, the GMM algorithm fails to converge. While the estimated moment conditions should be centered around zero, all of the estimated moment conditions turn out to be of the same sign. This means that there is not enough information in the data to accurately identify the model at short horizons, or that the model is not well specified. For short horizon returns, the CAPM fits the data much better than the loss aversion model.

4.3 Results without Bond Returns

It is possible that all of the loss aversion model's success in explaining the data is due to its ability to explain the equity premium. Hansen and Jaganathan (1991) show that standard models fail to explain the equity premium because the volatilities of their pricing

kernels are too low. Since the loss aversion model's pricing kernel is a function of the absolute value of the market return raised to a negative power, it is much more volatile than alternative kernels. If the explanatory power of the loss aversion model is due solely to its volatile pricing kernel then the model is of little use. One way to determine whether volatility is driving the model's fit is to estimate the model with assets that do not reflect the equity premium. To determine how well the model explains the returns of equity portfolios alone, Table 4 reports the results of estimating both the loss aversion model and the two benchmark models using only the size portfolios as assets.

The first four lines of Table 4 compare the loss aversion model to the two-factor model, the CAPM with *SMB*. The loss aversion model does not explain the data as well as the two-factor model. This is not surprising, considering that the *SMB* factor is constructed to explain the size effect. The parameter A in the loss aversion model is estimated to be -0.02, which violates no-arbitrage pricing. However, it is not estimated very precisely and is not statistically significant. Since A is close to zero, the representative agent seems only to care about downside risk when the size portfolios are used to estimate the model's parameters. Excluding the bond portfolios increases the curvature of the representative agent's value function. The parameter C goes from -0.242 to -0.50, corresponding to a square-root utility function over gains. The loss aversion model is not rejected by the data, and its pricing errors are not extremely large.

The last four lines of Table 4 compare the CAPM to a loss aversion model in which the parameter C is constrained to equal -0.242. These two models are directly comparable because they have the same number of degrees of freedom. The CAPM does not perform well; its estimates fail to converge in the sense described above (all estimated moment conditions are of the same sign). The loss aversion model again produces a negative but insignificant (and small) value for the parameter A and a large value for B . The pricing errors of the constrained model are not large, and the model is not rejected by the data.

The loss aversion model fits data that reflect the equity premium well. It explains most of the average returns of size portfolios, but it does not fit these data quite as well as the

combined bond and stock return data.

4.4 Economic Interpretation of the results

The factor model representation of the loss aversion model can be used to interpret the economic significance of the parameter estimates. Since the factor model splits each asset's covariance with the pricing kernel into an upside beta (or a beta on the part of m_t that corresponds to positive market returns) and a downside beta, the factor model can help determine whether positive or negative market returns are being priced disproportionately by the representative investor. Table 5 and Figure 4 examine this issue.

Table 5 contains regression coefficients from three time-series regressions of portfolio returns on different factors. The first regression is the standard market model. Betas from this model are listed in the third column of the table. The second regression includes the loss aversion factor for positive market returns, $m_+ = I_+|r_m|^{-0.242}$, and the third regression contains the loss aversion factor for negative market returns, $m_- = I_-|r_m|^{-0.242}$. Since these regressions are estimated with overlapping annual returns (observed monthly), they are estimated with the Yule-Walker correction for eleventh order autocorrelation. The values of β_{r_i, m_+} and β_{r_i, m_-} are highly correlated across assets; the sample correlation coefficient of the two betas is -0.88.

Figure 4 plots average portfolio returns against sensitivities to m_+ and m_- , using the data from the second, fourth and fifth columns of Table 5. The bond portfolios are identified in both plots by boxes while the stock portfolios appear as dots. The figure illustrates the loss aversion model's explanation of the equity premium. Sensitivities to both factors should be inversely related to expected returns, so the slopes of both plots should be negative. The lack of an obvious negative slope in the plot of returns on upside betas implies that upside betas do not have much explanatory power for expected returns. Restating equation (16), expected returns should satisfy

$$E(R_i) = \frac{1}{E(m)} - \beta_{r_i, m_+} \frac{A\sigma^2(m_+)}{E(m)} - \beta_{r_i, m_-} \frac{B\sigma^2(m_-)}{E(m)}. \quad (18)$$

Using the loss aversion parameters reported in Table 3 and some simple summary statistics, the expected returns relation is approximately

$$E(r_i) = 0.043 - 0.266\beta_{r_i, m_+} - 0.887\beta_{r_i, m_-} \quad (19)$$

In the plot of average returns versus upside beta (beta on m_+), the bonds appear as outliers near the origin of the graph. In this plot, bond betas do not appear to be very different than stock betas. In fact, the beta of the smallest stock portfolio lies between the betas of corporate bonds and long-term government bonds. In the plot of average returns on downside betas, the bond portfolios again appear as outliers. In this plot, however, bond betas are substantially higher than stock betas. In fact, the bond portfolios in the lower right hand corner of the plot produce a negative slope in a regression of average returns on downside betas. Thus, stocks and bonds appear to be priced primarily by their downside betas. This is consistent with the risk premiums in the approximate expected returns relation, equation (19).

Moreover, the equity premium may be explained by the finding that bonds have substantially higher downside betas than stocks while their upside betas are not very different from those of stock portfolios. If bonds are evaluated by their covariances with the market in both positive return and negative return states, they will appear too risky to pay the low returns that they have historically paid. Only by treating upside and downside betas asymmetrically does the equity premium make sense.

5 Conclusion

This paper develops and tests an asset-pricing model that is based on loss averse investors. The model is simple and testable, and it is consistent with no-arbitrage pricing and rational expectations. The model relies on the specification of the representative investor's evaluation period, the period over which investors determine whether their portfolios have produced gains or losses.

Estimating the loss aversion model produces results that are consistent with prior research. The evaluation period at which the model seems to fit the data is one year. The loss aversion parameter estimates calculated with annual data are fairly close to values estimated from experimental data. For annual returns, the loss aversion model performs substantially better than the traditional CAPM. It produces pricing errors that are on average 40% smaller than the CAPM's errors, and it is not rejected by the data according to GMM's test of overidentifying restrictions.

Beside providing a good fit for the data, the loss aversion model helps to explain the equity premium. In particular, according to the loss aversion model, the equity premium is driven by bonds' covariances with the pricing kernel when the market is realizing losses rather than bonds' covariances conditional on market gains.

While this study presents some evidence that favors the loss aversion model, important questions about loss aversion remain unanswered. It is important to find conditions under which aggregating loss averse agents with different evaluation periods or different threshold returns produces a loss averse representative agent. It is also important to derive an asset pricing model that fits returns measured at frequencies different than the evaluation period, like monthly returns. To know how economically significant loss aversion is, it is important to explore which asset pricing anomalies loss aversion can explain. These important questions are beyond the scope of this paper.

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Table 1: Summary Statistics

Table 1 presents summary statistics for the asset returns that are used in the tests in subsequent tables. All returns are calculated from the Center for Research in Security Prices (CRSP) database. SMB is created by subtracting the equally-weighted average return of size portfolios six through ten from the average return of portfolios one through five. The returns described are overlapping annual returns that are observed monthly. All samples are observed from 1963 to 1995, a total of 396 months.

Panel A: Means and Standard Deviations

| Portfolio | Mean | Std Dev | Minimum | Maximum |
|---------------|--------|---------|---------|---------|
| VW market | 0.1200 | 0.1567 | -0.4020 | 0.6417 |
| Size 1 | 0.2272 | 0.3720 | -0.4612 | 1.8231 |
| Size 2 | 0.1824 | 0.3127 | -0.4669 | 1.3744 |
| Size 3 | 0.1689 | 0.2859 | -0.4826 | 1.1484 |
| Size 4 | 0.1636 | 0.2595 | -0.4265 | 1.0432 |
| Size 5 | 0.1474 | 0.2450 | -0.4293 | 1.0902 |
| Size 6 | 0.1583 | 0.2292 | -0.4203 | 0.9894 |
| Size 7 | 0.1552 | 0.2111 | -0.3917 | 0.8901 |
| Size 8 | 0.1396 | 0.1925 | -0.3892 | 0.8073 |
| Size 9 | 0.1405 | 0.1759 | -0.3962 | 0.6733 |
| Size 10 | 0.1117 | 0.1502 | -0.4044 | 0.5661 |
| LT T-Bonds | 0.0764 | 0.1185 | -0.1710 | 0.5441 |
| LT Corp Bonds | 0.0796 | 0.1140 | -0.1817 | 0.4674 |
| T-Bills | 0.0642 | 0.0268 | 0.0274 | 0.1522 |
| SMB | 0.0300 | 0.1303 | -0.1894 | 0.5494 |

Panel B: Correlation Matrix

| | VWmkt | Size 1 | Size 2 | Size 3 | Size 4 | Size 5 | Size 6 | Size 7 | Size 8 | Size 9 | Size 10 | Tbond | CorpB | TBill |
|---------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|-------|-------|-------|
| Size 1 | 0.61 | | | | | | | | | | | | | |
| Size 2 | 0.68 | 0.98 | | | | | | | | | | | | |
| Size 3 | 0.73 | 0.94 | 0.98 | | | | | | | | | | | |
| Size 4 | 0.77 | 0.92 | 0.97 | 0.98 | | | | | | | | | | |
| Size 5 | 0.80 | 0.91 | 0.96 | 0.98 | 0.99 | | | | | | | | | |
| Size 6 | 0.85 | 0.87 | 0.93 | 0.96 | 0.98 | 0.98 | | | | | | | | |
| Size 7 | 0.88 | 0.82 | 0.89 | 0.93 | 0.95 | 0.96 | 0.98 | | | | | | | |
| Size 8 | 0.92 | 0.78 | 0.85 | 0.90 | 0.92 | 0.94 | 0.97 | 0.98 | | | | | | |
| Size 9 | 0.95 | 0.69 | 0.77 | 0.82 | 0.85 | 0.87 | 0.92 | 0.94 | 0.98 | | | | | |
| Size 10 | 0.98 | 0.49 | 0.56 | 0.60 | 0.64 | 0.68 | 0.75 | 0.78 | 0.84 | 0.89 | | | | |
| TBond | 0.42 | 0.05 | 0.09 | 0.15 | 0.13 | 0.18 | 0.24 | 0.28 | 0.35 | 0.39 | 0.45 | | | |
| CorpB | 0.48 | 0.11 | 0.15 | 0.22 | 0.20 | 0.25 | 0.31 | 0.34 | 0.41 | 0.43 | 0.51 | 0.96 | | |
| TBill | -0.02 | -0.13 | -0.08 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.02 | -0.02 | -0.03 | 0.09 | 0.05 | |
| SMB | 0.21 | 0.88 | 0.84 | 0.79 | 0.75 | 0.71 | 0.62 | 0.55 | 0.47 | 0.34 | 0.06 | -0.19 | -0.15 | -0.09 |

Table 2: GMM Estimation of Two Benchmark Models

Table 2 presents the results of GMM estimates of the parameters of two benchmark models: the traditional CAPM and the CAPM with an additional size factor, *SMB*. Standard errors are in parentheses. For quarterly, semi-annual, and annual returns, the estimates are calculated with overlapping returns that are observed monthly. The Newey-West method is used to estimate the weighting matrix used by GMM. The weighting matrix used by the monthly model is estimated with five lags, the quarterly with six lags, the six-month with twelve lags, and the annual with eighteen lags. The χ^2 tests of overidentifying restrictions for the CAPM have eleven degrees of freedom while those for the CAPM with SMB have ten degrees of freedom.

Panel A: The CAPM

$$E(m_t R_t) = 1, \quad m_t = A + B r_{mt}$$

| Period | <i>A</i> | <i>B</i> | χ^2 | P-Value | Ave Err % |
|-----------|----------------|-----------------|----------|---------|-------------|
| 1 month | 1.02 (0.02) | -2.45 (1.29) | 17.37 | 0.097 | 0.154 |
| 3 months | 1.08 (0.06) | -3.10 (1.22) | 20.56 | 0.038 | 0.536 |
| 6 months | 1.15 (0.09) | -2.77 (1.00) | 18.21 | 0.077 | 0.985 |
| 12 months | 1.24 (0.20) | -2.47 (1.12) | 14.62 | 0.200 | 1.430 |

Panel B: The CAPM with SMB

$$E(m_t R_t) = 1, \quad m_t = A + B r_{mt} + C \text{SMB}_t$$

| Period | <i>A</i> | <i>B</i> | <i>C</i> | χ^2 | P-Value | Ave Err % |
|-----------|----------------|-----------------|-----------------|----------|---------|-------------|
| 1 month | 1.02 (0.02) | -2.04 (1.20) | -2.87 (1.37) | 17.24 | 0.069 | 0.112 |
| 3 months | 1.07 (0.05) | -2.04 (1.15) | -2.29 (1.12) | 19.86 | 0.031 | 0.426 |
| 6 months | 1.10 (0.08) | -1.58 (1.00) | -2.12 (1.05) | 17.59 | 0.062 | 0.628 |
| 12 months | 1.16 (0.15) | -1.45 (0.98) | -1.56 (0.72) | 13.91 | 0.177 | 0.996 |

Table 3: GMM Estimation of the Evaluation Period Loss Aversion Model

Table 3 presents the results of GMM estimates of the parameters of the loss aversion model. Standard errors are in parentheses. For quarterly, semi-annual, and annual returns, the estimates are calculated with overlapping returns that are observed monthly. The Newey-West method is used to estimate the weighting matrix used by GMM. The weighting matrix used by the monthly model is estimated with five lags, the quarterly with six lags, the six-month with twelve lags, and the annual with eighteen lags. The χ^2 tests of overidentifying restrictions have ten degrees of freedom.

$$E(m_t R_t) = 1, \quad m_t = (AI_+ + BI_-) |r_{mt}|^C$$

| Period | A | B | C | χ^2 | P-Value | Ave Err % |
|-----------|----------------|----------------|-------------------|----------|---------|-------------|
| 1 month | 3.41 (3.42) | 4.61 (5.05) | 0.367 (0.294) | 16.54 | 0.085 | 2.149 |
| 3 months | 0.03 (0.02) | 0.01 (0.01) | -0.908 (0.134) | 8.02 | 0.626 | 8.307 |
| 6 months | 0.65 (0.71) | 1.46 (2.10) | -0.028 (0.445) | 17.93 | 0.056 | 1.680 |
| 12 months | 0.36 (0.21) | 1.12 (1.04) | -0.242 (0.286) | 14.35 | 0.157 | 0.844 |

Table 4: GMM Estimation of Models without Bond Returns

Table 4 presents results of estimating the models without bond returns. All models are estimated with overlapping annual returns (observed monthly) of the size decile portfolios returns from CRSP. Estimates are obtained by GMM, using the Newey-West weighting matrix with 18 lags.

Assets = overlapping annual returns of 10 size portfolios

| Model | A | B | C | χ^2 | P-Value | Ave Err % |
|------------------------------|-----------------|-----------------|-----------------|----------|---------|-------------|
| CAPM w/SMB | 1.02 (0.32) | -0.45 (2.37) | -2.03 (0.80) | 12.57 | 0.083 | 0.789 |
| Loss Aversion | -0.02 (0.06) | 1.16 (0.77) | -0.50 (0.18) | 3.36 | 0.850 | 1.340 |
| CAPM | 2.08 (0.63) | -8.99 (4.30) | - - | 11.80 | 0.160 | 10.89 |
| Loss Aversion w/fixed C | -0.01 (0.10) | 2.62 (0.54) | -0.24 - | 4.78 | 0.781 | 1.624 |

Table 5: Regression Coefficients

Table 5 presents the results of regressions estimating three types of factor models. All the regressions use overlapping annual returns. The Yule-Walker method is used to correct for eleventh order autocorrelation. Each regression is estimated separately; only univariate regressions are reported.

$$r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it}$$

$$r_{it} = \alpha_i + \beta_i^+ [I_+ |r_{mt}|^{-0.242}] + \epsilon_{it}$$

$$r_{it} = \alpha_i + \beta_i^- [I_- |r_{mt}|^{-0.242}] + \epsilon_{it}$$

| Portfolio | Ave r (%) | β_i | β_i^+ | β_i^- |
|---------------|-----------|-----------|-------------|-------------|
| Size 1 | 22.7 | 1.215 | 0.0023 | -0.0332 |
| Size 2 | 18.2 | 1.200 | 0.0047 | -0.0329 |
| Size 3 | 16.9 | 1.230 | 0.0062 | -0.0346 |
| Size 4 | 16.4 | 1.212 | 0.0068 | -0.0369 |
| Size 5 | 14.7 | 1.212 | 0.0066 | -0.0354 |
| Size 6 | 15.8 | 1.224 | 0.0072 | -0.0379 |
| Size 7 | 15.5 | 1.178 | 0.0080 | -0.0396 |
| Size 8 | 14.0 | 1.130 | 0.0077 | -0.0380 |
| Size 9 | 14.1 | 1.101 | 0.0084 | -0.0403 |
| Size 10 | 11.2 | 0.944 | 0.0072 | -0.0319 |
| LT T-Bonds | 7.6 | 0.231 | 0.0005 | -0.0042 |
| LT Corp Bonds | 8.0 | 0.216 | 0.0025 | -0.0047 |
| T-Bills | 6.4 | -0.002 | 0.0000 | 0.0001 |

Figure 1: Loss Averse Value Functions

Figure 1 depicts the value function proposed by Tversky and Kahneman (1992),

$$\begin{aligned} V(x) &= x^\alpha & x \geq 0 \\ &= -\lambda(-x)^\beta & x < 0, \end{aligned}$$

for two sets of parameter values. The solid line uses the parameter values $\alpha = \beta = 0.758$, and $\lambda = 3.11$. These parameter values come from GMM estimates of the loss aversion model with annual data (see Table 3). The dashed line uses parameter values from Tversky and Kahneman (1992); $\alpha = \beta = 0.88$, and $\lambda = 2.25$.

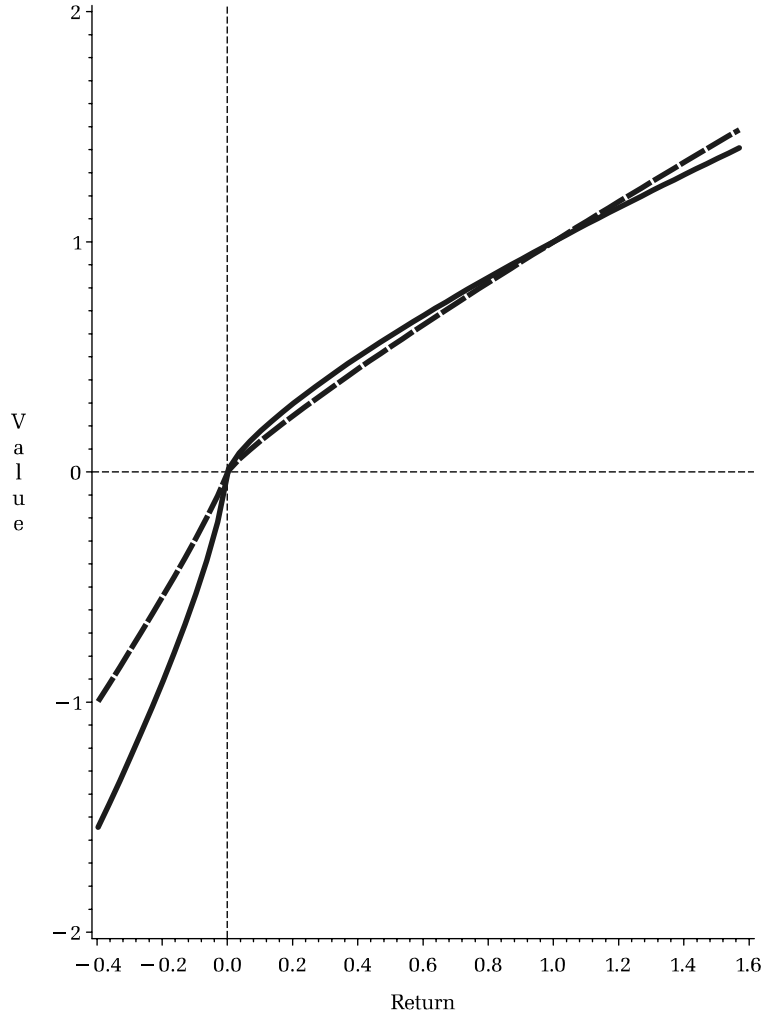


Figure 2: The Pricing Kernel Function

Figure 2 depicts the pricing kernel function implied by the loss aversion model,

$$m(r_m) = |r_m|^C (AI_+ + BI_-)$$

for the parameter values $A = 0.36$, $B = 1.12$, and $C = -0.242$. These parameter values come from GMM estimates of the loss aversion model with annual data (see Table 3). The range is truncated at 2.5 for presentation purposes only; the pricing kernel approaches infinity as the market return approaches zero.

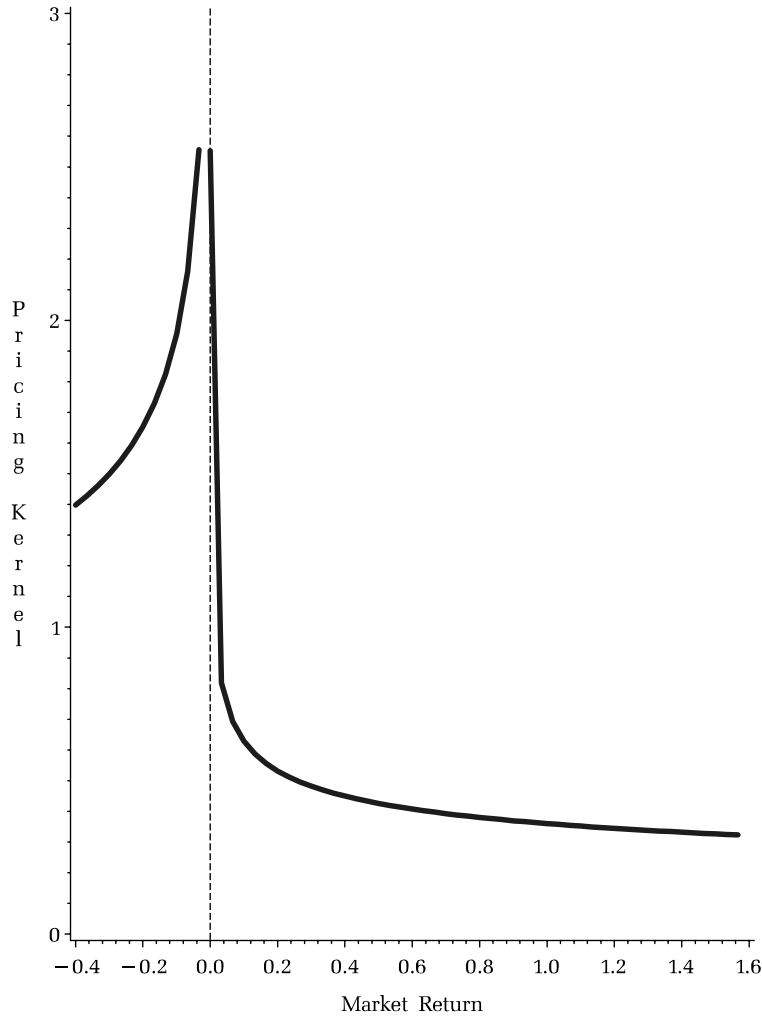


Figure 3: Nonparametric Pricing Kernel Function

Figure 3 depicts the pricing kernel function estimated nonparametrically by Bansal and Viswanathan (1993). Although Bansal and Viswanathan use monthly returns, their pricing kernel looks remarkably similar to the kernel in Figure 2.

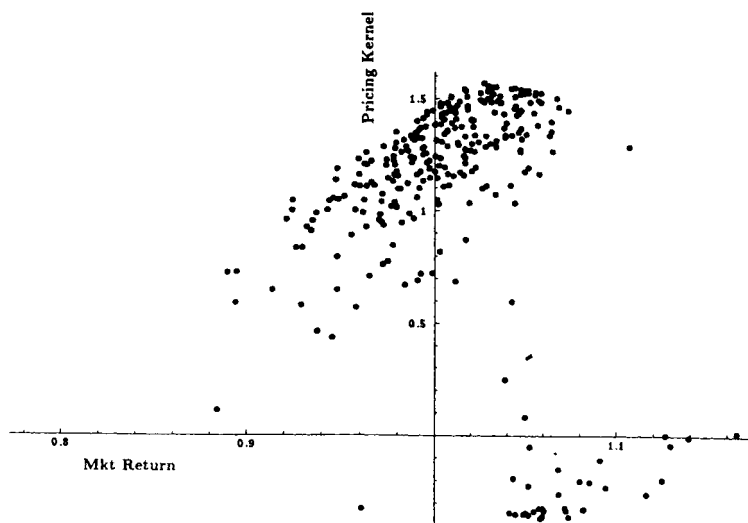


Figure 2. Estimated pricing kernel versus actual market returns: Instrument set A. This figure shows the scatterplot of the estimated pricing kernel (using instrument set A) against the one-month market return.

Source: Bansal, R., and S. Viswanathan, 1993, No Arbitrage and Arbitrage Pricing: A New Approach, *Journal of Finance*, XLVIII, 1231-1262.

Figure 4: Betas versus Average Returns

This figure plots the betas on the two factors in (19), restated here,

$$E(r_i) = 0.043 - 0.266\beta_{r_i, m_+} - 0.887\beta_{r_i, m_-}$$

The parameter C is set to -0.242, based on the GMM estimates of the loss aversion model with annual data (see Table 3). The data for these plots are available in Table 4. The boxes in the plots represent bond portfolio observations while the dots represent stock portfolios.

