THE PERMANENT INCOME HYPOTHESIS AND
THE REAL INTEREST RATE
Some Evidence From Panel Data

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A test of the permanent income hypothesis in panel data is formulated taking into account both the time-series and cross-section variation in the rate of return. The over-identifying restrictions of the theory are rejected.

1. Introduction

Hall (1978) shows that if consumers determine their consumption according to the permanent income hypothesis (PIH), consumption should approximately follow a random walk. In his paper, Hall assumes that the real rate of return is constant. Mankiw (1981) and Hall (1981) extend the analysis to the case of a variable real rate of return. Under this extension, consumption should grow, in expectation, at a rate proportional to the real rate of return.

Those studies, as well as others, ¹ use data on aggregate consumption. Because the theory is based on the behavior of individuals or households, data on individuals or households may provide a more convincing test. The Panel Study of Income Dynamics provides data on consumption, income, and other characteristics of a large panel of households. In

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¹ See Mankiw (1981) for a brief discussion of other papers.

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aggregate data, the identification comes from the time-series variation in the real rate of return. This study also exploits the substantial cross-section variation in real rates of return caused by variation in the marginal tax rates among households.

Hall and Mishkin (1982) perform the original Hall (1978) random-walk test for data on households. They do not account for either the time-series or cross-section variation in the rate of return. The rate of return must vary across households because of the progressivity of the tax system, so it seems important to exploit this feature of household data in testing the PIH.

2. Theory

The textbook’s consumer solves the optimization problem of maximizing expected utility \( V = \mathcal{U}(C_1, \ldots, C_t, \ldots) \). If the household can borrow and lend at the real rate of return, \( r_{i,t+1} \), the first order condition for optimal intertemporal consumption is

\[
\mathbb{E}[\exp(r_{i,t+1})(\partial V/\partial C_{i,t+1})/(\partial V/\partial C_{i,t})] I_{it} = 1. \tag{1}
\]

Under rational expectations and market clearing, the error in forecasting the product of the marginal rate of substitution and the rate of return must be uncorrelated with all information \( I_{it} \) available to household \( i \) at time \( t \). If not, the forecast is suboptimal. Eq. (1) can be expressed as

\[
\exp(r_{i,t+1})(\partial V/\partial C_{i,t+1})/(\partial V/\partial C_{i,t}) = 1 + \epsilon_{i,t+1}, \tag{2}
\]

where \( \epsilon_{i,t+1} \) is the forecast error. That is, the first-order condition holds \textit{ex post} except for an error term uncorrelated with information available to the household at time \( t \).

To implement (2) empirically, define the utility function \( V = \sum_{t=1}^{\infty} \exp(-\delta)(1/(1-A))C_{it}^{1-A} \). The time separable, constant relative risk aversion specification leads to a convenient regression formulation. To obtain it, differentiate the utility function, substitute into (2), take logs and a Taylor approximation, and rearrange. This yields the regression

\[
g_{i,t+1} = \alpha_t + \beta r_{i,t+1} + u_{i,t+1}. \tag{3}
\]
where $g_{i,t+1} = \log(C_{i,t+1}/C_{it})$, $\alpha_i = (\omega_T/2 - \delta)/A$, $\beta = 1/A$, and $u_{i,t+1} = (\omega^2_{i,t+1}/2 - \epsilon^2_{i,t+1})/A$. The left-hand side variable is the growth in consumption. The intercept $\alpha_i$ is a linear combination of the discount rate and the variance of the forecast error, $\omega_T^2$. The variance is likely to differ because, for example, an hourly worker might face more uncertainty than a salaried worker. The coefficient is the elasticity of intertemporal substitution. Its reciprocal is the coefficient of relative risk aversion. In this paper, it is not allowed to vary across individuals, but that restriction is testable. The error term, $u_{i,t+1}$, is uncorrelated with variables in $I_{it}$.

Households face different real rates of return even if they face the same nominal rates of return. The ex post real after-tax rate of return is

$$r_{i,t+1} = R_i (1 - \tau_{i,t+1}) - \pi_{t+1},$$

where $R_i$ is the nominal rate of return, $\tau_{i,t+1}$ the marginal tax rate, and $\pi_{t+1}$ the actual rate of inflation. The nominal rate of return is dated at $t$ because households know it before making the $t + 1$ consumption decision. The inflation in year $t + 1$ is not, however, predetermined.

The error term $u_{i,t+1}$ is serially uncorrelated under the null hypothesis of rational expectations. Moreover, suppose that the error term for a given household can be decomposed into a macroeconomic component that is the same across households, and a household-specific term that is uncorrelated across households. Households are independent except for macroeconomic shocks that affect all households equally. Thus

$$u_{i,t+1} = \nu_{i,t+1} + \mu_{t+1},$$

where the $\nu$ are the uncorrelated household shocks and the $\mu$ the macroeconomic shocks. Substituting (4) and (5) into (3) yields

$$g_{i,t+1} = \alpha_i - \beta \pi_{t+1} + R_i (1 - \tau_{i,t+1}) + \nu_{i,t+1} + \mu_{t+1}.$$

An appropriate estimator for this model is the within-between groups fixed-effects estimator. Each household is regarded as an individual group. The transformation of the right-hand side variables needed to

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4 In these data I could not find significant differences according to the age or education of the head of the household [Shapiro (1982)].
carry out the estimation is given by Mundlak (1978). Suppose \( x_{it} \) is a right-hand side variable. The transformed variable \( \tilde{x}_{it} \) is given by

\[
\tilde{x}_{it} = x_{it} - \frac{1}{T} \sum_{j=1}^{T} x_{ij} - \frac{1}{N} \sum_{k=1}^{N} x_{ik} + \frac{1}{NT} \sum_{j=1}^{T} \sum_{k=1}^{N} x_{ijk},
\]

where \( N \) is the number of households and \( T \) the number of time periods. That is, the transformed variable is the variable minus the household and time means plus the total mean.

Let \( X_{i,t+1} = R_t(1 - \tau_{i,t+1}) \). Applying the transformation given in eq. (7) to eq. (6) yields

\[
g_{i,t+1} = \beta \tilde{X}_{i,t+1} + \tilde{\nu}_{i,t+1}.
\]

Note that the transformation annihilates the \( \alpha_i \), the \( \pi_{i+1} \), and the \( \mu_{i+1} \). Furthermore, the covariance matrix of the transformed system, \( E \tilde{\nu} \tilde{\nu}' \), is diagonal. The covariance matrix is nonetheless likely to be heteroskedastic because the variance of different household forecast errors are likely to differ. Consequently, the standard covariance estimator is inconsistent. White (1980) gives a procedure for calculating a consistent covariance matrix under heteroskedasticity. Such heteroskedasticity-consistent covariance estimates are reported in table 1, and used in the tests in this paper.

Table 1
Regression of \( \log(C_{i,t+1}/C_{i,t}) \). Household-time fixed effects, \( N = 1253, T = 3. \)

<table>
<thead>
<tr>
<th>Estimation technique</th>
<th>Independent variables</th>
<th>After-tax rate of return</th>
<th>Log of lagged real disposable income</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS (1a)</td>
<td>2.08</td>
<td>(3.27)</td>
<td></td>
</tr>
<tr>
<td>(1b)</td>
<td>2.00</td>
<td>-0.158</td>
<td>(0.043)</td>
</tr>
<tr>
<td>2SLS (2a)</td>
<td>26.5</td>
<td>(28.6)</td>
<td></td>
</tr>
<tr>
<td>(2b)</td>
<td>20.4</td>
<td>-0.157</td>
<td>(0.043)</td>
</tr>
</tbody>
</table>

* White’s (1980) heteroskedasticity-consistent standard errors are given in parentheses.
The rate of return variable, $\bar{X}$, may be correlated with the error term. To allow for this possibility, eq. (8) is estimated by ordinary least squares and by two stage least squares. A Hausman (1978) specification test provides a measure of the consistency of the OLS estimates.

The addition of a variable from the information available to the household at time $t$ provides a test against the specification of the model. No information from $I_t$ should help in forecasting $g_{t,t+1}$ given the rate of return has been taken into account. If a variable in $I_t$ is significant in eq. (8), one can reject the hypothesis that the specification is correct. Lagged income is an obvious candidate for the specification test.

3. Data

Data from the Panel Study of Income Dynamics are used to make the estimates and the tests. The PSID contains detailed information on the income, employment, and other characteristics of the families surveyed. Unfortunately, expenditure on food is the only category of non-durable consumption recorded. Food consumption obeys the equation of the model if it is separable from other consumption. The consumption measure is the sum of food at home and food away from home after each has been divided by the respective deflators from the National Income and Product Accounts.

For four of the thirteen years of the PSID, federal marginal tax rates are available. This figure is used for $\tau$ in the regressions. The interest rate used in the estimates is the one year treasury bill rate in the December preceding the year for which consumption is being forecast.

The disposable income for each family, namely, taxable income plus transfers minus federal taxes, is deflated by the price index for total personal consumption expenditure from the National Income and Product Accounts. Both the consumption and the income figures are normalized by the number of members of the household.

This study uses a subset of the PSID data. According to the theory, each household is represented by a unidimensional utility function. To select households that might be presumed to act as a unit and consistently across time, only families with an unchanging couple or individual as head, and no other adults, are included. The oversampled poor

$^5$ State taxes are neglected. Because they are small, the induced bias should be small.
families in the original PSID data base are excluded to yield a representa-
tive sample.

4. Estimates

Table 1 reports household-time fixed effects regressions for eq. (8). The first two equations are estimated with OLS, and the second two with 2SLS. Line (1a) gives the OLS estimate of the coefficient of the rate of return. The point estimate of the elasticity of substitution is about 2, which is in the middle of the range of estimates from aggregate data. Mankiw’s (1981, p. 310) estimate in aggregate data is 0.25, indicating less substitutability or, equivalently, greater risk aversion. In another study in aggregate data, Mankiw, Rotemberg and Summers (1982, p. 29) find the elasticity of intertemporal substitution to be as high as 5.7.

The heteroskedasticity-consistent standard errors of the coefficient estimates in table 1 are high. The large year-to-year variation in consumption makes it difficult to get precise estimates. Line (1b) of table 1 reports the regression including lagged income. Under the null hypothesis that members of the panel obey the first order condition for optimal intertemporal allocation of consumption, this coefficient should be zero. Despite the large variance of the left-hand side variable, the hypothesis can be strongly rejected. The negative sign of the coefficient is consistent with the view that the rejection of the permanent income hypothesis is caused by liquidity constraints. If income of a liquidity constrained household is transitorily low, its consumption is expected to grow more than would be predicted by the permanent income theory. Therefore, one would expect to observe a negative correlation between lagged income and consumption growth if liquidity constraints caused the PIH to be violated. 7

The table also gives 2SLS estimates of the equations. The results are

6 The consistent standard error is about 30 percent higher than the standard one. Standard errors of estimate are not given in table 1 because they cannot be extracted using White’s procedure. White’s (1980) test rejects the null hypothesis of homoskedasticity at the 0.99 level. The large variance of the growth rate of consumption is probably due to error in the measurement of consumption. As long as the error is not correlated with other variables, the coefficient estimates are consistent. See Shapiro (1982) for a more detailed discussion of the measurement error problem.

7 Restrictions on the utility function, such as time separability, could also be responsible for the rejection.
qualitatively the same as the OLS estimates. The utility function parameter is of the expected positive sign but imprecisely estimated. The coefficient of lagged income is again negative and strongly significant, indicating a rejection of the PIH. A Hausman test of the OLS versus the 2SLS specification indicates that one cannot reject at traditional significance levels the hypothesis that the two estimates are the same. In any case, both estimators yield a strong rejection of the PIH.

5. Conclusion

Previous tests of the PIH have relied on aggregate data or the assumption of a constant real interest rate. In household data where the rate of return is allowed to vary, the over-identifying restrictions of the theory are still rejected. The nature of the rejection suggests the existence of liquidity constraints, but before these are taken too seriously, a structural model of liquidity constraints should be formulated and tested in data on households or individuals.

References


The statistic is $(b_{2SLS} - b_{OLS})(V_{2SLS} - V_{OLS}) - (b_{2SLS} - b_{OLS})$ where the $b$ and the $V$ are the estimated coefficients and covariances. It is distributed as chi-squared with 1 degree of freedom. The statistics are 1.46 for eqs. (1a) versus (2a) and 2.23 for (1b) versus (2b), which have values 0.77 and 0.86. Because these are high enough not to imply clear acceptance of the null hypothesis that the estimators are equal, both estimates are reported.
