Approximation Algorithms for Capacitated Stochastic Lot-sizing Inventory Control Models

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Introduction. We study a class of capacitated stochastic lot-sizing inventory control problems with stochastic, non-stationary and correlated demands that evolve over time. Economies of scale and capacity constraints exist in many practical scenarios. However, models with fixed ordering costs and capacity constraints are typically computationally intractable, and even the structure of the optimal policies is not well understood. Thus, computing provably good policies is usually very challenging. In this paper, we develope new provably near-optimal approximation algorithms for a class of core inventory management models with fixed ordering costs and capacity constraints.

The models. Stochastic inventory theory provides streamlined models with the following common setting. The goal is to coordinate a sequence of orders over a planning horizon of finitely many discrete periods, aiming to satisfy a sequence of random demands with minimum expected cost. The cost consists of a fixed ordering cost incurred in each period, where a positive quantity of supply units is ordered regardless of the size of the order; a per-unit holding cost for carrying excess inventory from one period to the next; and a per-unit backlogging penalty cost that is incurred in each period for each unit of unsatisfied demand. We assume that the holding and backlogging cost could be changing in different period. Without loss of generality, we assume the proportional ordering cost is zero and the discount factor is one. At the beginning of each period, we need to decide how many unit to order, whereas the maximum possible order quantity in a period is bounded by a given capacity limit which is identical for all periods. If an order is placed then the fixed ordering cost is incurred and the order arrives after a given lead time of several periods. Then the demand in this period is realized and satisfied to the maximum extend possible from the on hand inventory, a unit holding costs for excess inventory or backlogging cost for unsatisfied demand is incurred at the end of period. The model allow very general demand models. In particular, demands in different periods can be correlated and the information about the joint distribution of future demands can evolve over time as more information becomes available to the decision maker. We also have no assumptions on the type of distribution that the demands have to take.

Contributions and results. To analysis the capacitated stochastic lot-sizing models, this paper extends the model studied in Levi and Shi [13]. It is known that the constraint on capacity makes future costs heavily dependent on current decisions. To overcome this difficulty, we adopt the concept of marginal cost accounting where the total cost is decomposed into different parts that are determined sololy by decision made in each period. That is, after one decision was made in a period, the cost associated with this decision will not be affected by any future decisions. In this way, the costs associated with different decisions would be independent of each other. To decompose the backlogging cost, we employs the concept of forced backlogging cost accounting introduced in Levi, Roundy, Shmoys and Truong [12]. The forced backlogging cost captures the long-term impact of a decision in the presence of capacity constraints. With all these concepts, we propose a new policy called randomized 1/2-balancing policy for the capacitated stochastic lot-sizing models. Under our policy, we balance different type of costs incurred by our policy, namely, the marginal holding cost, the forced backlogging cost and the fixed ordering cost. Then we proved that the optimal policy have to encounter part of our policy’s cost. Since the costs were balanced in our policy, we can amortize the total cost of our policy with the cost of optimal policy. In this way, we can prove that
the randomized 1/2-balancing policy has a minimum worst-case performance guarantee of 4. That is, the expected cost of the policy is guaranteed to be at most 4 times the optimal expected cost, regardless of the specific instance.

Our model can also be extended to capacitated stochastic lot-sizing problem with batch ordering constraint. With the batch order constraint, every order quantity has to be a multiple of a pre-specified batch size. We refer the reader to [2, 9] for details concerning batch order. Without loss of generality, we can assume the capacity is also a multiple of batch size. In this model, we can extend the idea of randomized 1/2-balancing policy and it can also be proven that the worst-case performance guarantee is still 4.

In practice, we show how these policies can be parameterized to create a broader class of policies. A simulation based optimization is used to find the ‘best’ parameters per a given instance of the problem. This clearly preserves the same worst-case guarantees. Moreover, computational experiments that we conducted indicate that it can lead to near-optimal policies that perform near optimal, significantly better than the worst-case performance guarantees. For the computational complexity, our policy has a computational complexity of $O(T^2)$ which is very efficient compared to obtaining the optimal policy by dynamic programming approach due to the well-known "curse of dimensionality". The average empirical performance of our algorithms is within 20% - 30% of the optimal policies (using benchmark models in [6] and [14]), which is significantly better than the worst-case performance guarantees.

Literature review. Stochastic lot-sizing problems have attracted the attention of many researchers over the years. The dominant paradigm in the existing literature has been to formulate these problems using a dynamic programming paradigm. For many of the uncapacitated lot-sizing models it can be shown that state-dependent (s,S) policies are optimal (see, for example, [15, 17]). Gallego and Özer [6] have established their optimality for models with correlated demands.

However, capacitated problems are inherently harder structurally and computationally compared to their uncapacitated counterparts. The constraint on capacity makes future cost heavily dependent on current decisions. Myopic policies seem to perform well for some scenarios in uncapacitated systems and are even optimal in some specific settings (see Veinott [16], Ignall and Veinott [10] and Iida and Zipkin [11]). However, when applied to capacitated problems, they generally perform poorly because they ignore the limitations caused by capacity in future periods. Numerous heuristics have been developed under various demand and cost assumptions, both for capacitated and uncapacitated models. Federgruen and Zipkin [5] have proposed an algorithm to compute the optimal (s, S) policy in an infinite horizon model with independent and identically distributed demands. Federgruen and Zheng [4] described a simple and efficient algorithm to compute the infinite horizon optimal policy in a continuous-reviewed system when the demand is generated by a renewal process. Bollapragada and Morton [1] proposed a simple, myopic heuristic for computing the policies where the demands are assumed to be in different periods have the same form of distribution function but with different means, and the coefficient of variation of the demands is assumed to be stationary. Gavirneni [7] designed an efficient heuristic to compute (s, S) policies for nonstationary and capacitated model. Guan and Miller [8] proposed polynomial-time algorithms for the uncapacitated stochastic ELS problem in terms of the number of nodes in the stochastic programming scenario tree that is usually exponentially large. Chen and Lambrent [3] considered the lot-sizing model with uniform capacities. They provided an example to show that (s, S) policies are not optimal. They showed that the optimal policy follows a $X - Y$ band structure. That is, if the inventory level is below $X$, order the full capacity; If the inventory is higher than $Y$, order nothing. To the best of our knowledge, the existing algorithms for capacitated stochastic lot-sizing problems either lack any performance guarantees or are applicable only under restrictive assumptions of the demand distributions.

The main prior work relevant to our work is due to Levi, Roundy, Shmoys and Truong [12] who have introduced the forced marginal backlogging cost-accounting scheme to analysis the capacitated models without fixed ordering costs and Levi and Shi [13] who have proposed the randomized cost-balancing
policy to solve uncapacitated stochastic lot-sizing problems. They have shown that the algorithm in the uncapacitated model has a worst-case performance guarantee of 3.

References


